'Dark Matter Tomography'
Measuring the DM velocity distribution with directional detection

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University of Sheffield - 15th June 2016

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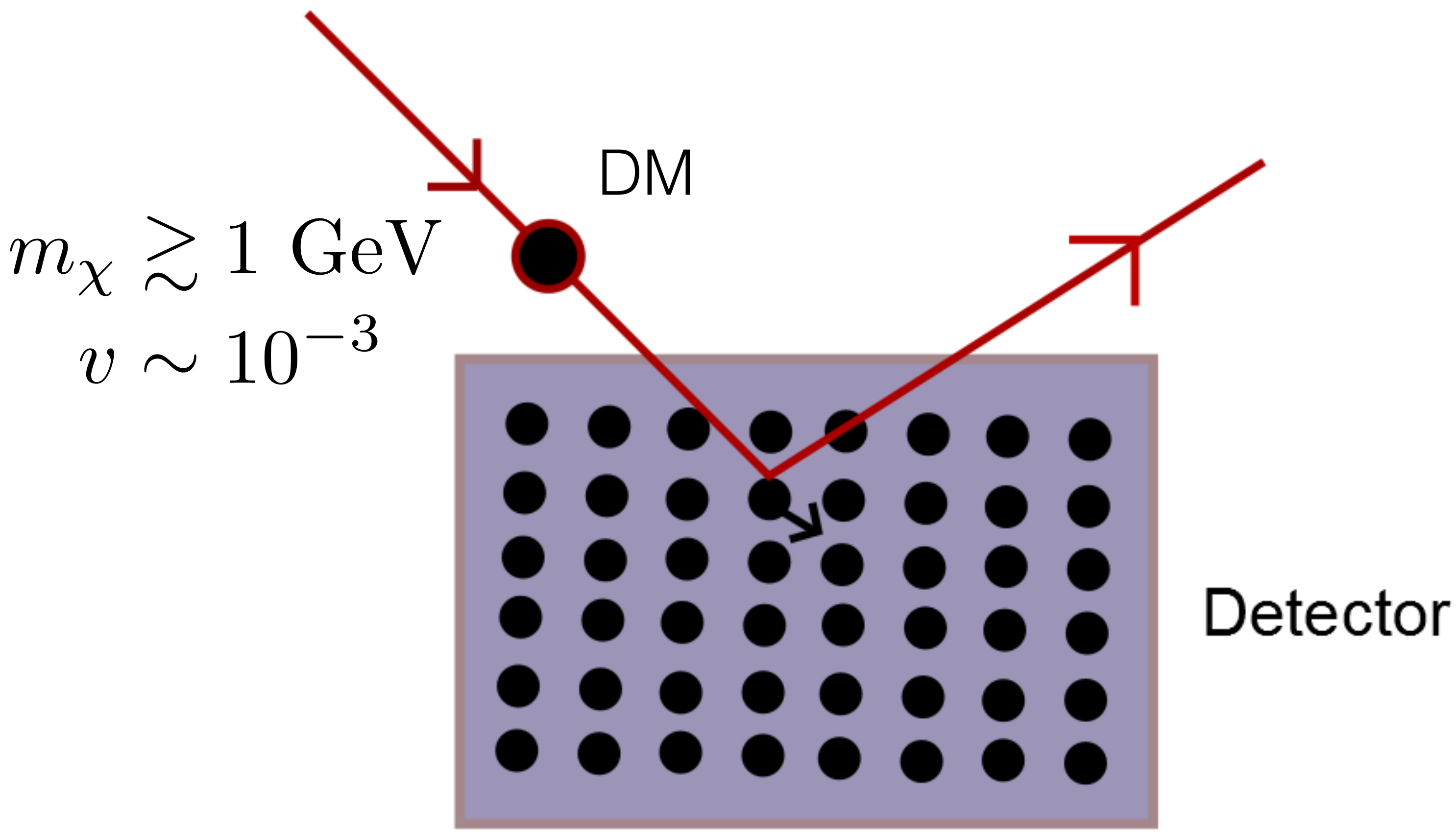
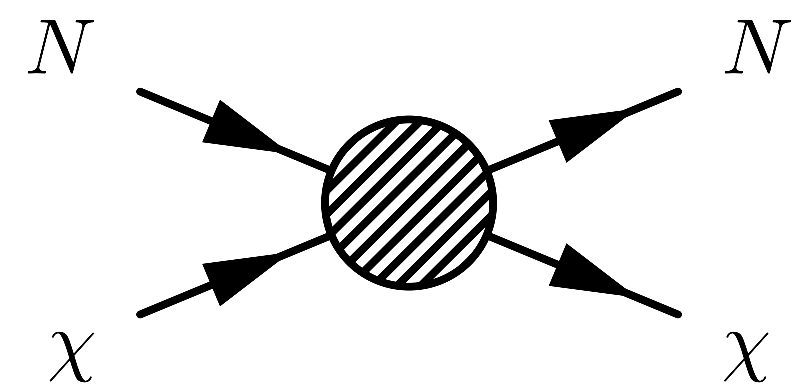
Direct detection





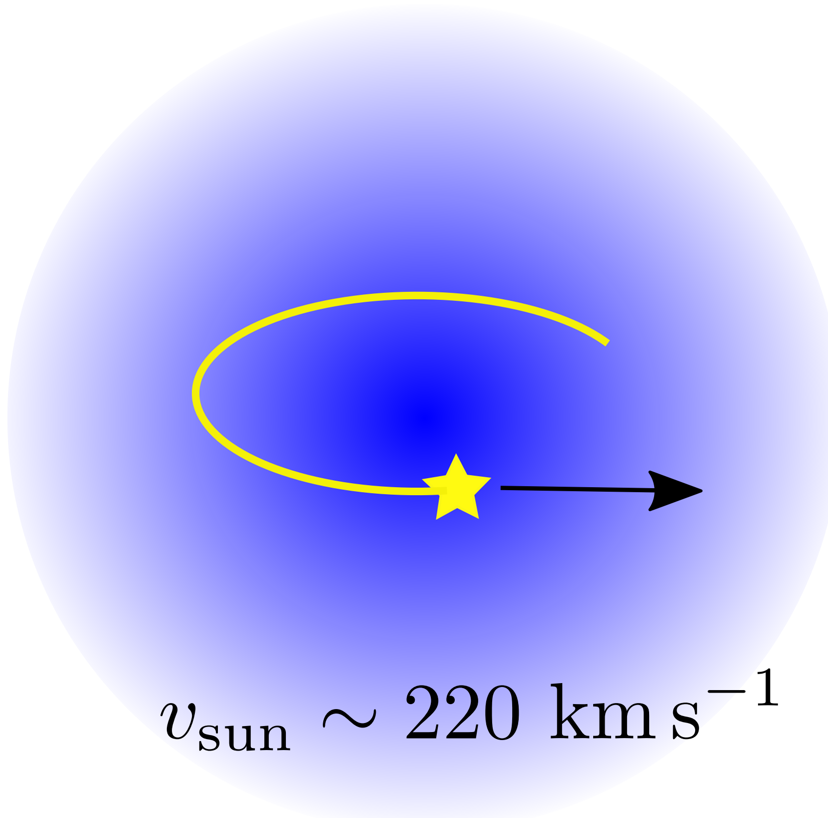
Measure energy (and possibly direction) of recoiling nucleus

Reconstruct the mass and cross section of DM?

Need to know the velocity distribution of the DM particles.

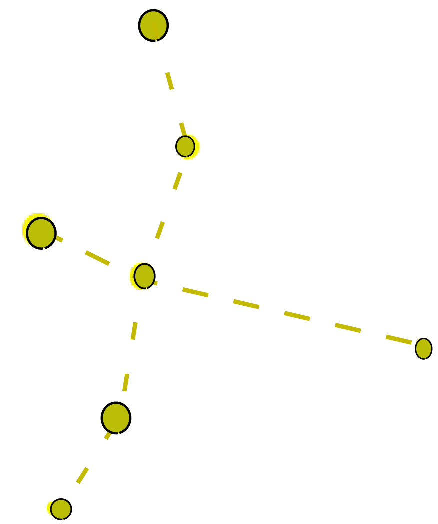
The WIMP wind

In the halo:



$v_{\rm sun} \sim 220\ {\rm km\,s^{-1}}$

WIMP: Weakly Interacting Massive Particle

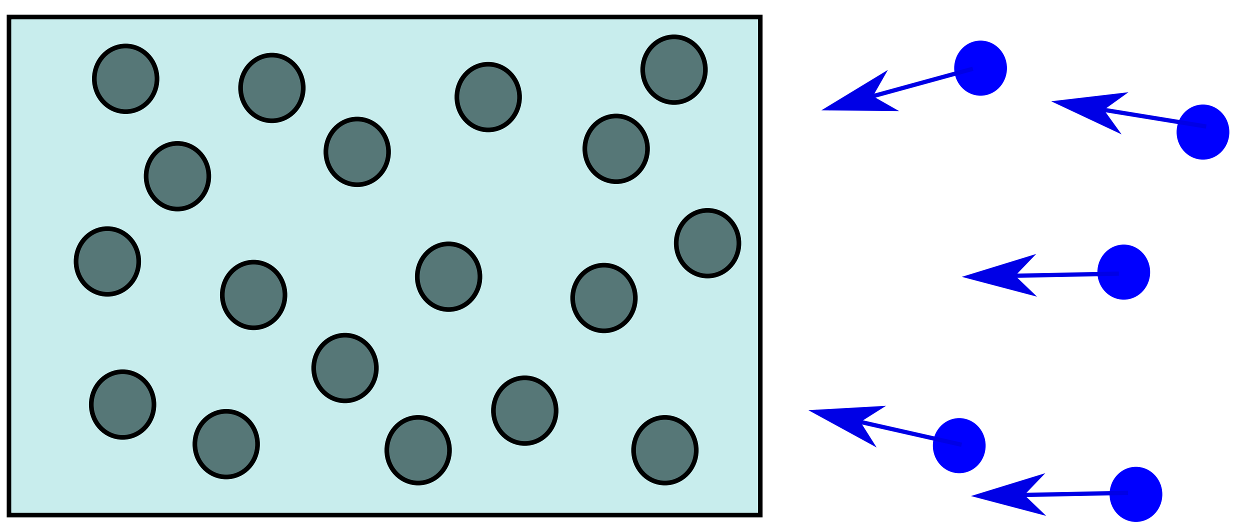


Cygnus constellation

In the lab:

Detector

$v_{\rm DM} \sim 220\ {\rm km\,s^{-1}}$

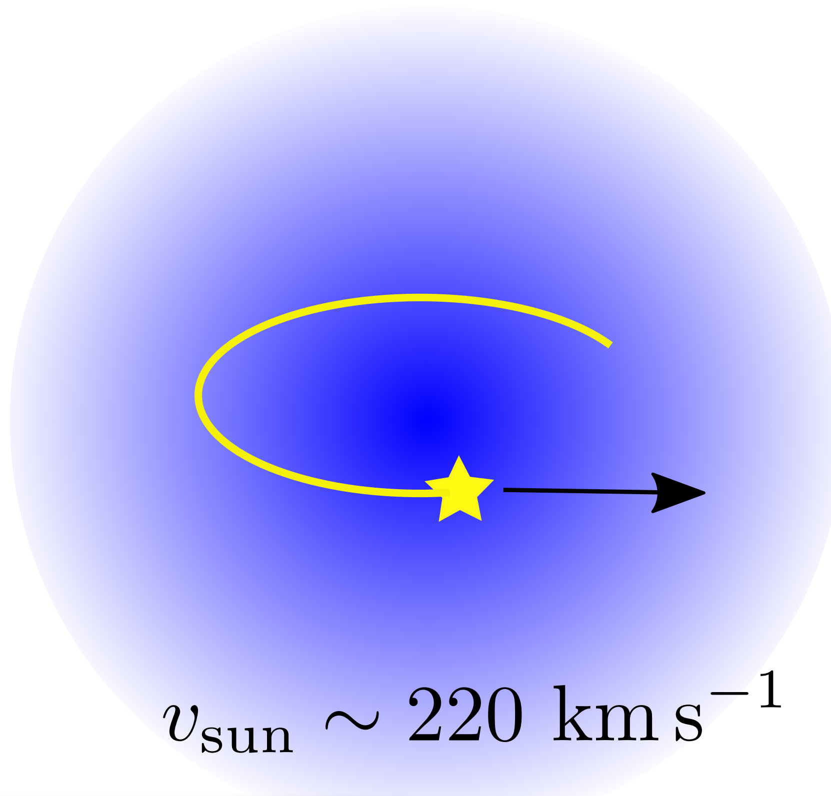


'WIMP wind from Cygnus'

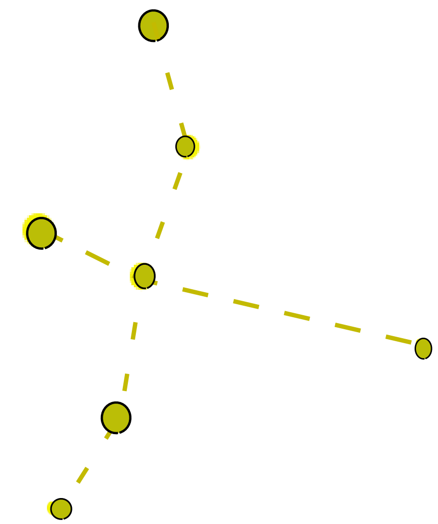
The WIMP wind

WIMP: Weakly Interacting Massive Particle

In the halo:



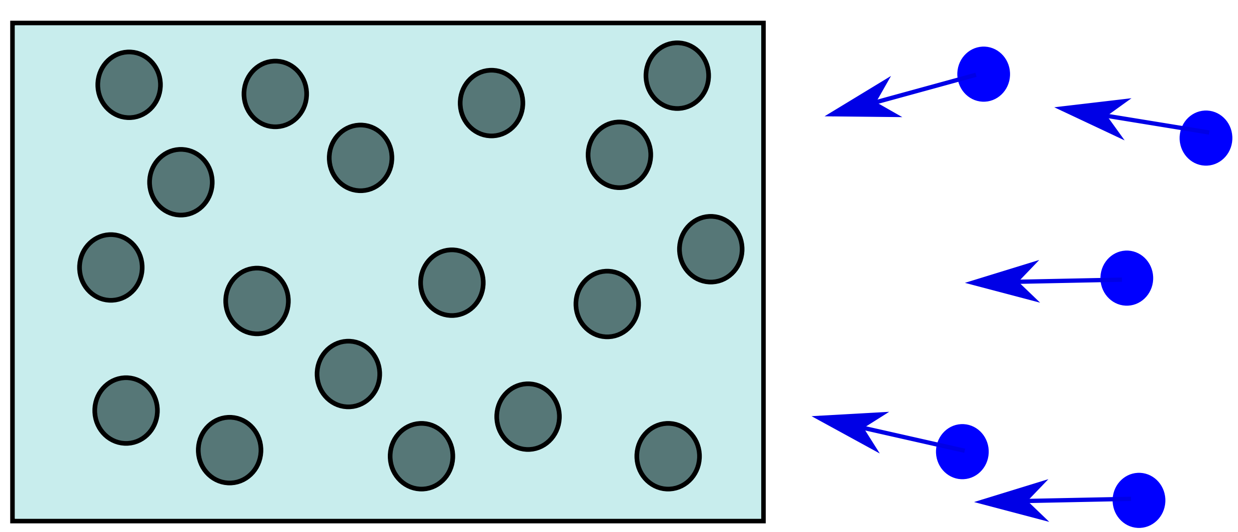
$$v_{\text{sun}} \sim 220\ \text{km}\,\text{s}^{-1}$$



Cygnus constellation

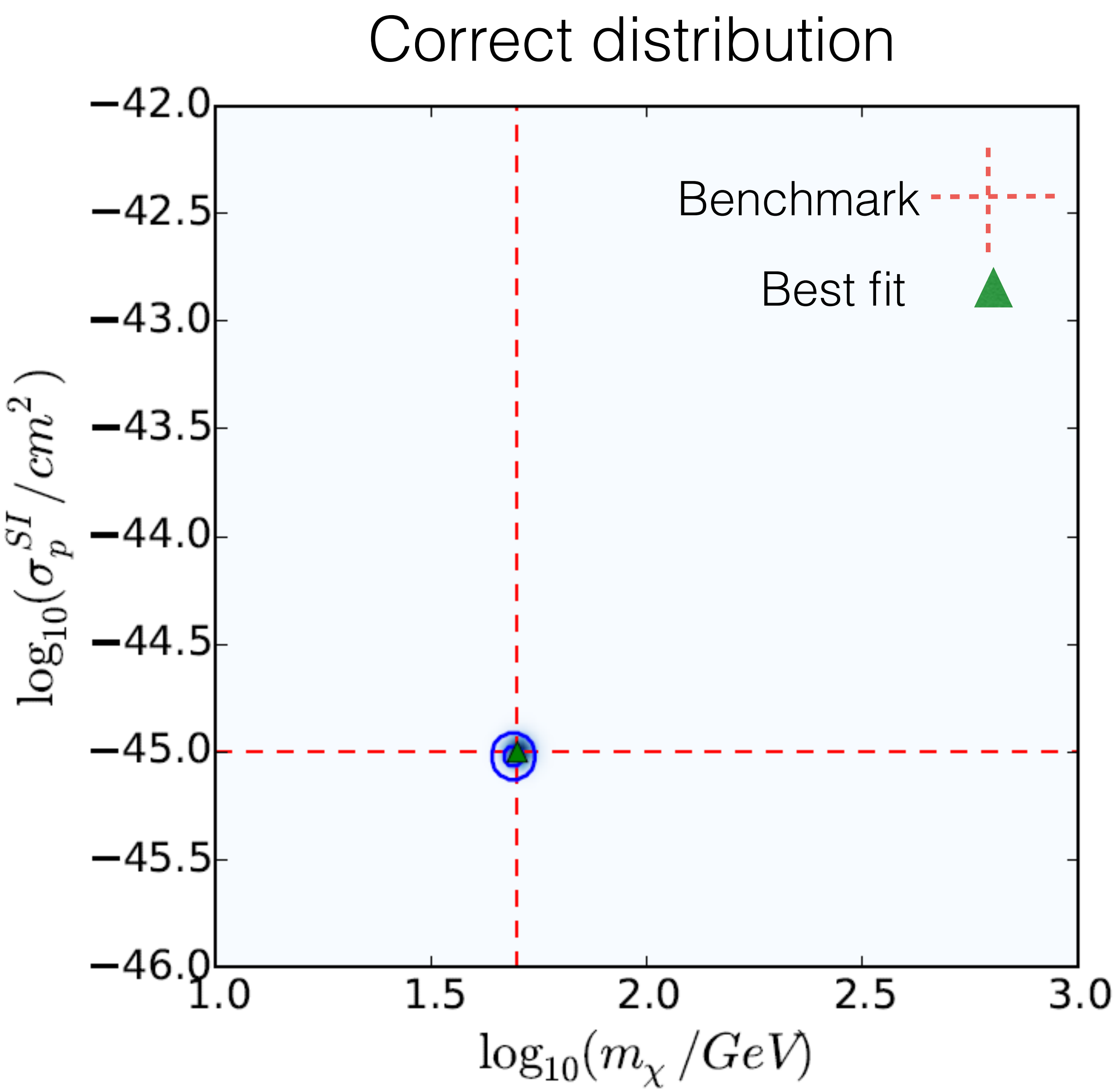
But we don't know the velocity distribution exactly!

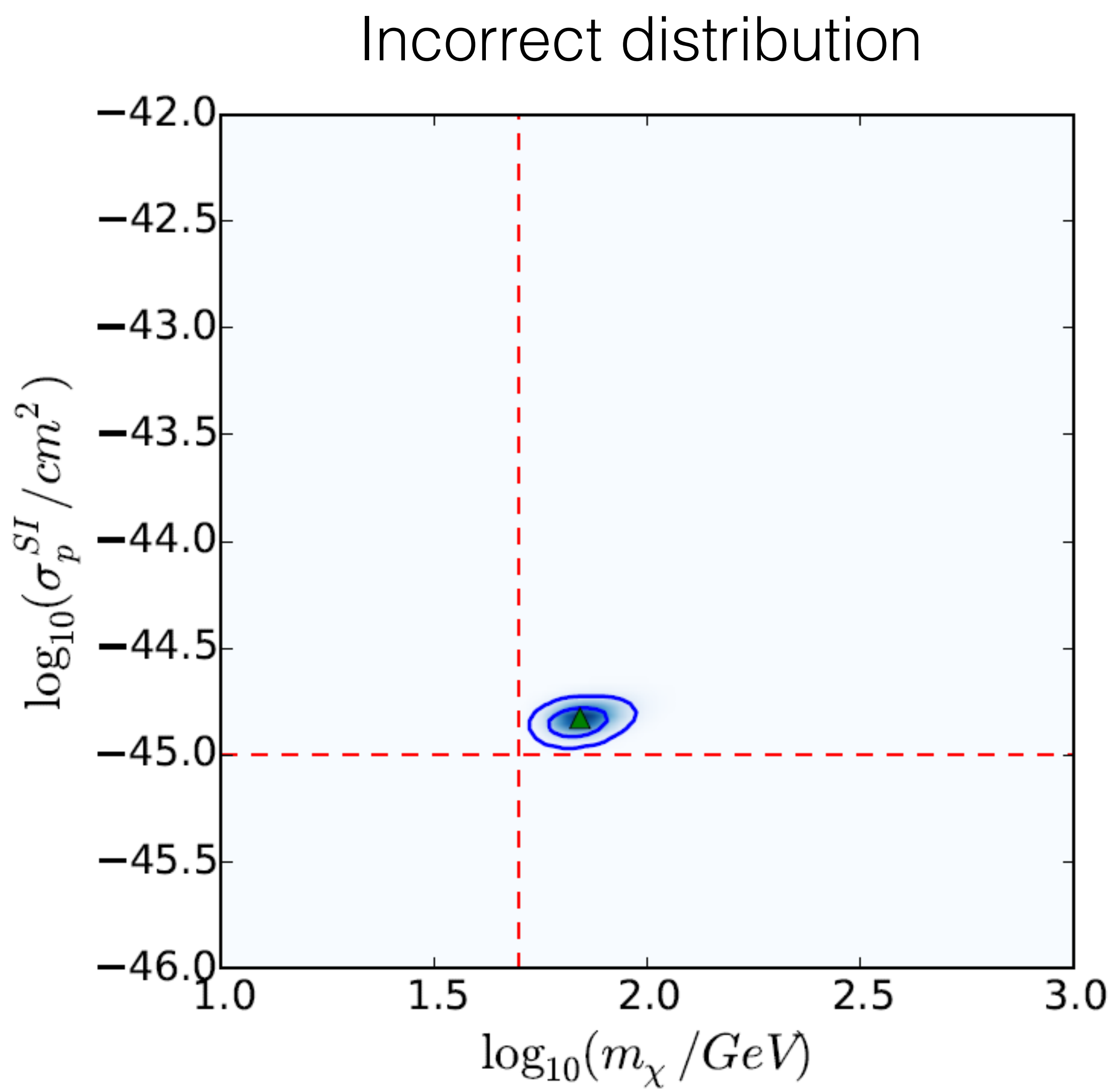
In the lab:



'WIMP wind from Cygnus'

What could go wrong?





Astrophysical uncertainties need to be accounted for!

While we're at it, why not try to reconstruct the velocity distribution too?!

Need directionality!

Outline

Directional event rate in DD

Mayet et al. [1602.03781]

Reconstructing f(v) in non-directional experiments

BJK, Green [1207.2039, 1303.6868,1312.1852]; BJK, Fornasa, Green [1410.8051]

Discretising the DM velocity distribution

BJK [1502.04224]

Reconstructing f(v) in directional experiments

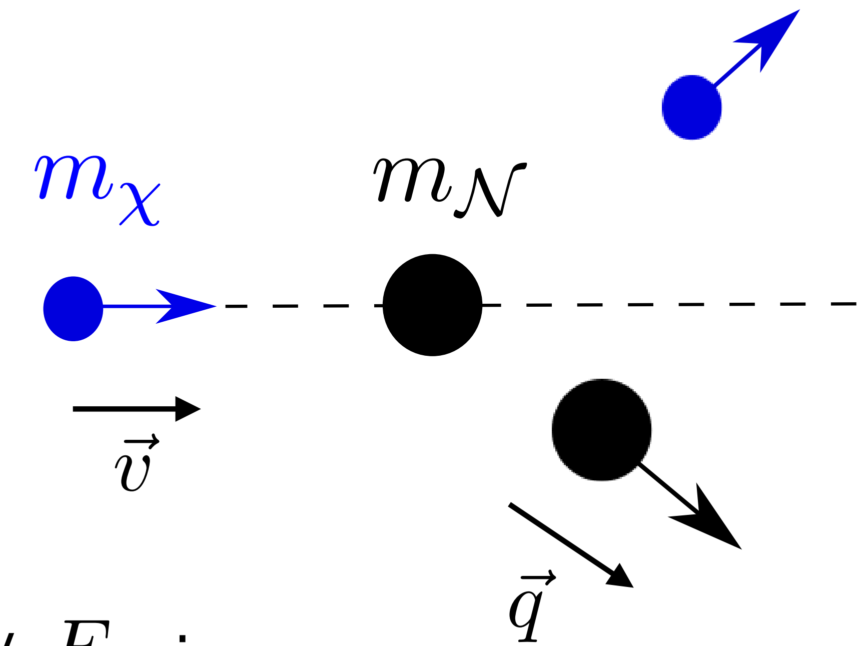
BJK, O'Hare [in preparation]

Directional recoil rate

Directional recoil rate

Flux of particles with velocity $\mathbf{v}$:

$$v\left(\frac{\rho_\chi}{m_\chi}\right) f(\mathbf{v})\,\mathrm{d}^3\mathbf{v}$$



Differential cross section for recoil energy E_R :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_R} \sim \frac{1}{v^2}$$

Kinematic constraint for recoil with momentum $\mathbf{q}$:

$$\hat{\mathbf{v}} \cdot \hat{\mathbf{q}} = v_\mathrm{min}/v$$

where $\quad v_\mathrm{min} = \sqrt{\frac{m_\mathcal{N} E_R}{2\mu_{\chi\mathcal{N}}^2}}$

$\rho_\chi \sim 0.2$–$0.6\ \mathrm{GeV\,cm}^{-3}$

Read (2014) [arXiv:1404.1938]

Directional recoil spectrum

$$\frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} = \frac{\rho_0}{4\pi \mu_{\chi p}^2 m_\chi} \sigma^p \mathcal{C}_\mathcal{N} F^2(E_R) \hat{f}(v_\mathrm{min}, \hat{\mathbf{q}})$$

$$v_\mathrm{min} = \sqrt{\frac{m_\mathcal{N} E_R}{2\mu_{\chi\mathcal{N}}^2}}$$

Directional recoil spectrum

$$\frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} = \frac{\rho_0}{4\pi\mu_{\chi p}^2 m_\chi} \underbrace{\sigma^p \mathcal{C}_\mathcal{N} F^2(E_R)}_{} \hat{f}(v_{\min}, \hat{\mathbf{q}})$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_R}$$

$$v_{\min} = \sqrt{\frac{m_\mathcal{N} E_R}{2\mu_{\chi\mathcal{N}}^2}}$$

Enhancement for nucleus $\mathcal{N}$:

$$\mathcal{C}_\mathcal{N} = \begin{cases} |Z + (f^p/f^n)(A - Z)|^2 & \text{SI interactions} \\ \frac{4}{3}\frac{J+1}{J} |\langle S_p \rangle + (a^p/a^n)\langle S_n \rangle|^2 & \text{SD interactions} \end{cases}$$

NB: May get interesting directional signatures from other operators BJK [1505.07406]

Form factor: $F^2(E_R)$

Directional recoil spectrum

$$\frac{\mathrm{d}R}{\mathrm{d}E_R \mathrm{d}\Omega_q} = \frac{\rho_0}{4\pi\mu_{\chi p}^2 m_\chi}\sigma^p \mathcal{C}_\mathcal{N} F^2(E_R)\hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}})$$

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Form factor: $F^2(E_R)$

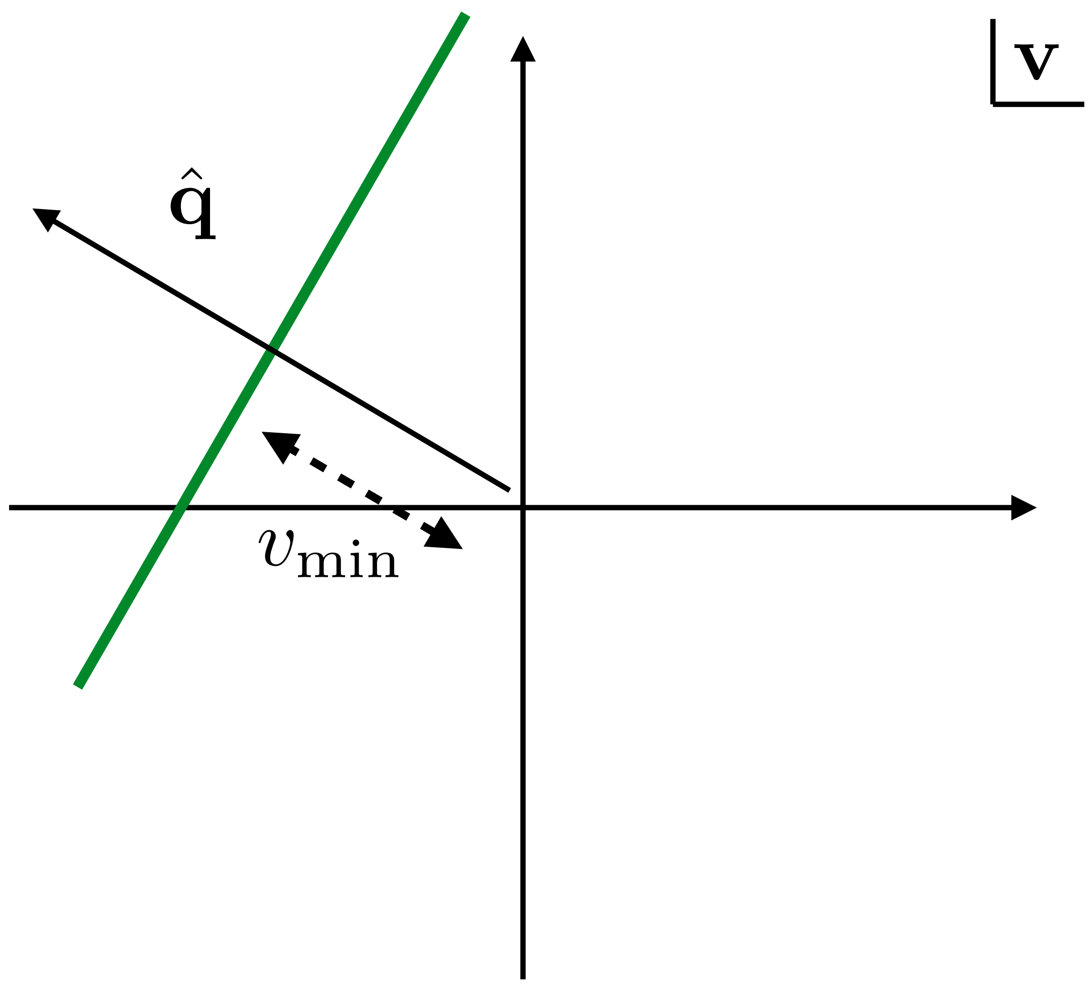
Radon Transform (RT):

$$\hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}}) = \int_{\mathbb{R}^3} f(\mathbf{v})\delta\left(\mathbf{v}\cdot\hat{\mathbf{q}} - v_{\mathrm{min}}\right)\mathrm{d}^3\mathbf{v}$$

Radon Transform

Radon Transform (RT):

$$\hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}}) = \int_{\mathbb{R}^3} f(\mathbf{v})\delta\left(\mathbf{v}\cdot\hat{\mathbf{q}} - v_{\mathrm{min}}\right)\mathrm{d}^3\mathbf{v}$$



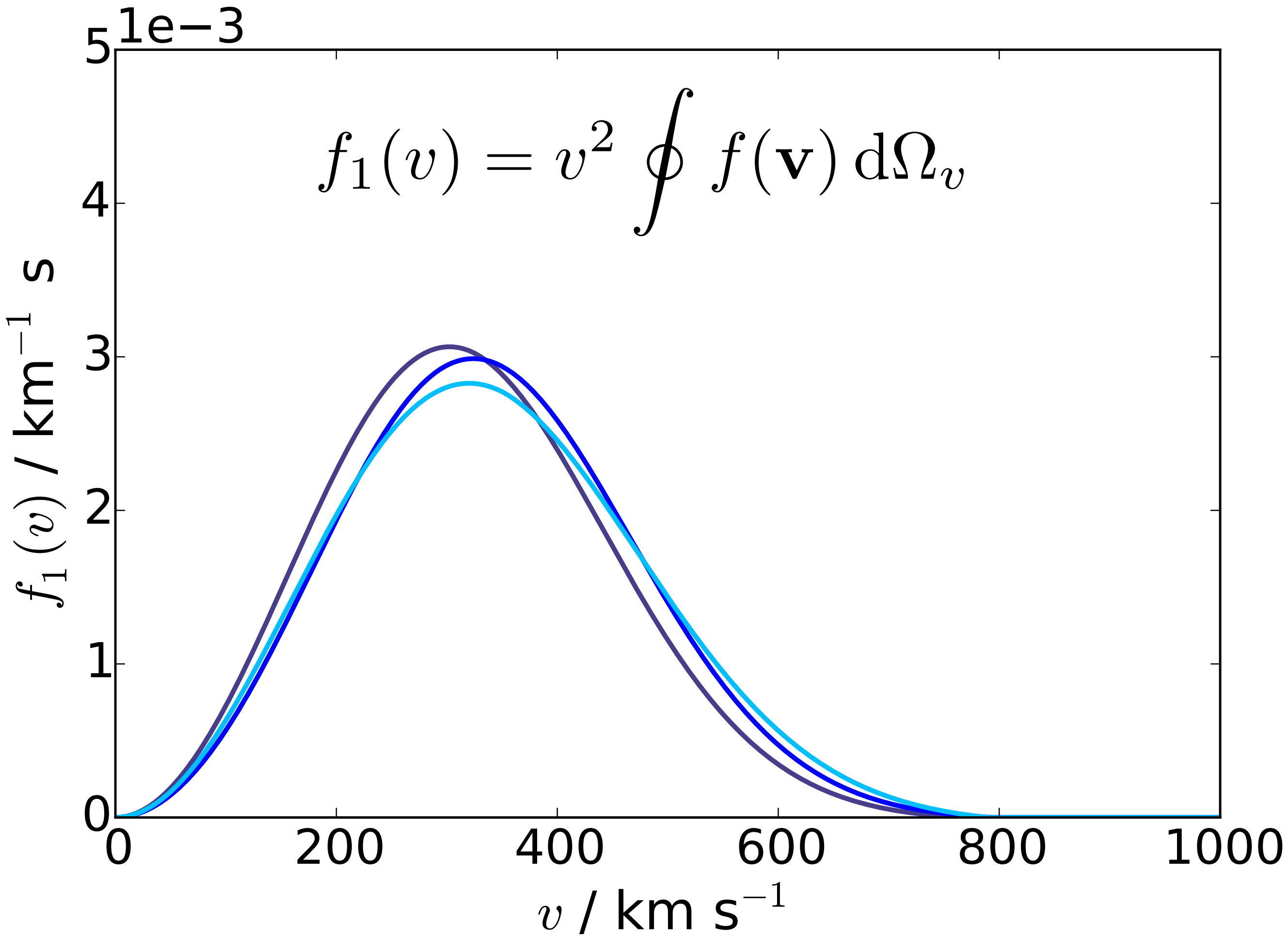
What do we know about the velocity distribution?

Standard Halo Model

Standard Halo Model (SHM) is typically assumed: isotropic, spherically symmetric distribution of particles with $\rho(r) \propto r^{-2}$.

Maxwell-Boltzmann distribution:

$$f_{\mathrm{Lab}}(\mathbf{v}) = (2\pi\sigma_v^2)^{-3/2} \exp\left[-\frac{(\mathbf{v}-\mathbf{v}_\mathrm{e})^2}{2\sigma_v^2}\right] \Theta(|\mathbf{v}-\mathbf{v}_\mathrm{e}| - v_\mathrm{esc})$$



$\mathbf{v}_e$ - Earth's Velocity

$$v_e \sim 220 - 250\,\mathrm{km\,s^{-1}}$$

$$\sigma_v \sim 155 - 175\,\mathrm{km\,s^{-1}}$$

Feast et al. [astro-ph/9706293], Bovy et al. [1209.0759]

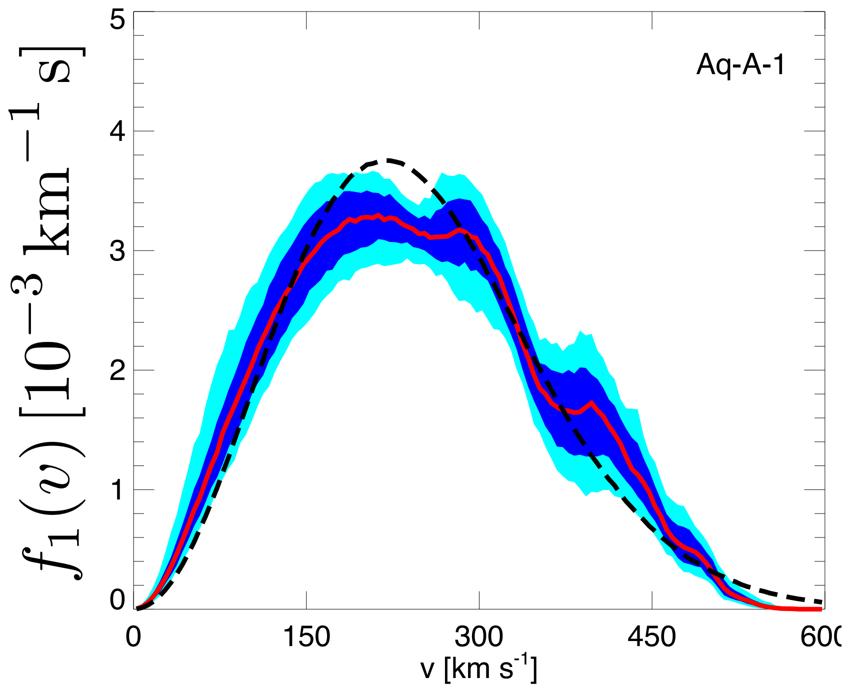
$$v_\mathrm{esc} = 533^{+54}_{-41}\,\mathrm{km\,s^{-1}}$$

Piffl et al. (RAVE) [1309.4293]

Astrophysical uncertainties

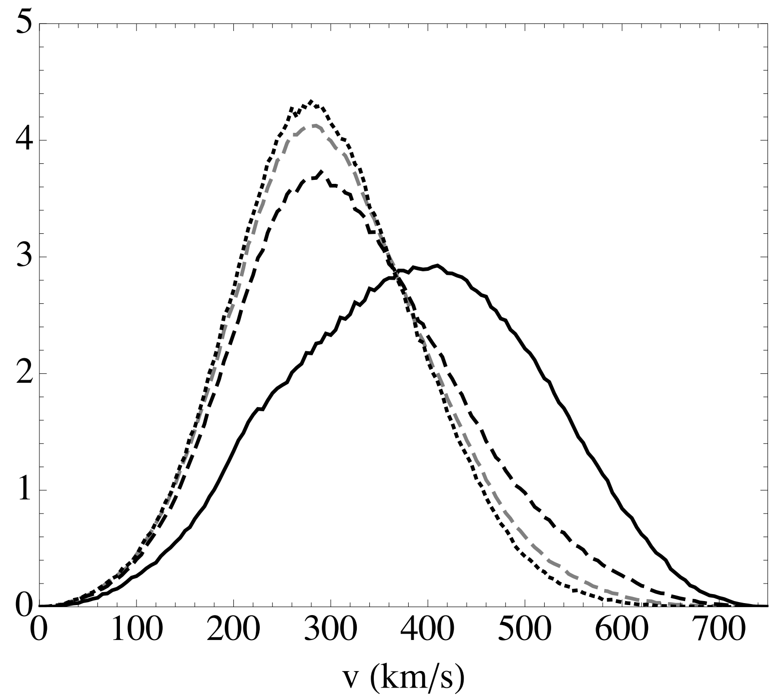
High resolution N-body simulations can be used to extract the DM speed distribution

Non-Maxwellian structure



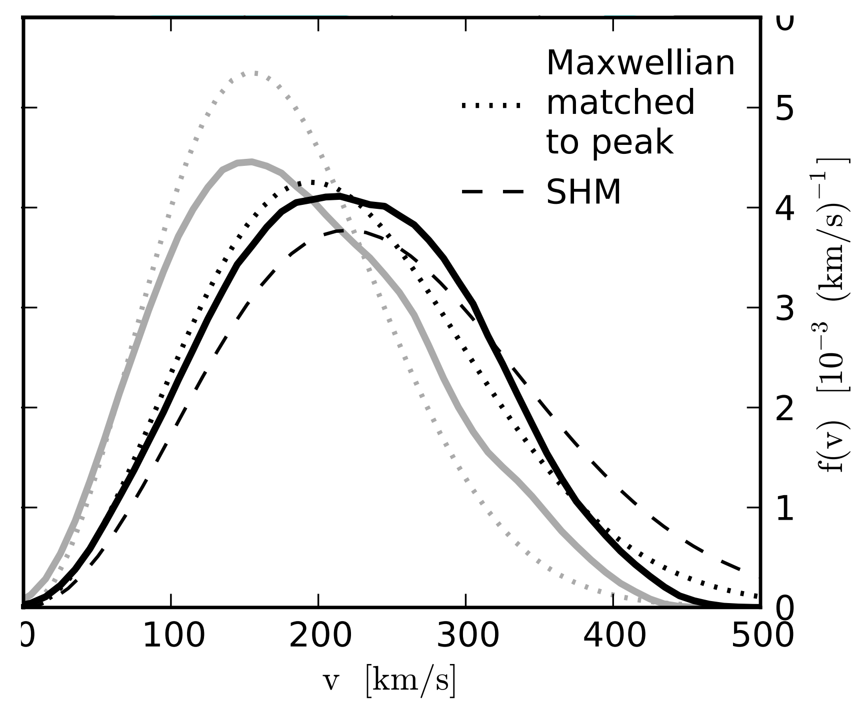
Vogelsberger et al. [0812.0362]

Debris flows



Kuhlen et al. [1202.0007]

Dark disk



Pillepich et al. [1308.1703], Schaller et al. [1605.02770]

However, N-body simulations cannot probe down to the sub-milliparsec scales probes by direct detection…

Local substructure

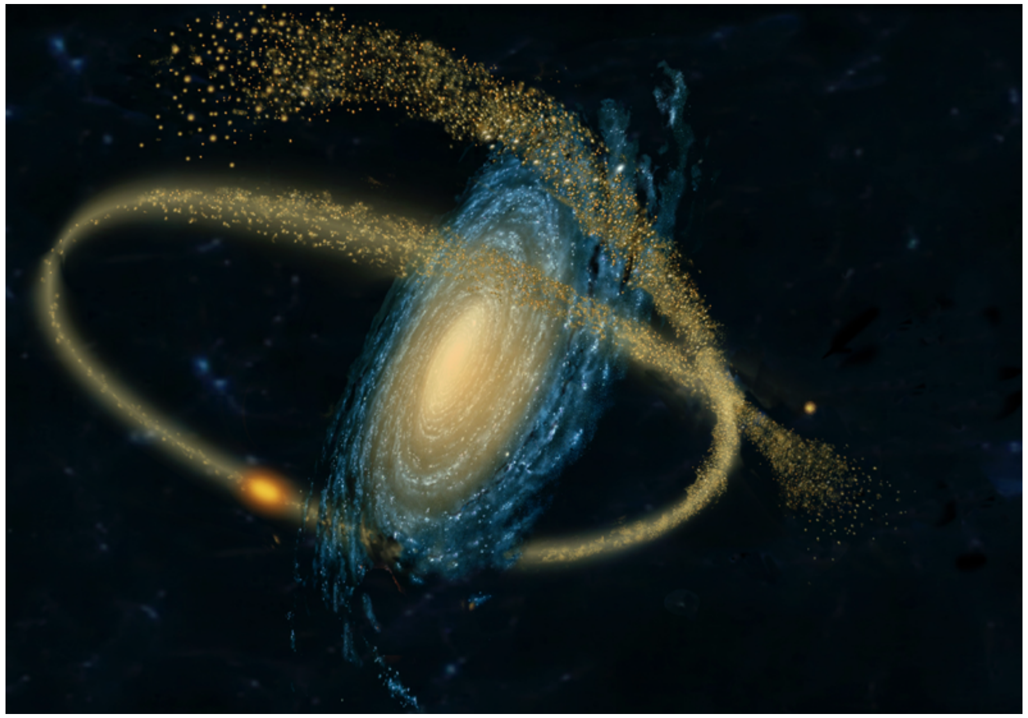
May want to worry about ultra-local substructure - subhalos and streams which are not completely phase-mixed.

But from N-body simulations, expect lots of 'sub-streams' to form a smooth halo.

Helmi et al. [astro-ph/0201289], Vogelsberger et al. [0711.1105]

However, this does not exclude the possibility of a stream - e.g. due to the ongoing tidal disruption of the Sagittarius dwarf galaxy.

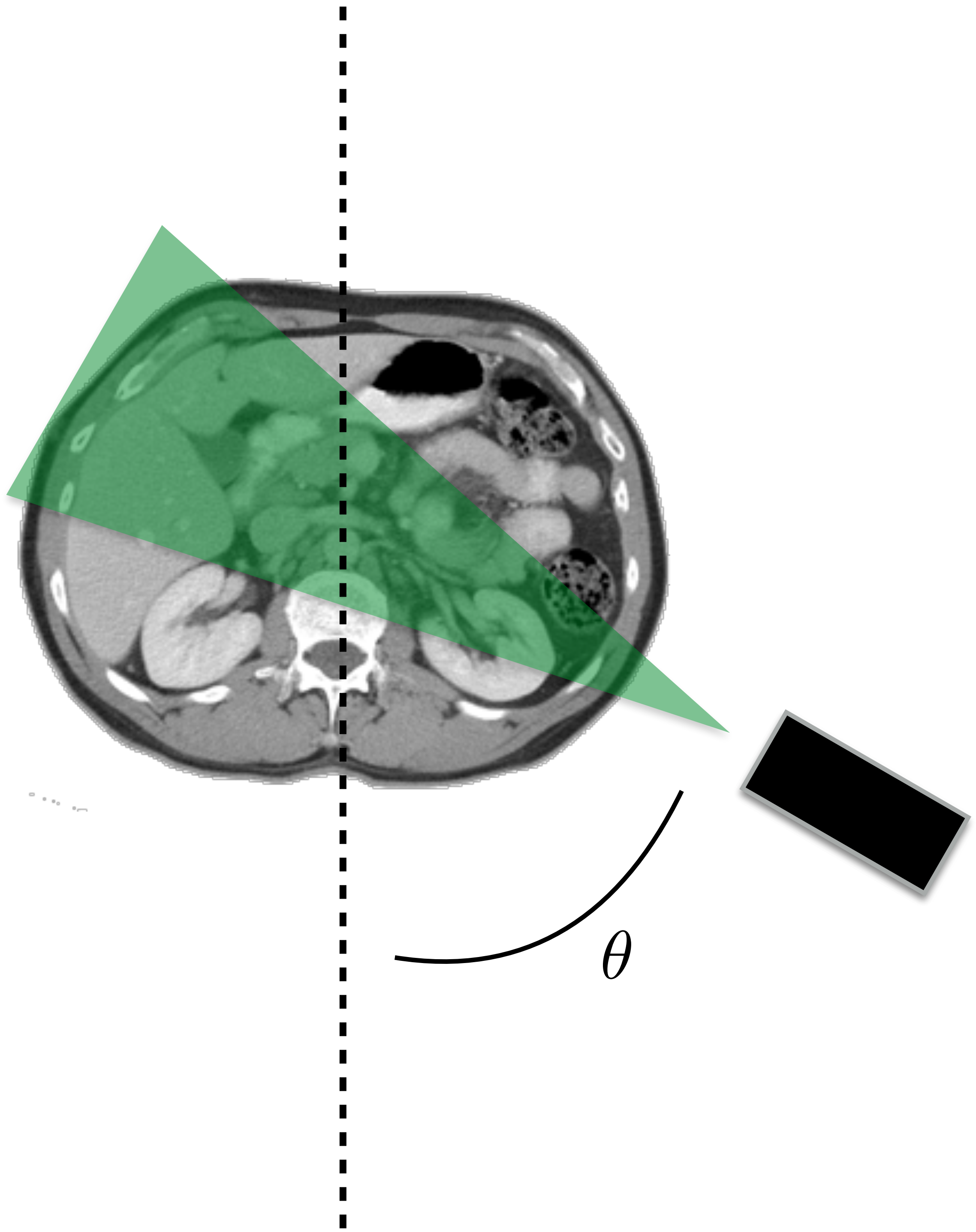
Freese et al. [astro-ph/0309279, astro-ph/0310334]

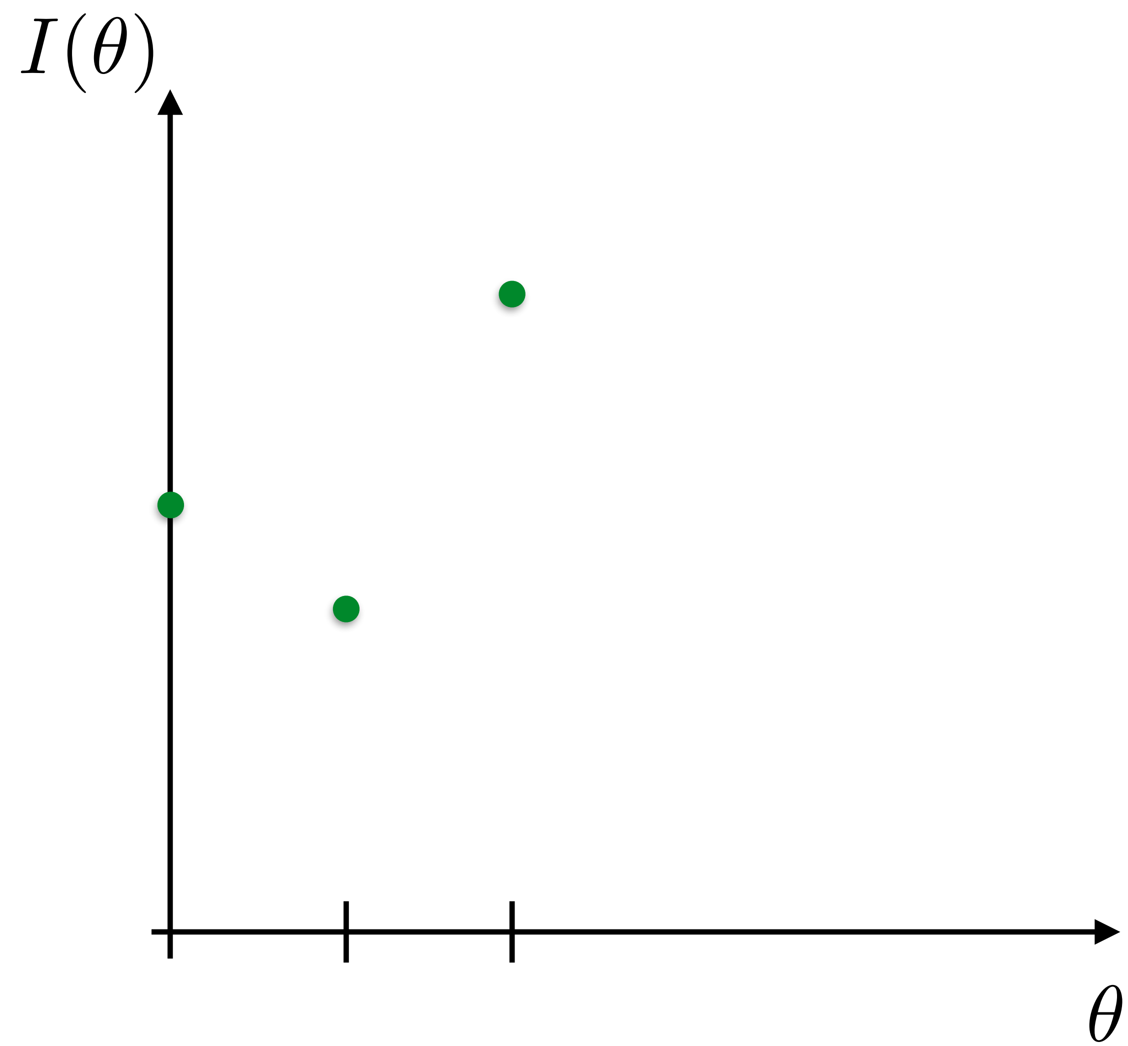


www.cosmotography.com

Measuring f(v) may tell us something about galaxy formation and the history of our Milky Way!

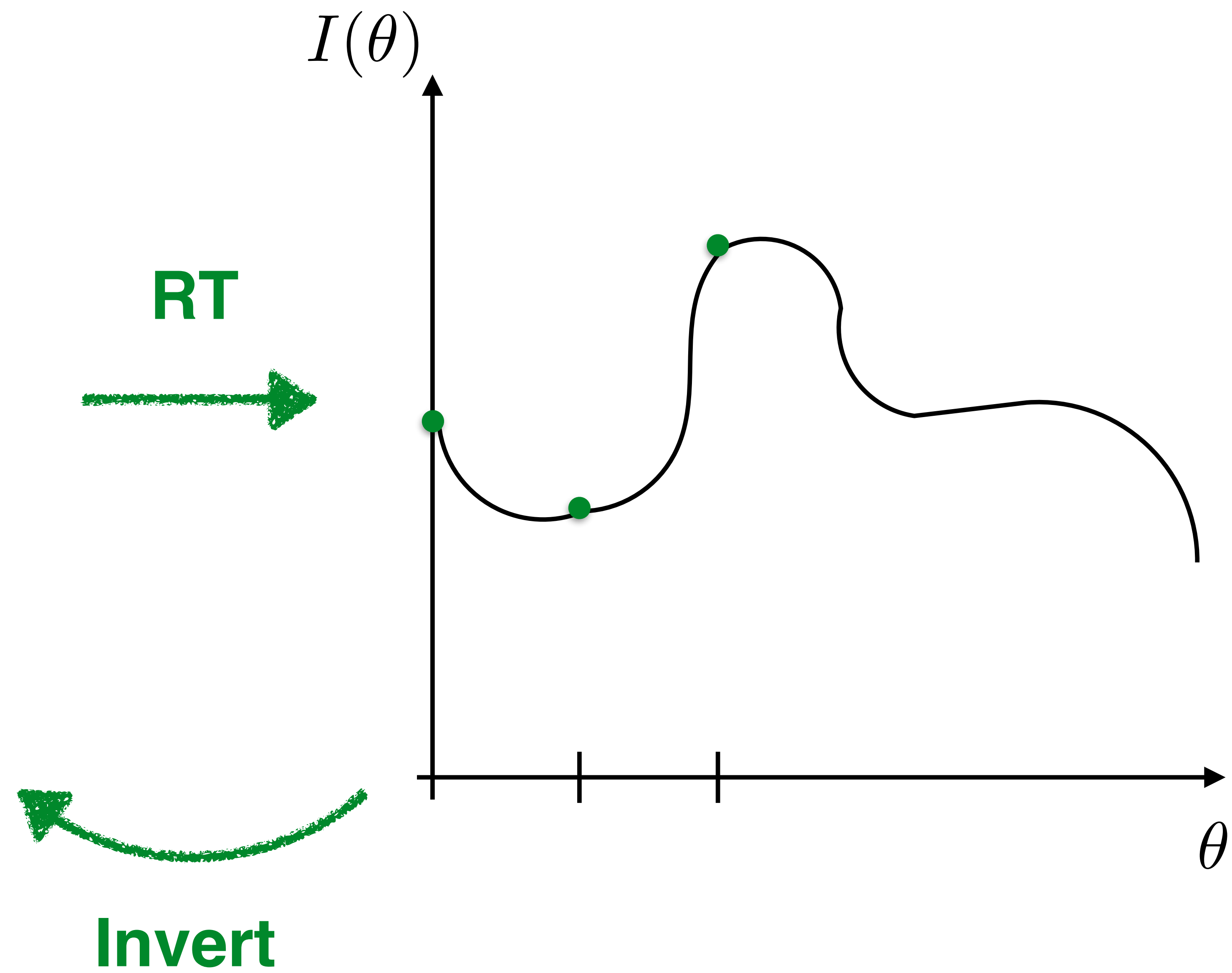
Tomography





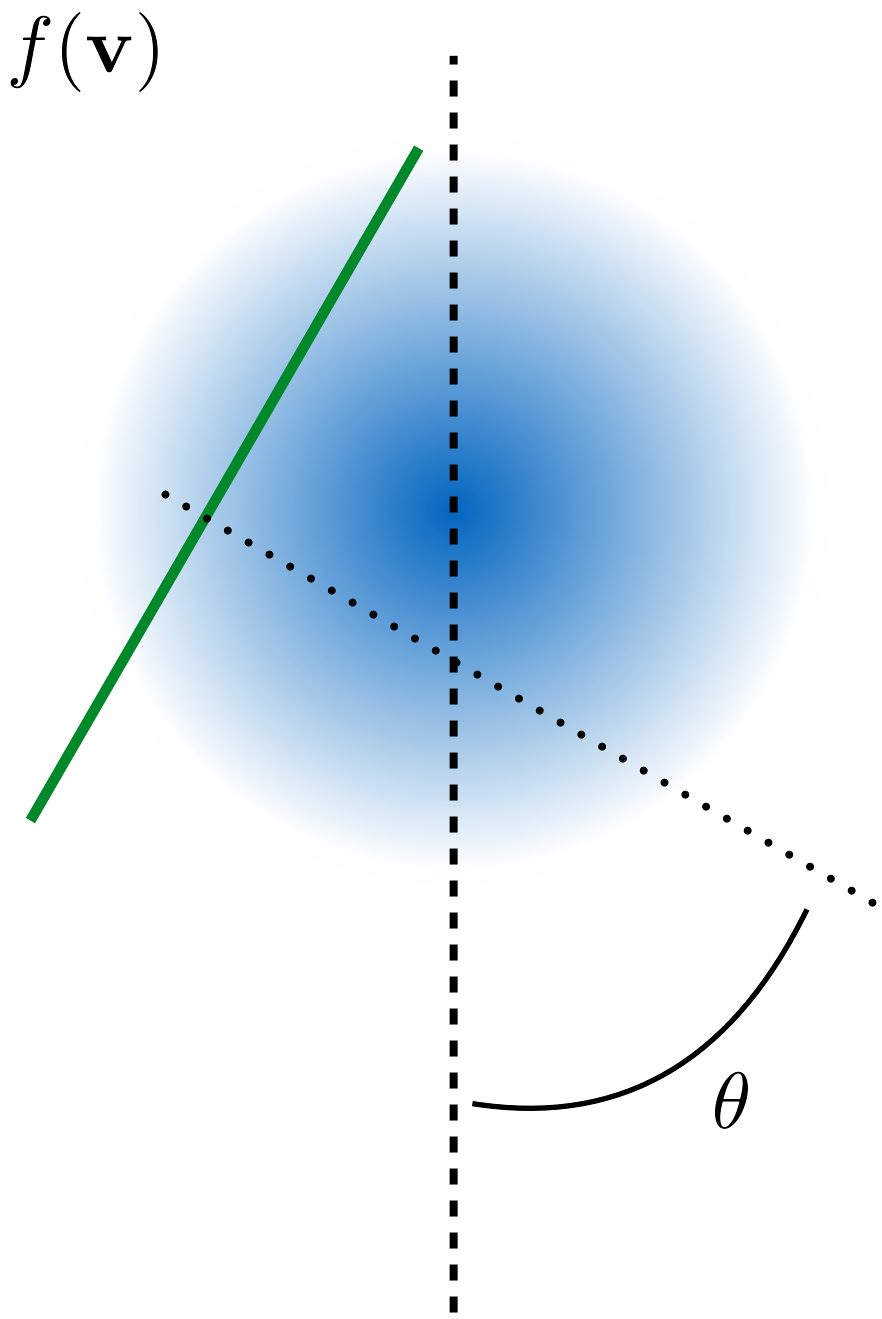
www.fda.gov

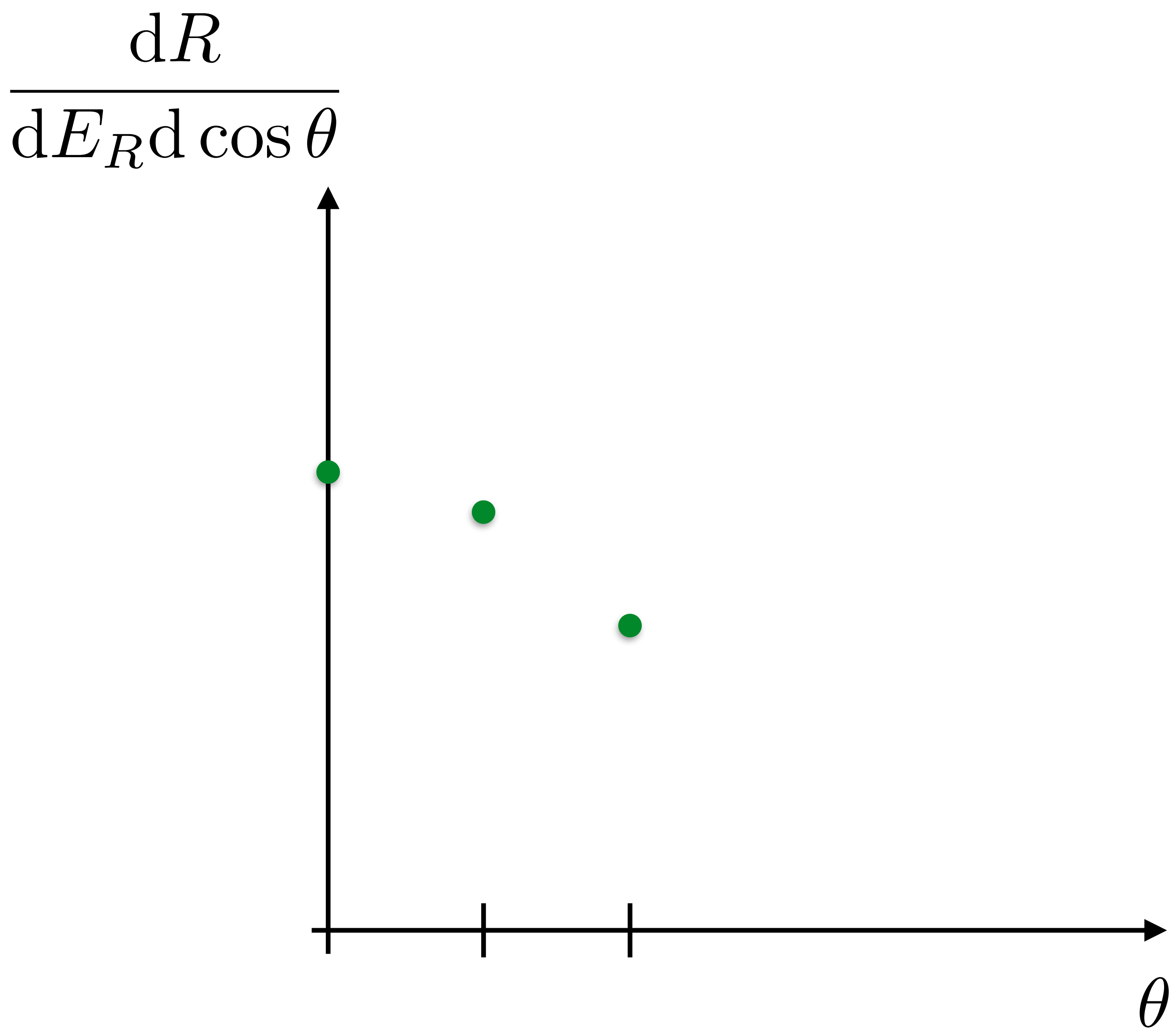
Tomography



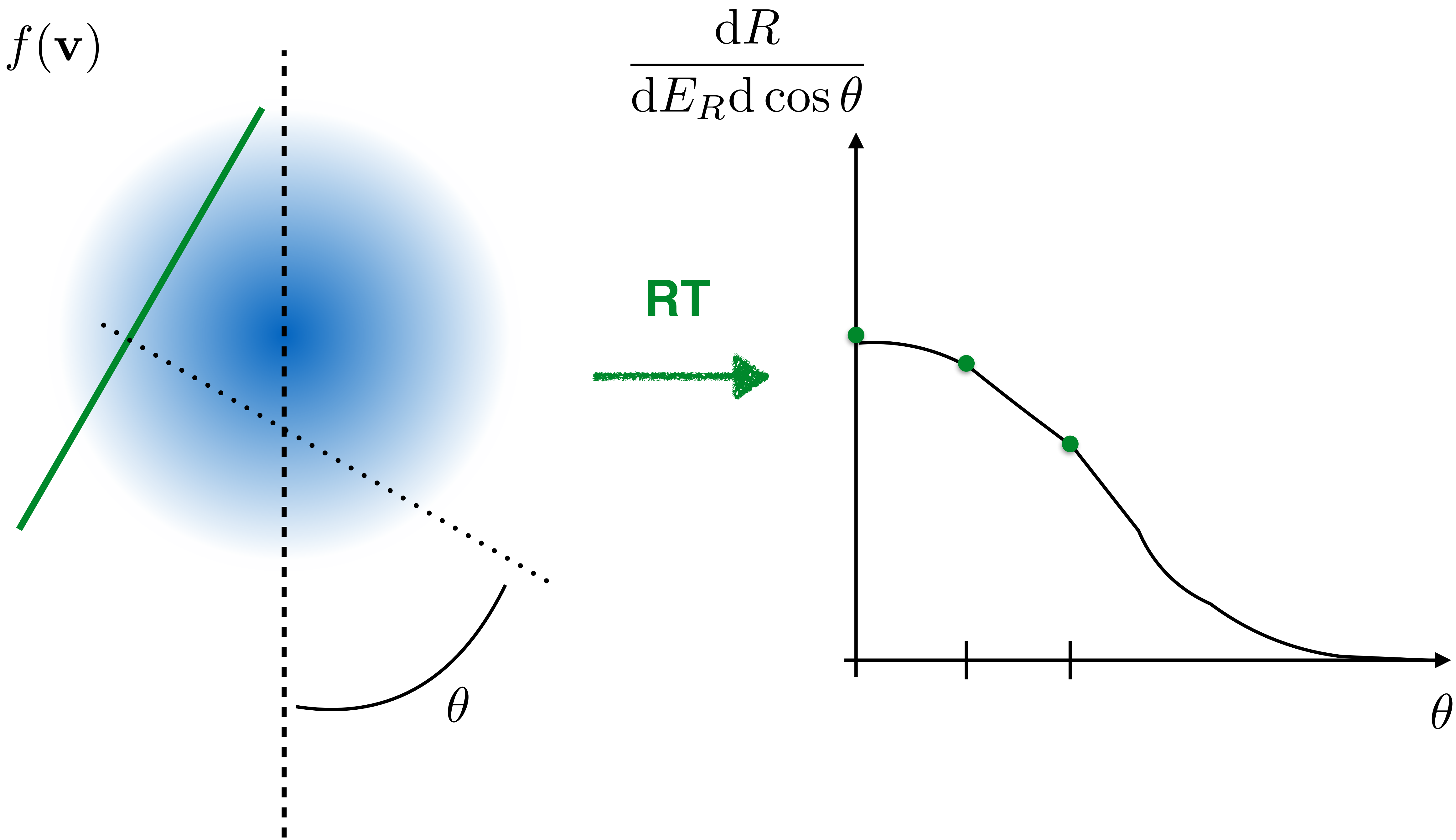
www.fda.gov

DM Tomography

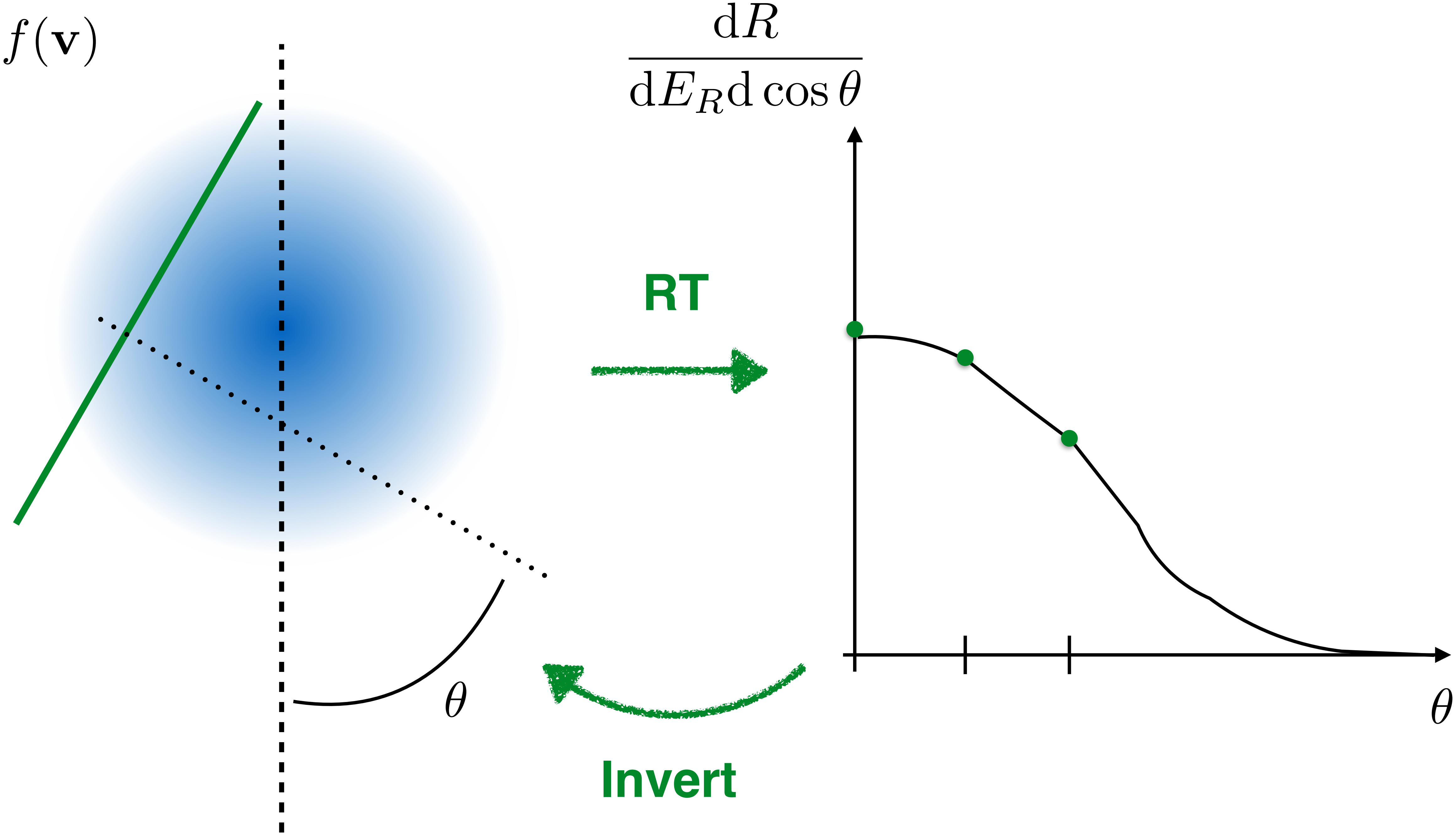




DM Tomography



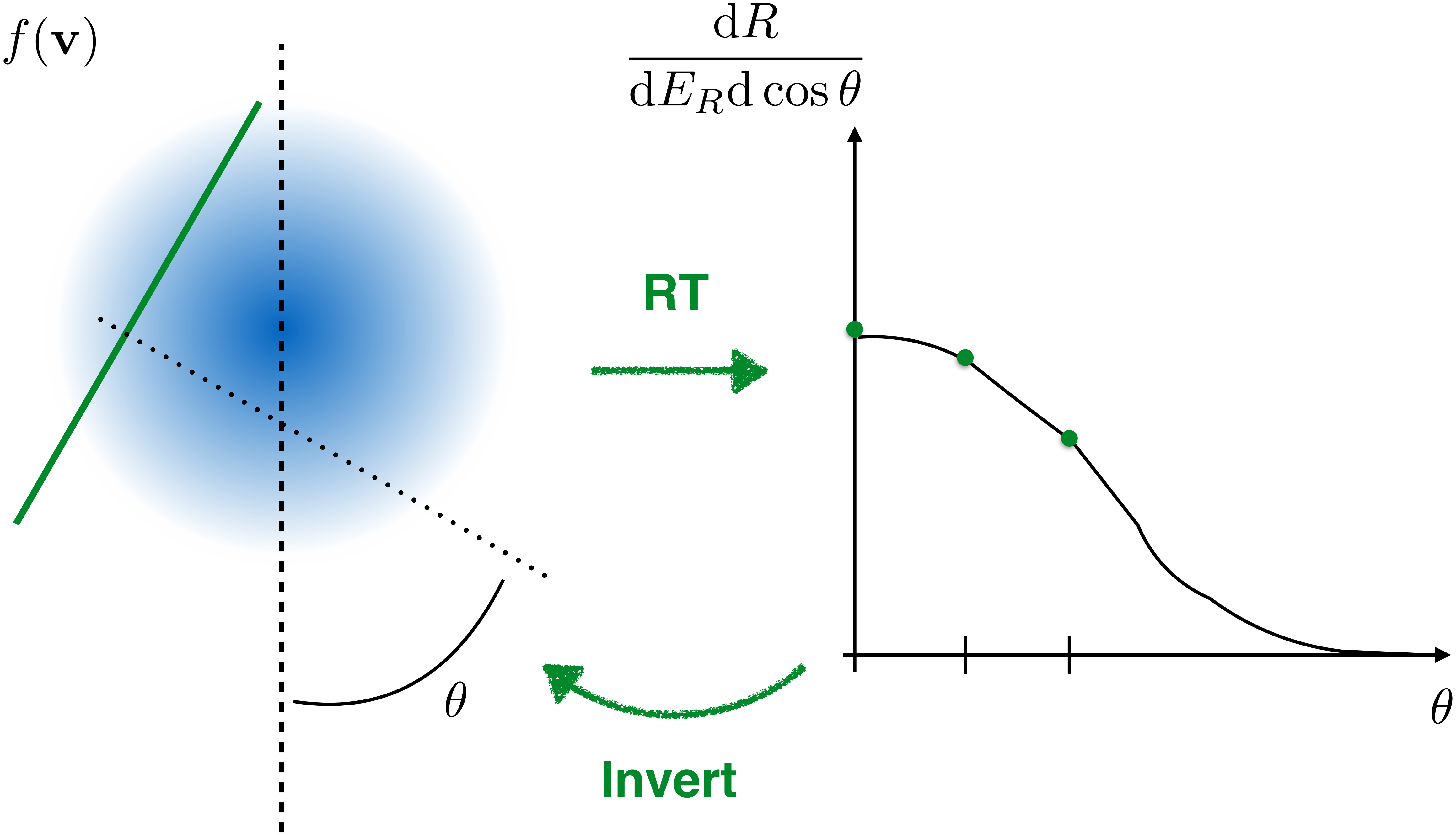
DM Tomography



$$f(\mathbf{v}) = -\frac{1}{8\pi^2}\int \frac{\mathrm{d}^2}{\mathrm{d}(\mathbf{v}\cdot\hat{\mathbf{q}})^2}\hat{f}(\mathbf{v}\cdot\hat{\mathbf{q}}, \hat{\mathbf{q}})\,\mathrm{d}\Omega_q$$

Gondolo [hep-ph/0209110]

DM Tomography



But we don't get to choose where to scan, we just get random samples!

1-D reconstructions (Energy only)

Reconstructing f(v)

Many previous attempts to tackle this problem:

Numerical inversion ('measure' f(v) from the data)

Fox, Liu, Weiner [1011.915], Frandsen et al. [1111.0292], Feldstein, Kahlhoefer [1403.4606]

Include uncertainties in SHM parameters in the fit

Strigari, Trotta [0906.5361]

Add extra components to the velocity distribution (and fit)

Lee, Peter [1202.5035], O'Hare, Green [1410.2749]

But can we be more general?

General empirical parametrisation

Write a *general parametrisation* for the speed distribution:

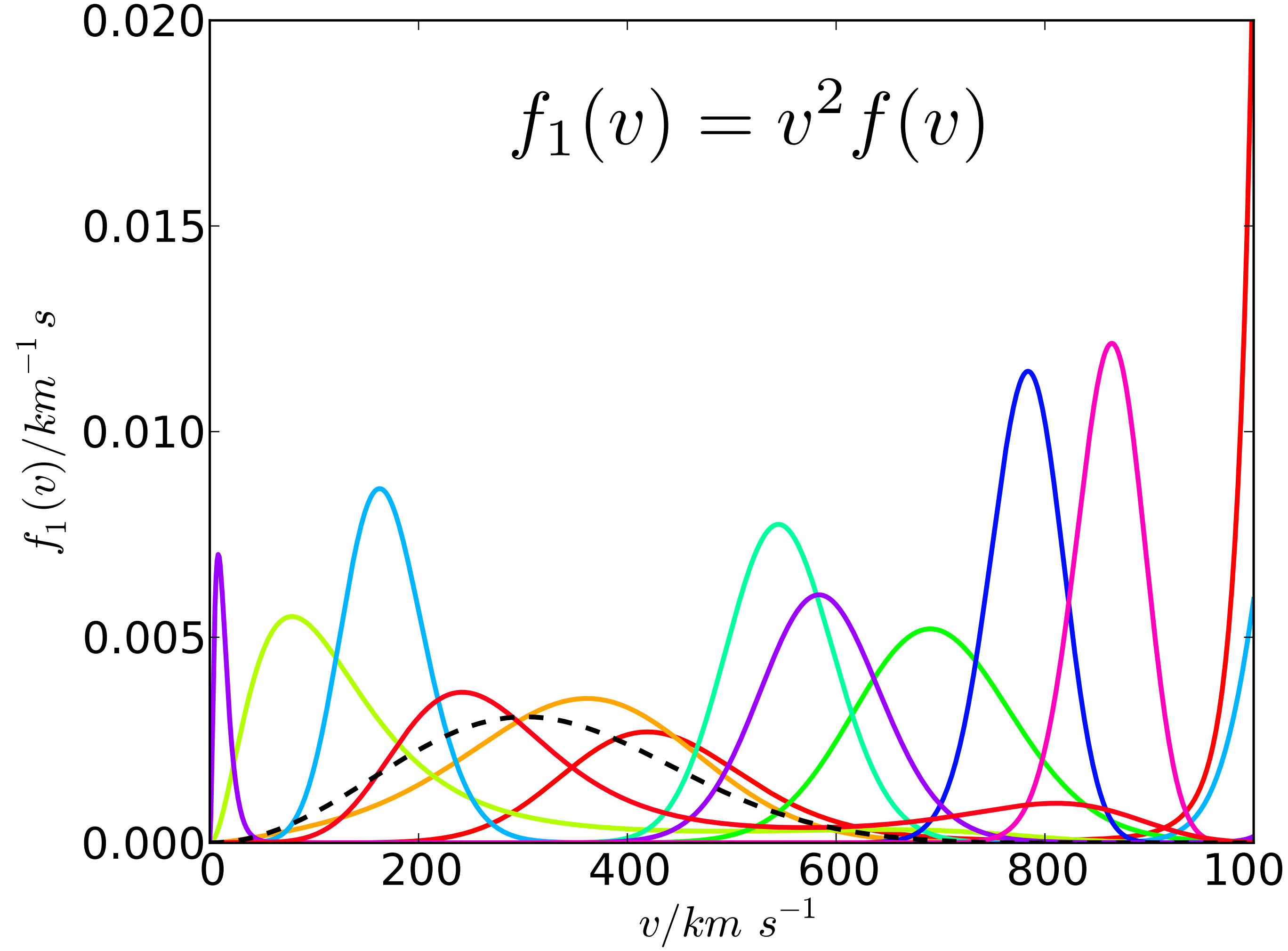
Peter [1103.5145]

$$\boxed{f(v) = \exp\left(-\sum_{k=0}^{N-1} a_k v^k\right)}$$

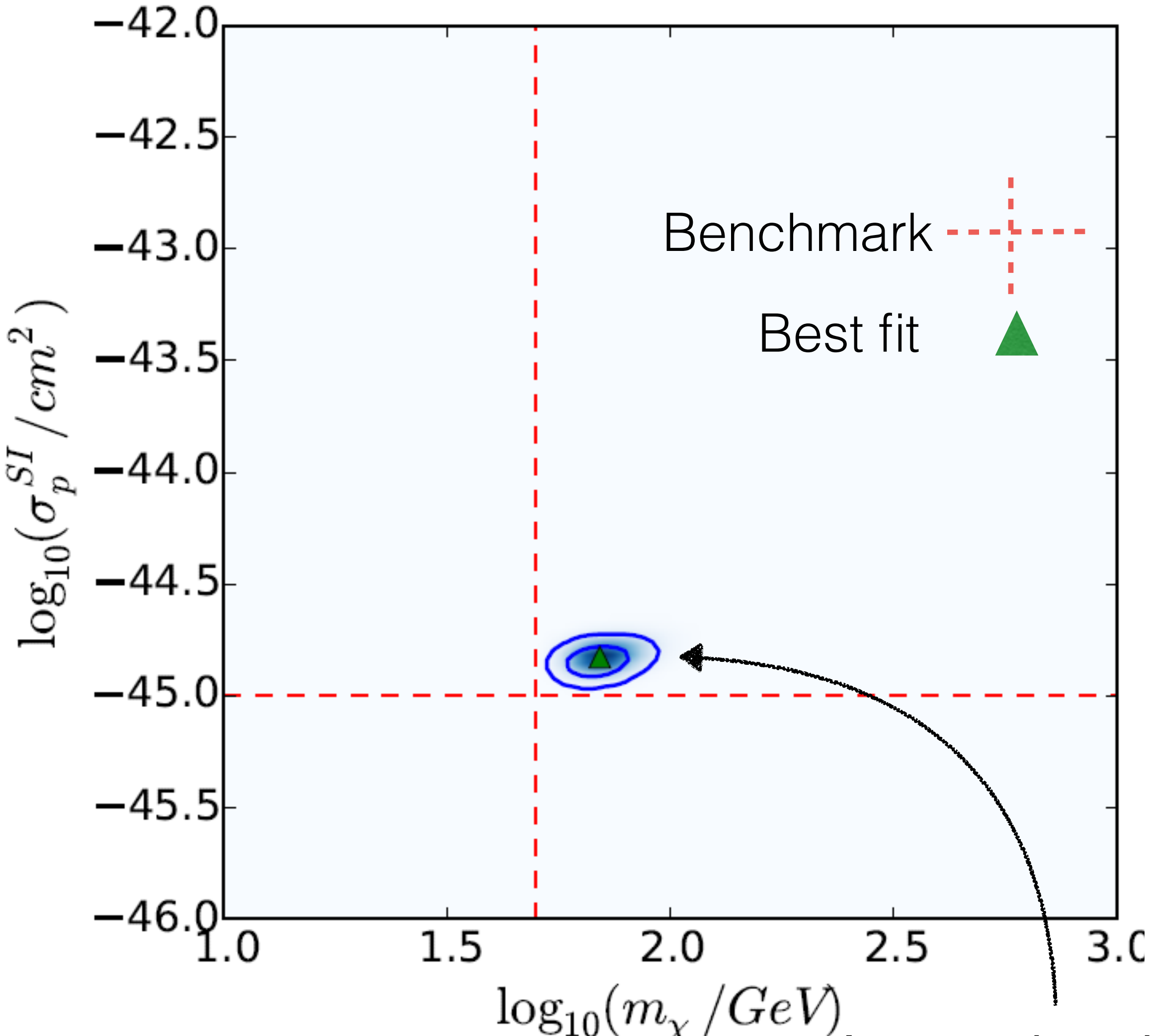
BJK & Green [1303.6868,1312.1852]

This form guarantees a positive distribution function.

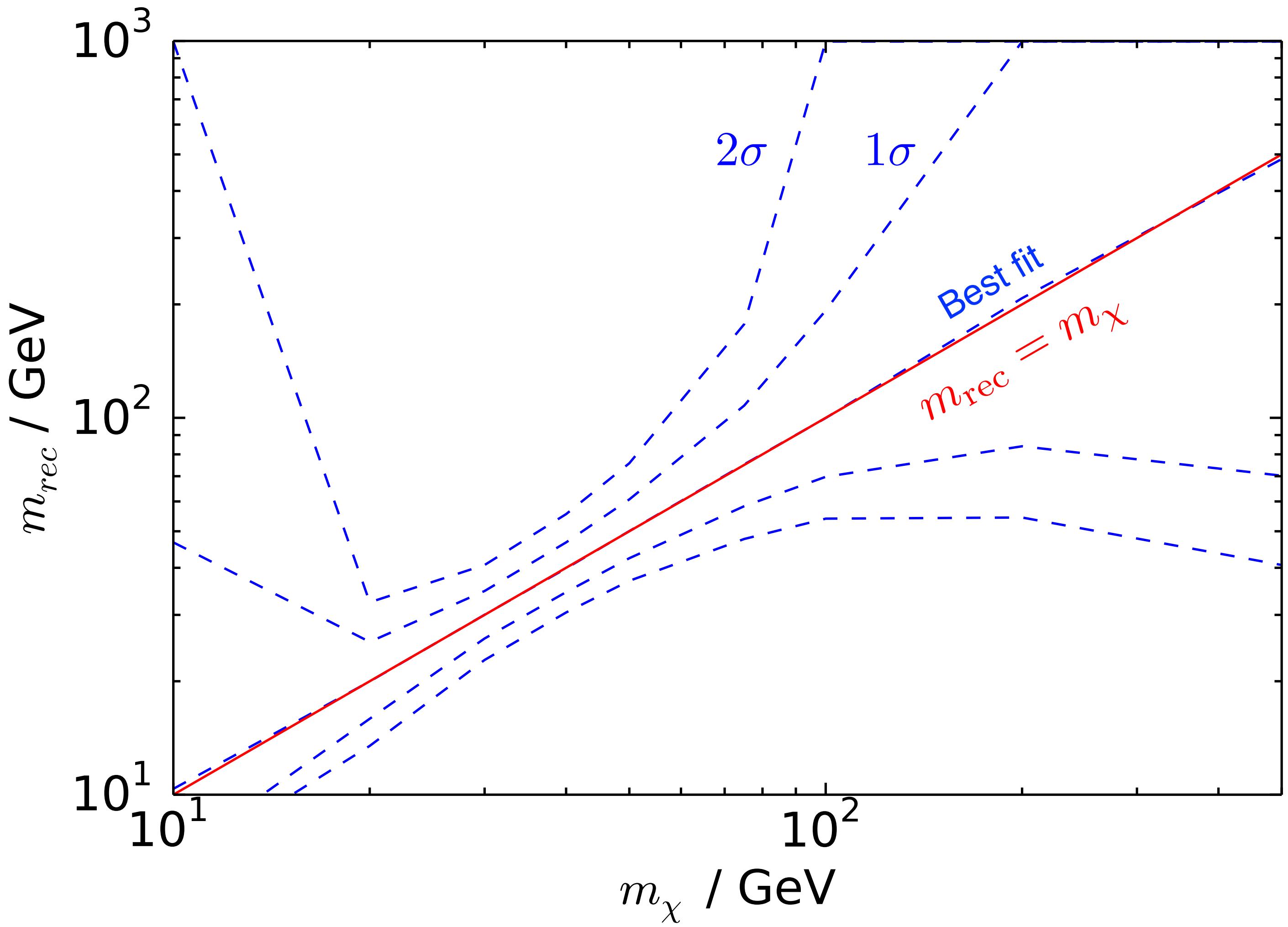
Now we attempt to fit the particle physics parameters (m_χ, σ^p), as well as the astrophysics parameters $\{a_k\}$.



Testing the parametrisation

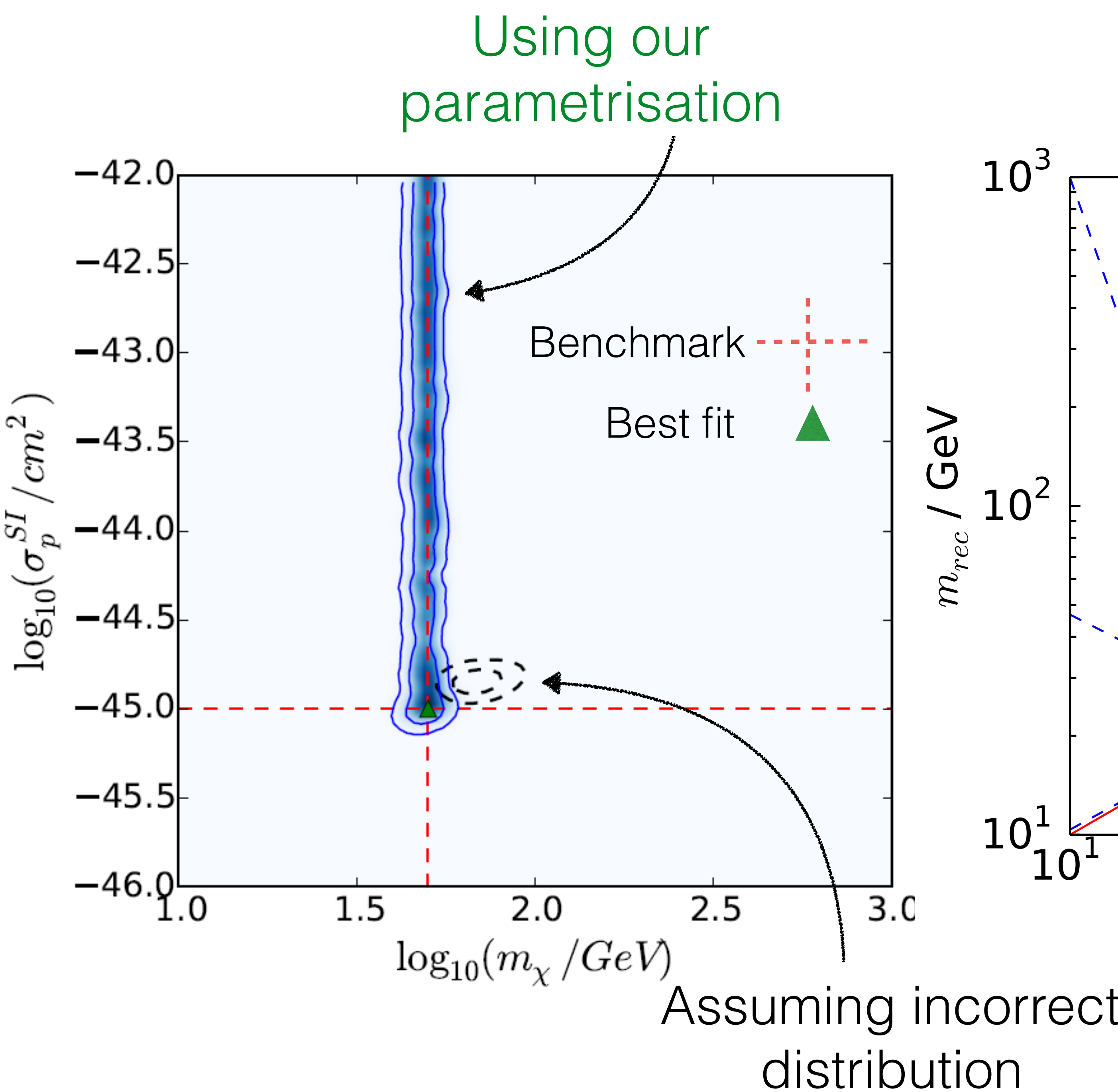


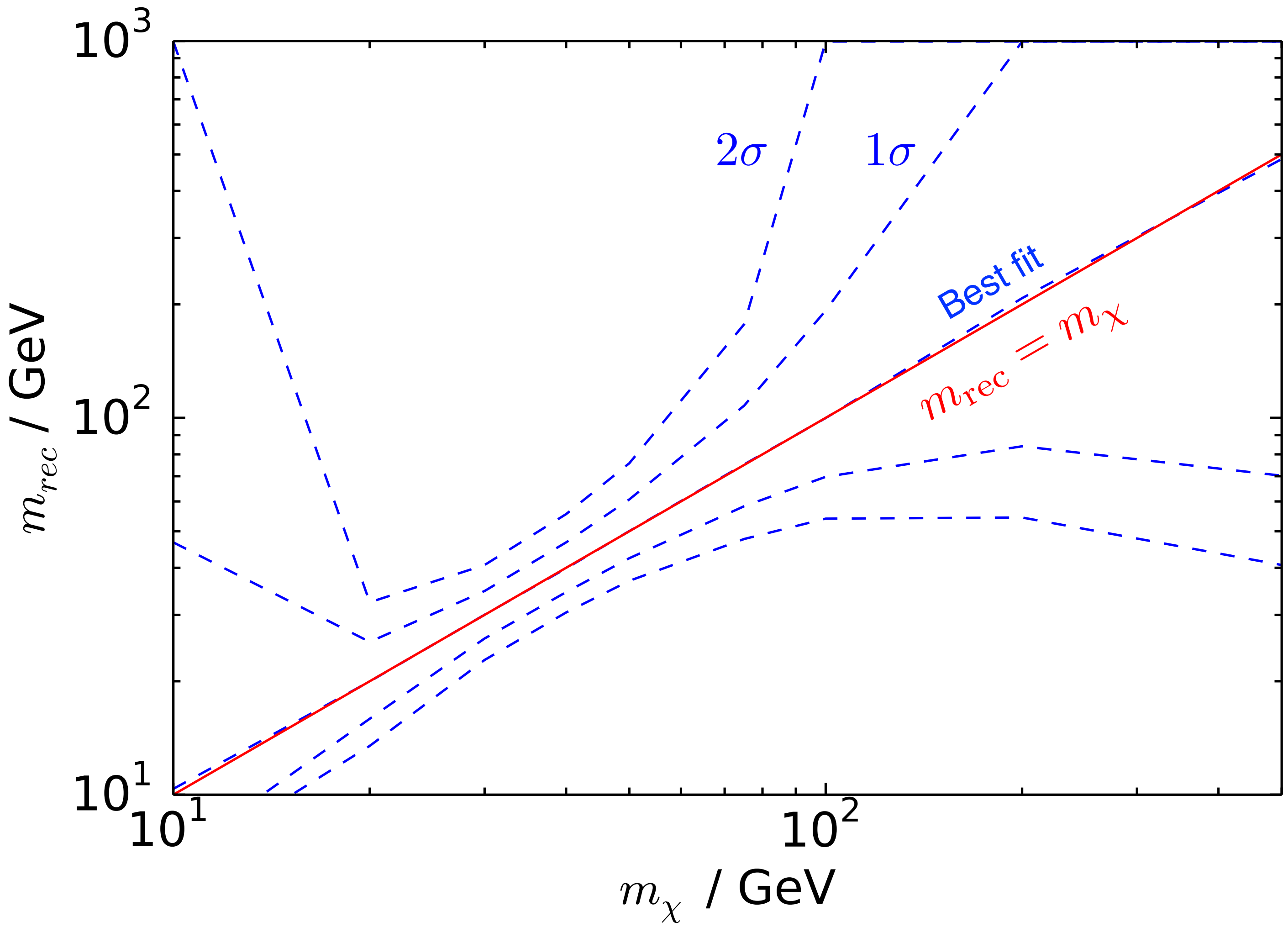
Assuming incorrect distribution



Tested for a number of different underlying speed distributions

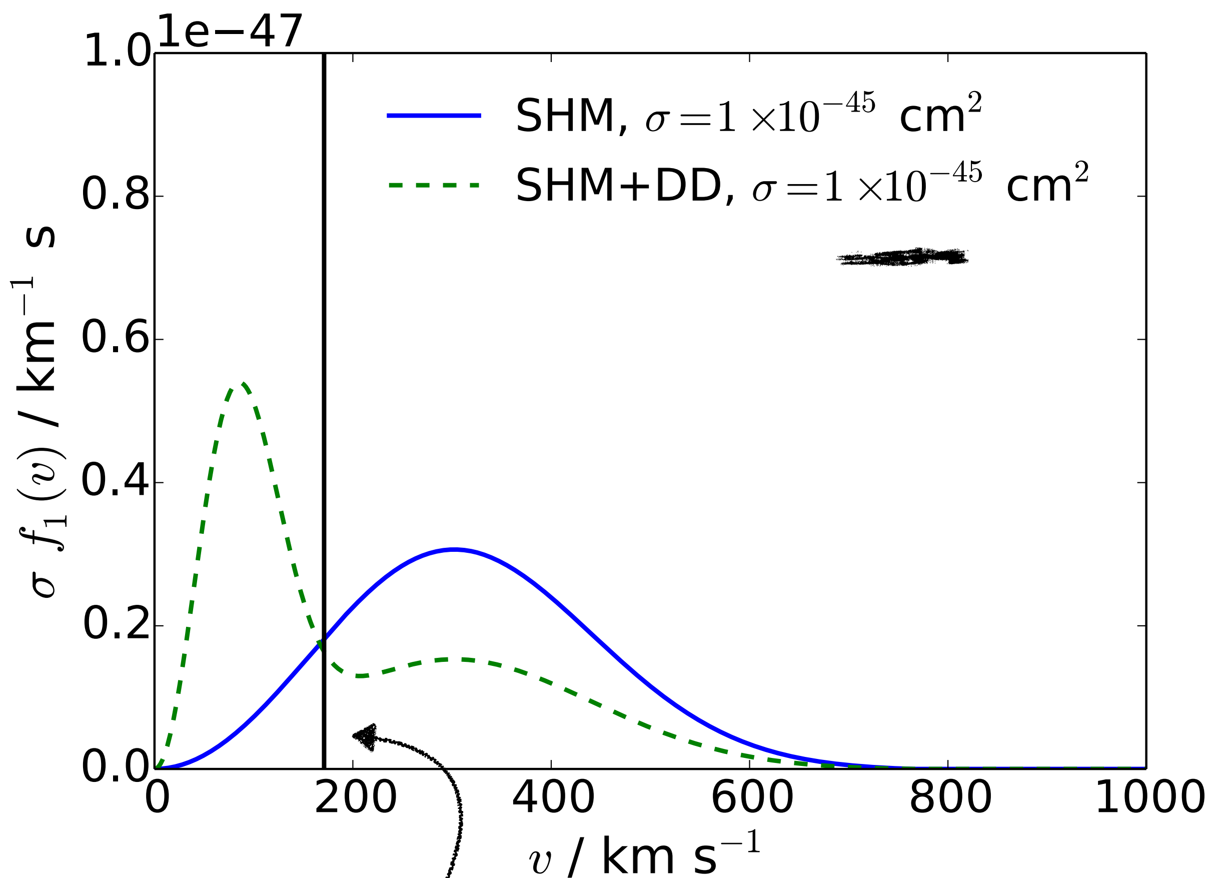
Testing the parametrisation





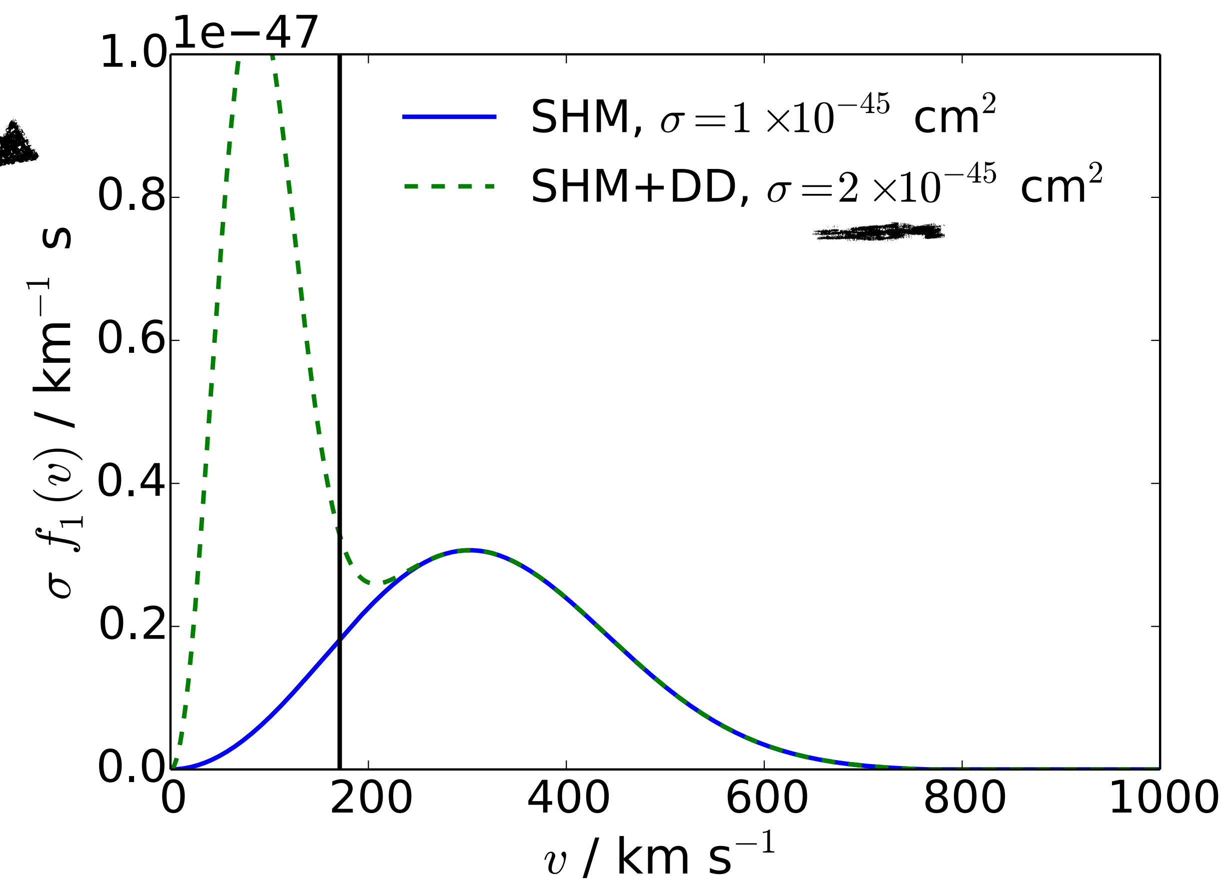
Tested for a number of different underlying speed distributions

Cross section degeneracy



Minimum DM speed probed by a typical Xe experiment

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} \propto \sigma \int_{v_{\mathrm{min}}}^{\infty} \frac{f_1(v)}{v}\,\mathrm{d}v$$



This is a problem for *any* astrophysics-independent method!

Can be solved by including data from Solar Capture of DM - sensitive to low speed DM particles BJK, Fornasa, Green [1410.8051]

1-D reconstructions

This parametrisation allows us to fit the 1-D *speed distribution* in a general way. This means we can reconstruct the DM mass without bias!

Can also reconstruct the form of the speed distribution itself from the parameters (but we'll leave that for later in the talk…)

But if we want to parametrise the full 3-D velocity distribution, we would need an infinite number of parameters!

But how do we extend this to directional detection?

A directional parametrisation

From 1-D to 3-D

$$f(\mathbf{v}) = f^1(v)A^1(\hat{\mathbf{v}}) + f^2(v)A^2(\hat{\mathbf{v}}) + f^3(v)A^3(\hat{\mathbf{v}}) + \ldots.$$

One possible basis is spherical harmonics:

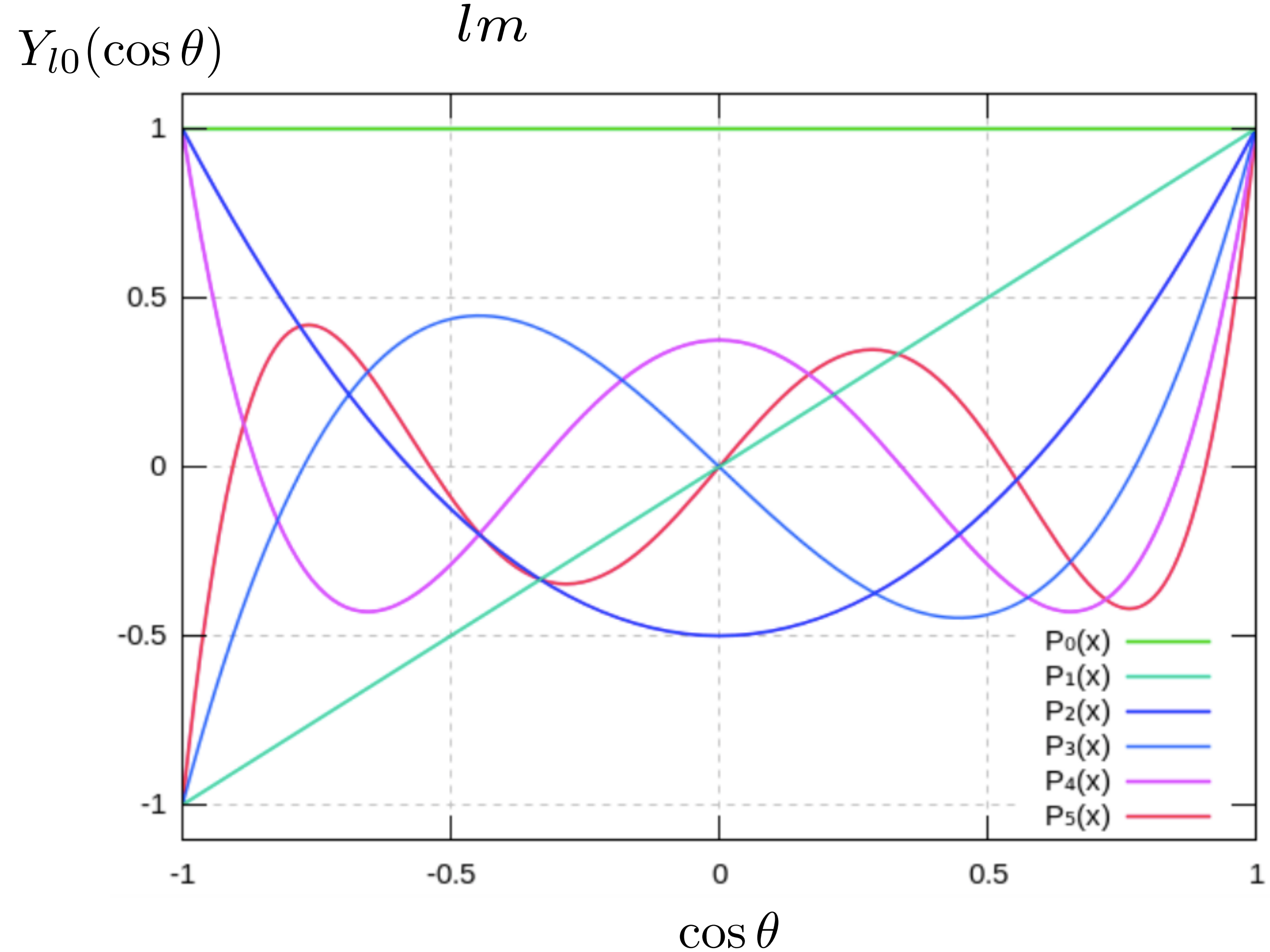
Alves et al. [1204.5487], Lee [1401.6179]

$$f(\mathbf{v}) = \sum_{lm} f_{lm}(v) Y_{lm}(\hat{\mathbf{v}})$$

$$\Rightarrow \hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}}) = \sum_{lm} \hat{f}_{lm}(v_{\mathrm{min}}) Y_{lm}(\hat{\mathbf{q}})$$

However, they are not strictly positive definite!

If we try to fit with spherical harmonics, we cannot guarantee that we get a physical distribution function!



A discretised distribution

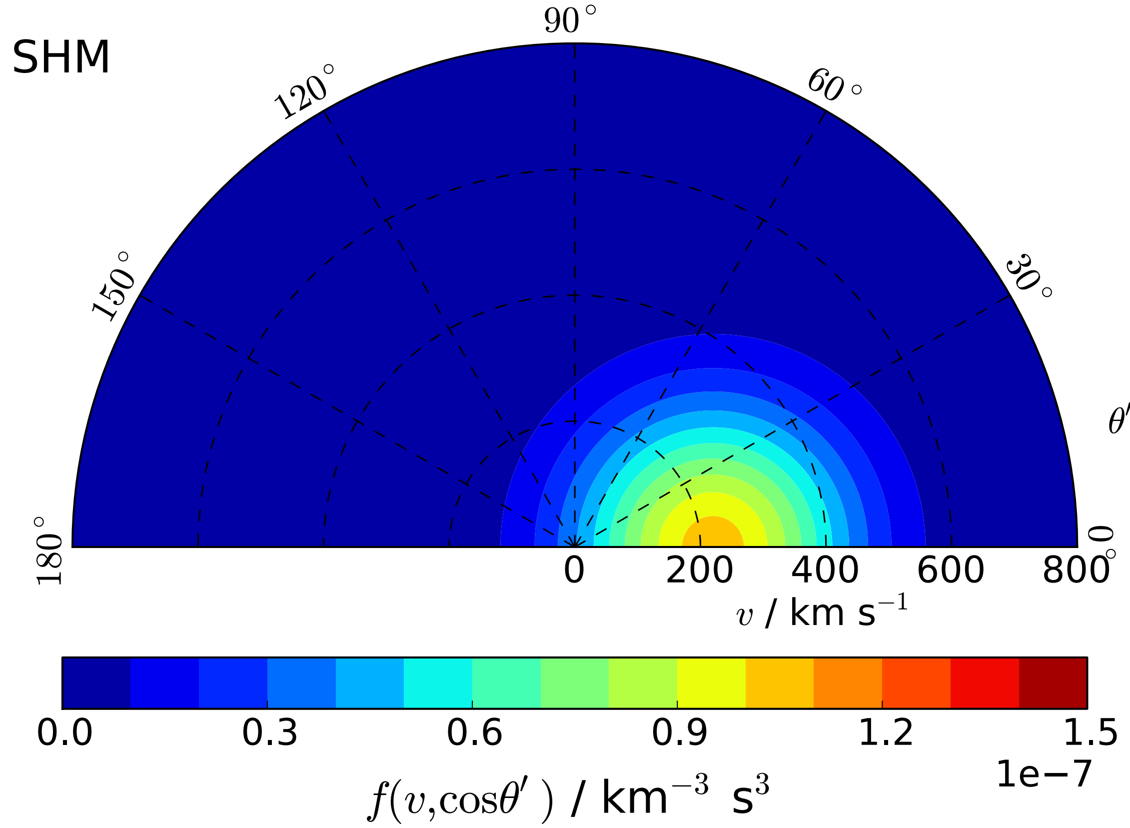
Divide the velocity distribution into N angular bins:

$$f(\mathbf{v}) = f(v, \cos\theta', \phi') = \begin{cases} f^1(v) & \text{for } \theta' \in [0, \pi/N] \\ f^2(v) & \text{for } \theta' \in [\pi/N, 2\pi/N] \\ & \vdots \\ f^k(v) & \text{for } \theta' \in [(k-1)\pi/N, k\pi/N] \\ & \vdots \\ f^N(v) & \text{for } \theta' \in [(N-1)\pi/N, \pi] \end{cases}$$

…and then we can parametrise $f^k(v)$ within each angular bin.

In principle, we could also discretise in ϕ', but assuming $f(\mathbf{v})$ is independent of ϕ' does not introduce any error.

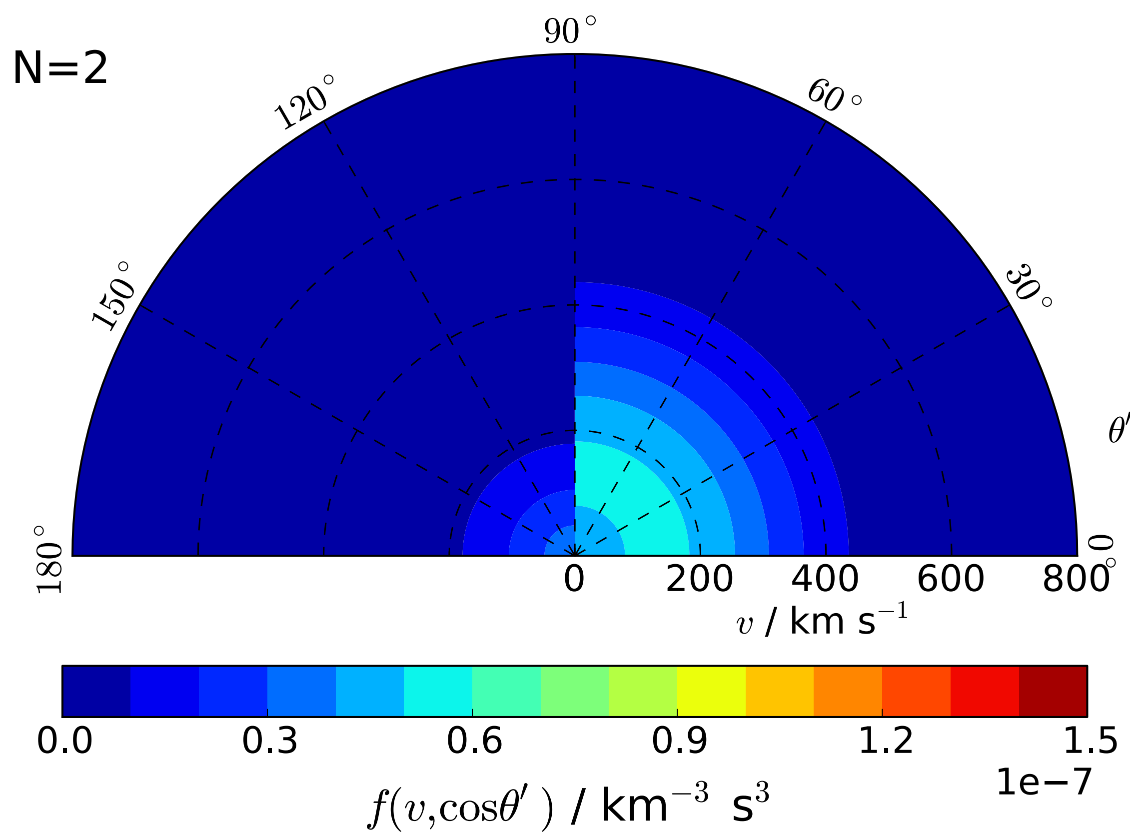
Example: SHM

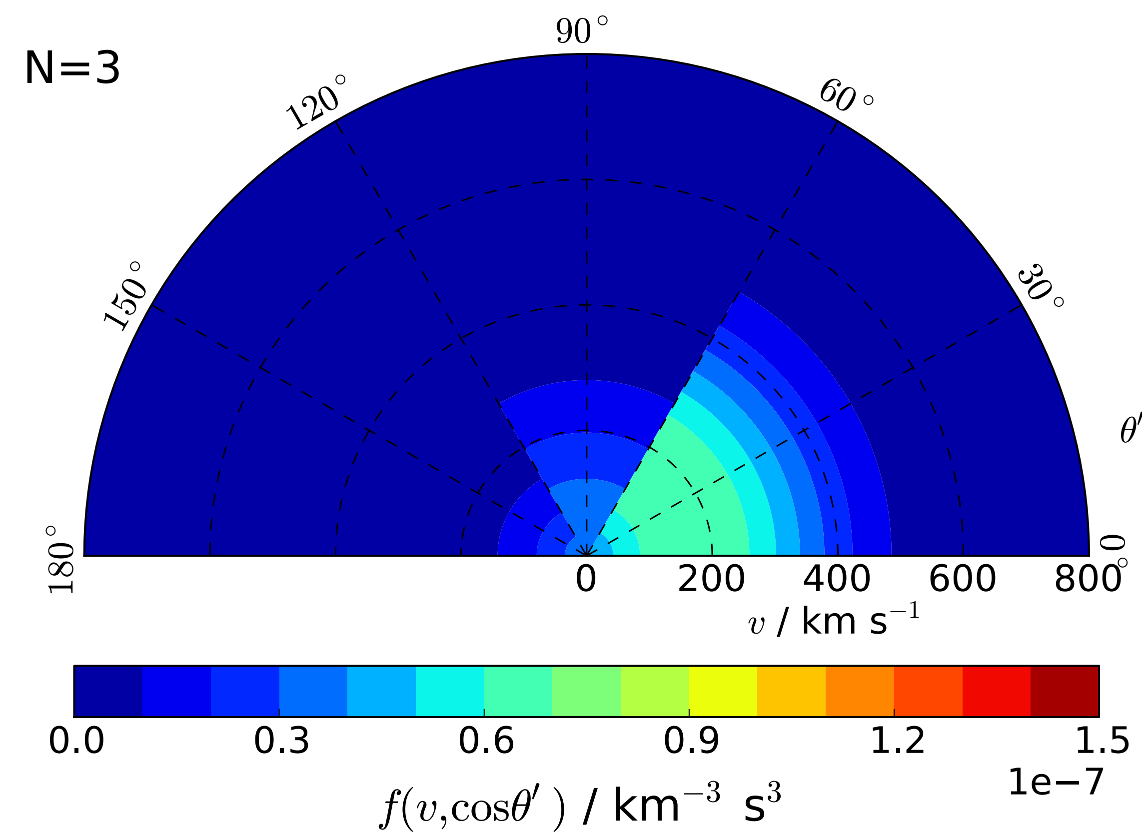


$f(\mathbf{v})$

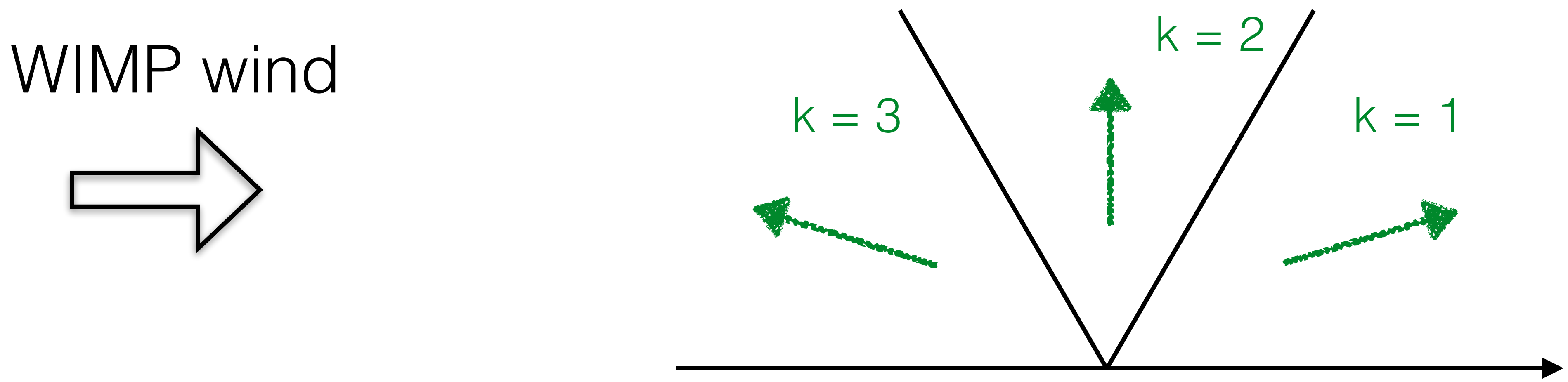
$$v_{\rm lag} = 220\ {\rm km\,s^{-1}}$$

$$\sigma_v = 156\ {\rm km\,s^{-1}}$$

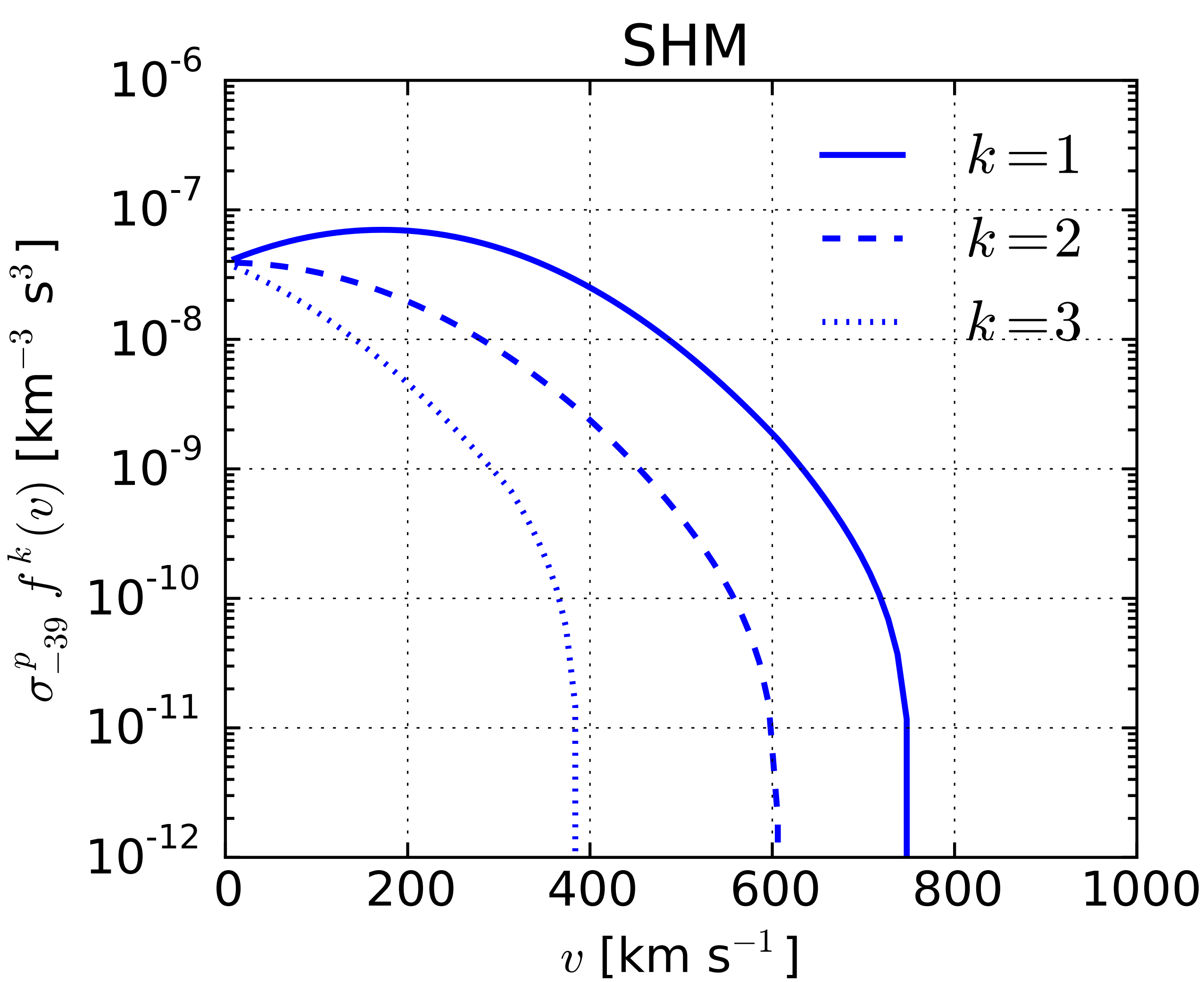




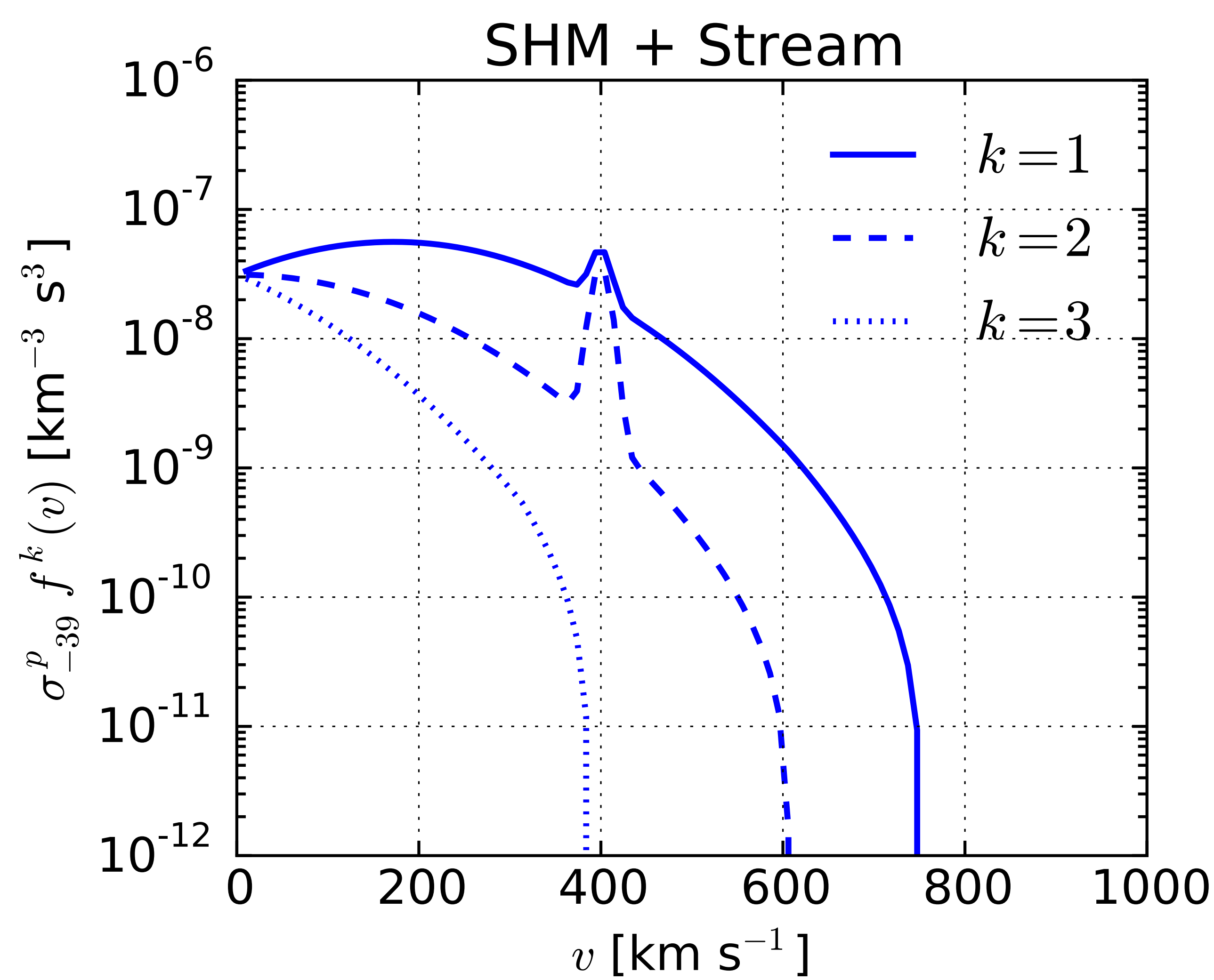
Examples: N = 3



WIMP wind
k = 3
k = 2
k = 1



SHM
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
0
200
400
600
800
1000
$k=1$
$k=2$
$k=3$
$\sigma^p_{-39}\, f^k(v)$ [km^{-3} s^3]
v [km s^{-1}]



SHM + Stream
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
0
200
400
600
800
1000
$k=1$
$k=2$
$k=3$
$\sigma^p_{-39}\, f^k(v)$ [km^{-3} s^3]
v [km s^{-1}]

Binned event rate

We want to try and calculate the event rate, binned in the same angular bins.

Need to calculate the integrated Radon Transform (IRT):

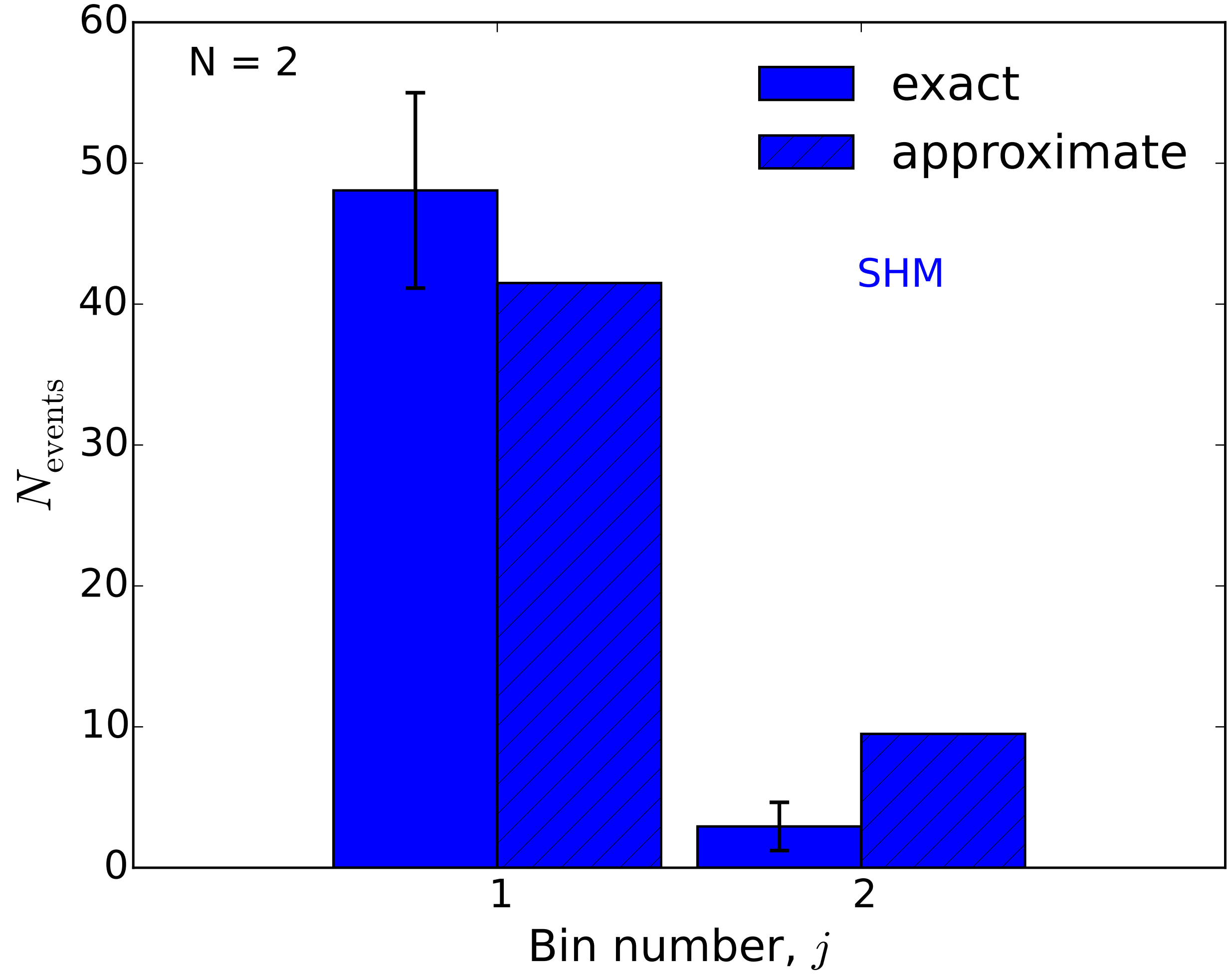
$$\hat{f}^j(v_{\mathrm{min}}) = \int_{\phi=0}^{2\pi} \int_{\cos(j\pi/N)}^{\cos((j-1)\pi/N)} \hat{f}(v_{\mathrm{min}}, \hat{\mathbf{q}}) \, \mathrm{d}\cos\theta \mathrm{d}\phi \,,$$

The calculation of the Radon Transform is rather involved, but it can be carried out analytically in the angular variables for an arbitrary number of bins N, and reduced to N integrations over the speed v .

BJK [1502.04224]

So how well does this 'approximation' work?

Event numbers



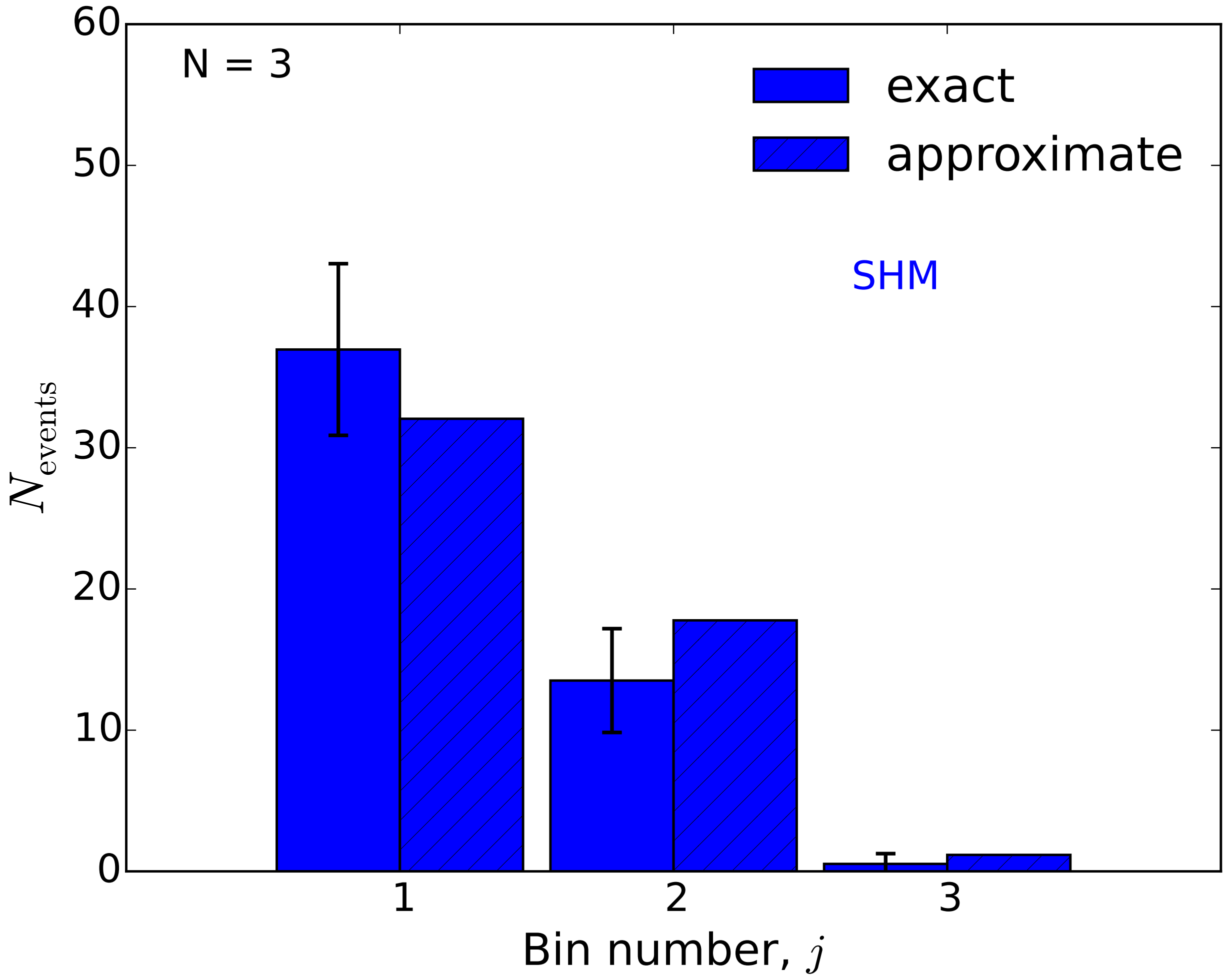
CF$_4$ detector

$E_{\rm th} = 20$ keV

$m_\chi = 100$ GeV

$N_S = 50$

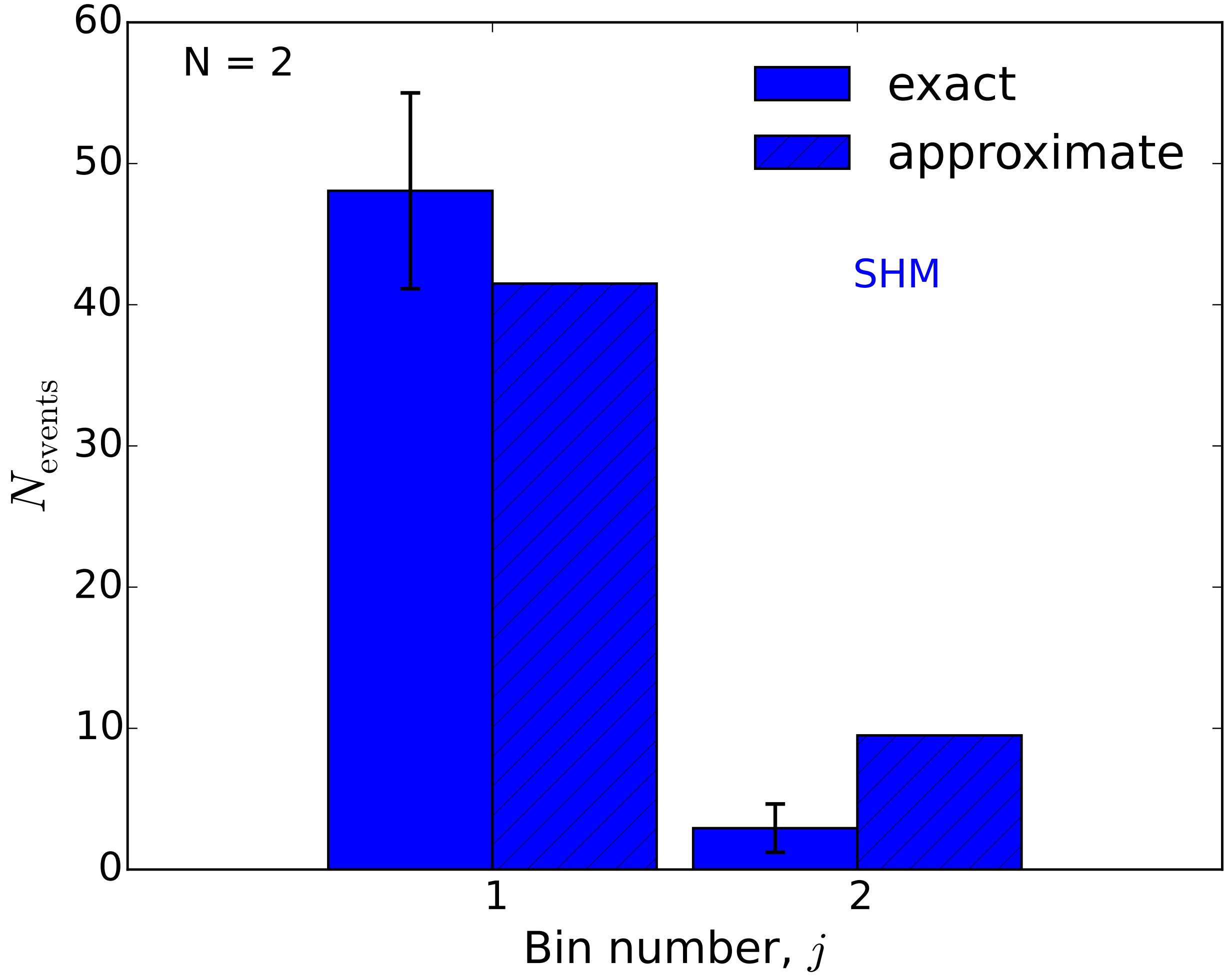
$N_{BG} = 1$



N_j

total number of events expected in each angular bin

Event numbers



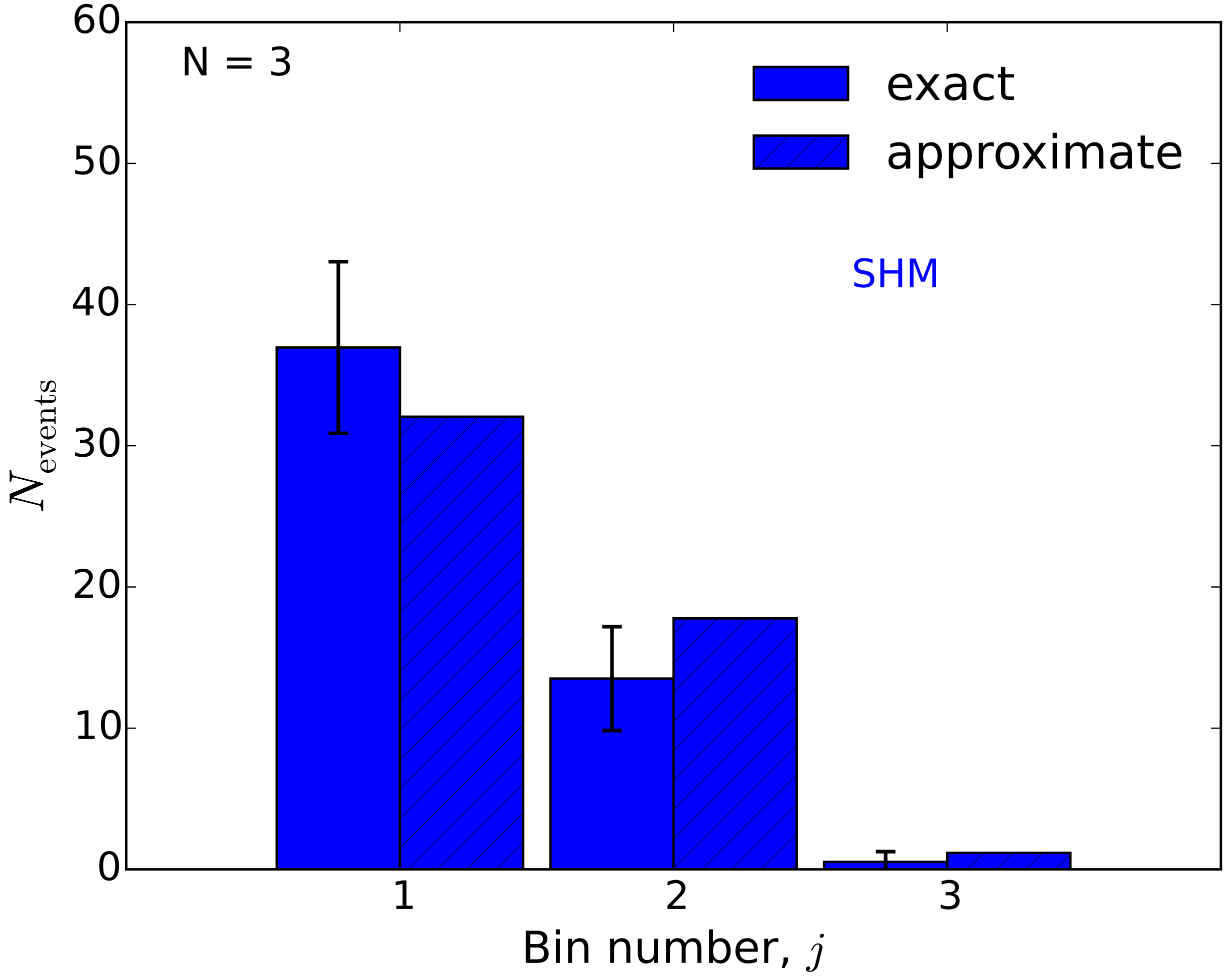
CF$_4$ detector

$E_{\mathrm{th}} = 20$ keV

$m_\chi = 100$ GeV

$N_S = 50$

$N_{BG} = 1$



Could keep increasing N!

For now, try **N = 3** angular bins

Procedure

Bin the data in each experiment, depending on the direction of the recoil, into N = 3 bins

Simultaneously fit (m_χ, σ^p), and N = 3 sets of $\{a_k\}$ describing the speed distribution in each angular bin

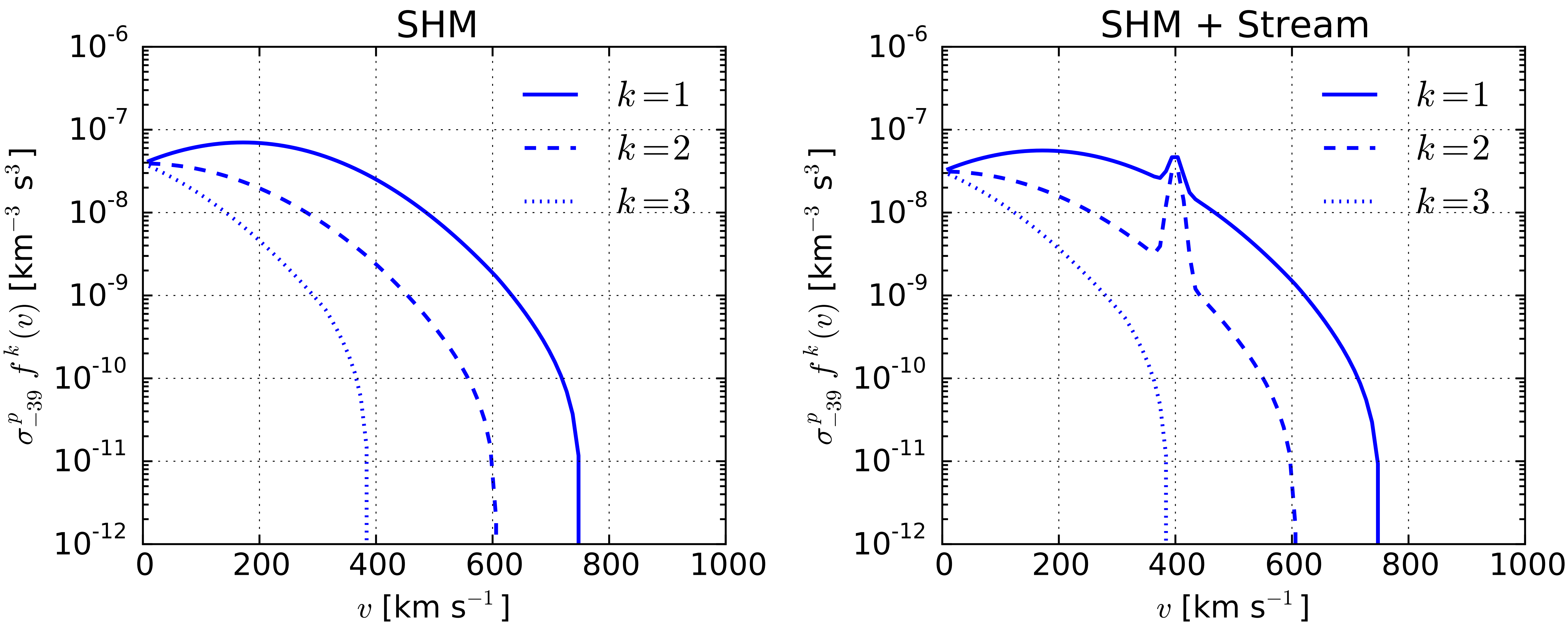
If an experiment is not directionally sensitive, just sum the three speed distributions to get the total

We'll use 4 terms to describe each of the 3 speed distributions. Some are fixed by normalisation, giving a total of 11 parameters for the fit.

Directional reconstructions

PRELIMINARY

Benchmarks



Xe detector

$E_{\rm th} = 5$ keV

1000 kg yr

~ 900 events

Mohlabeng et al. [1503.03937]

Mock data from 2 ideal experiments

Consider with and without directionality

F detector

$E_{\rm th} = 20$ keV

10 kg yr

~ 50 events

DRIFT [1010.3027]

Reconstructions

Best Case

Assume underlying velocity distribution is known exactly.

Fit $m_\chi,\ \sigma_p$

Reasonable Case

Assume functional form of underlying velocity distribution is known.

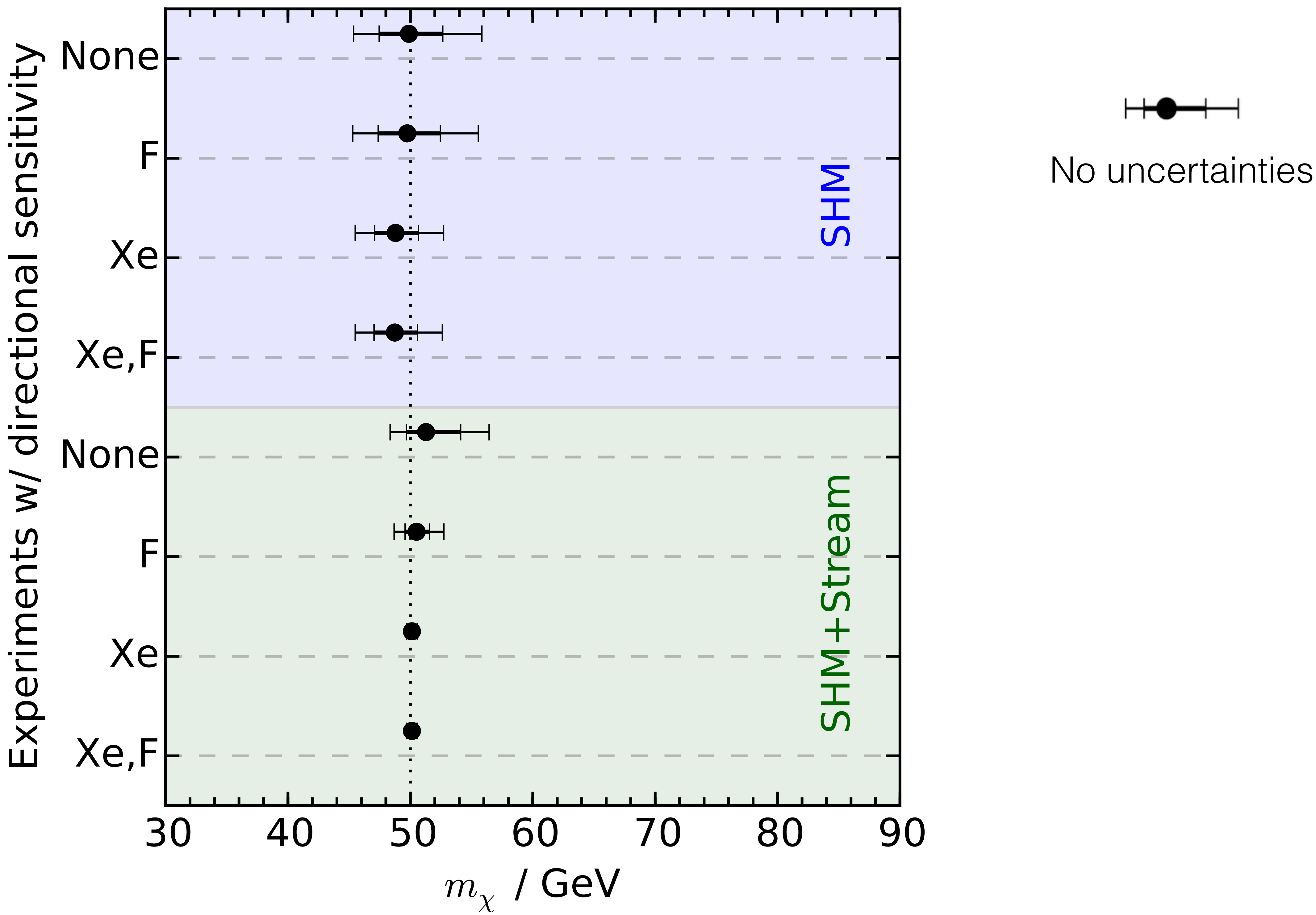
Fit $m_\chi,\ \sigma_p$ and theoretical parameters of f(v)

Worst Case

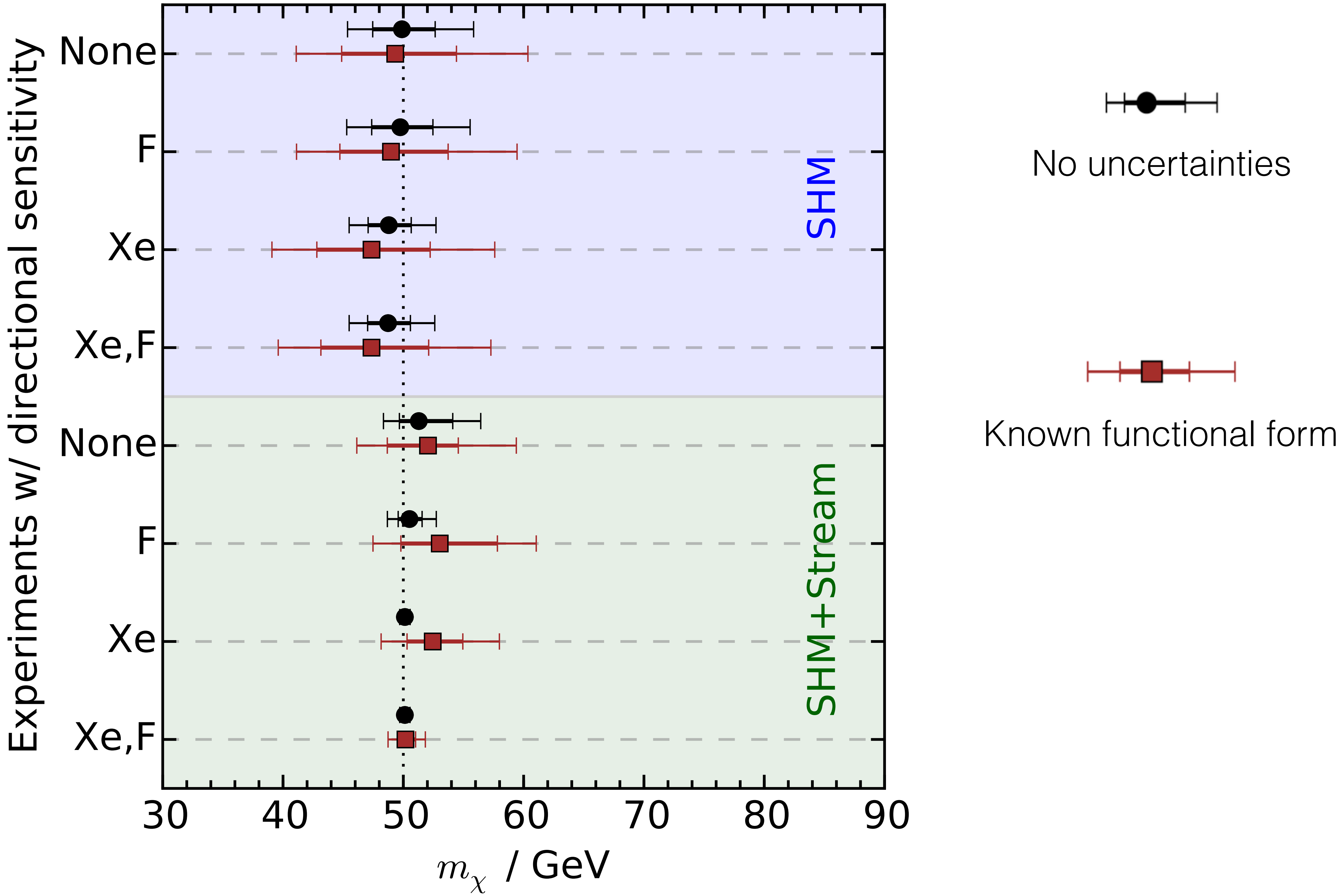
Assume nothing about the underlying velocity distribution.

Fit $m_\chi,\ \sigma_p$ and empirical parameters of f(v)

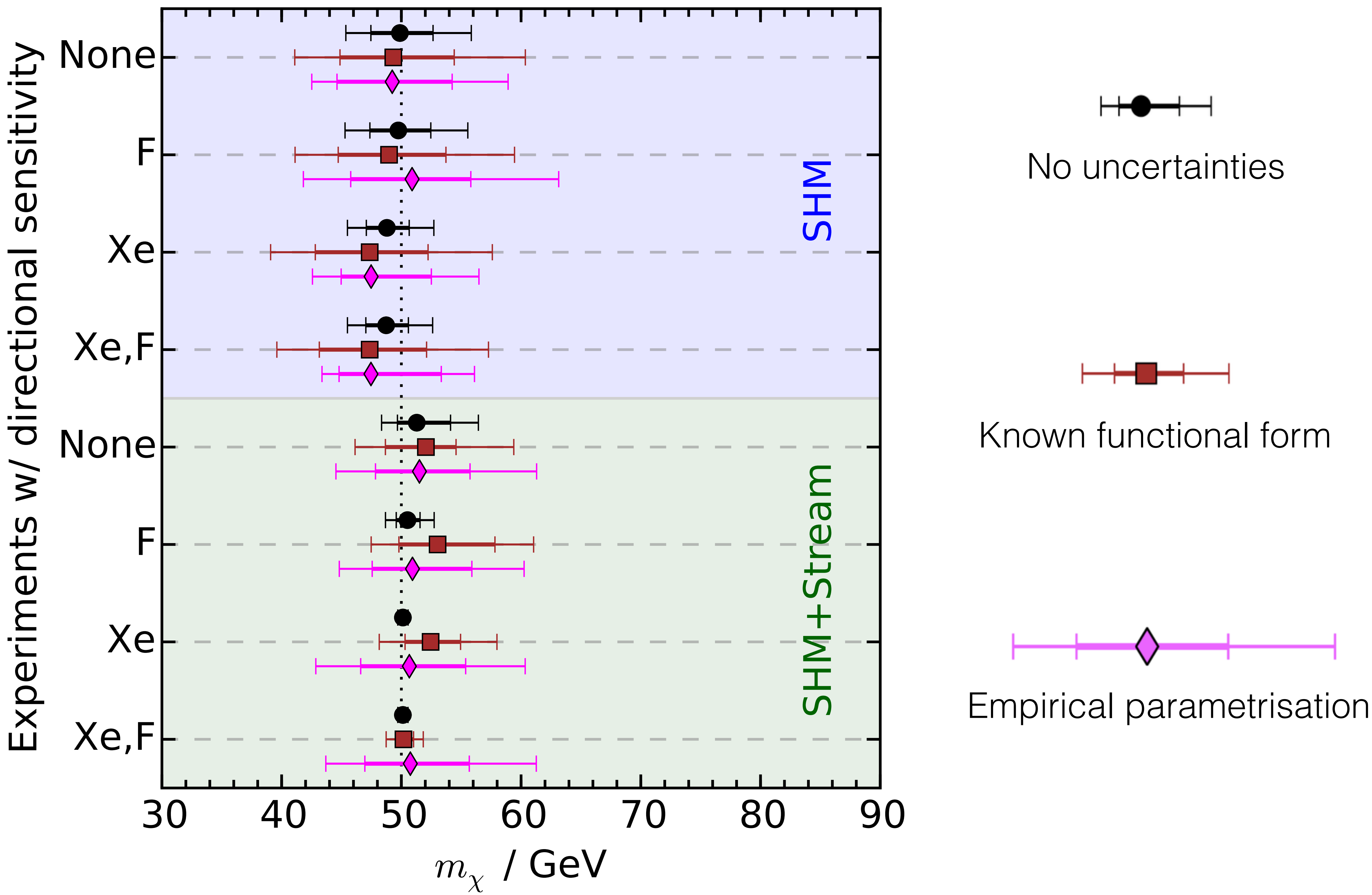
Reconstructing the DM mass



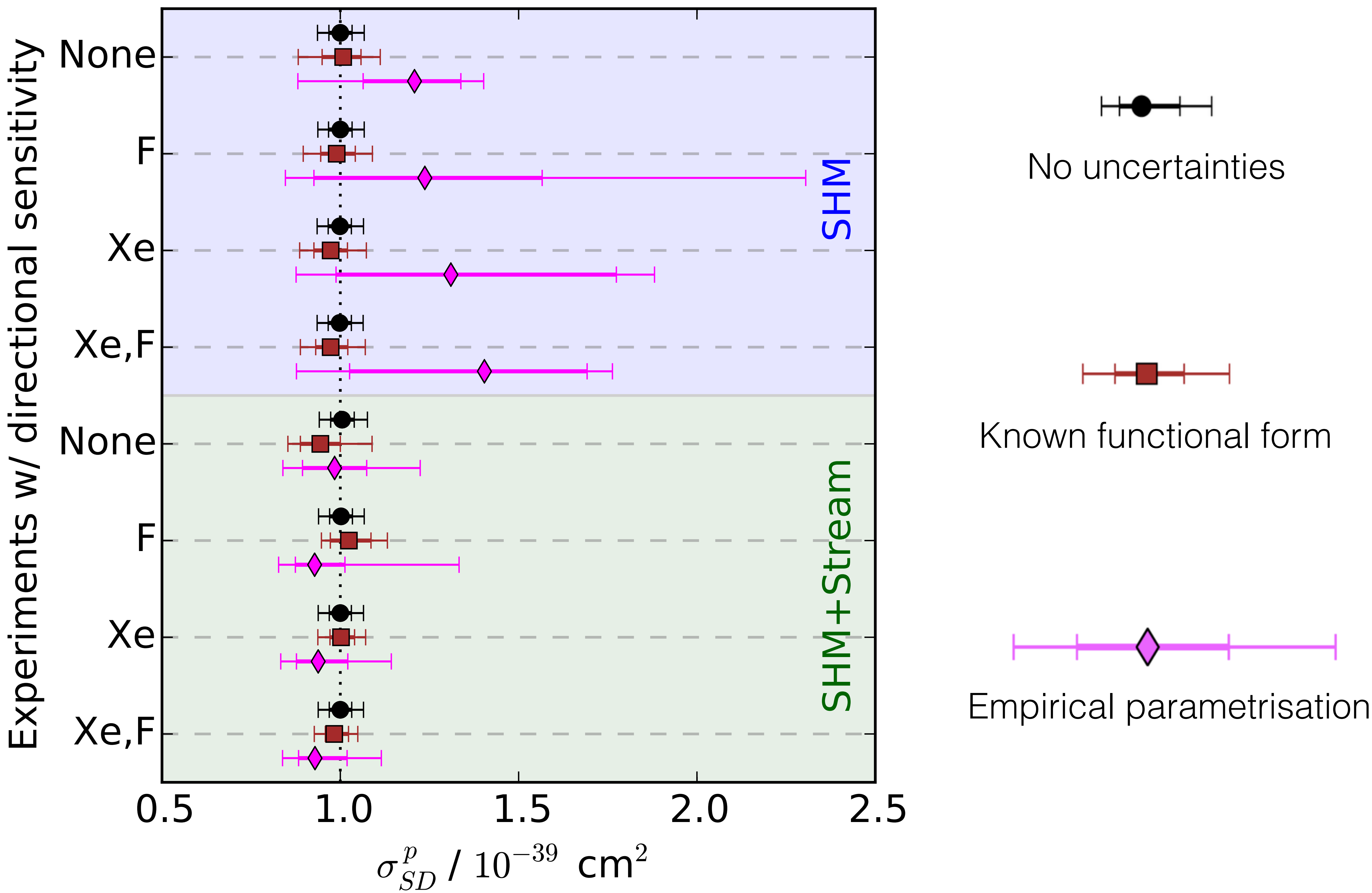
Reconstructing the DM mass



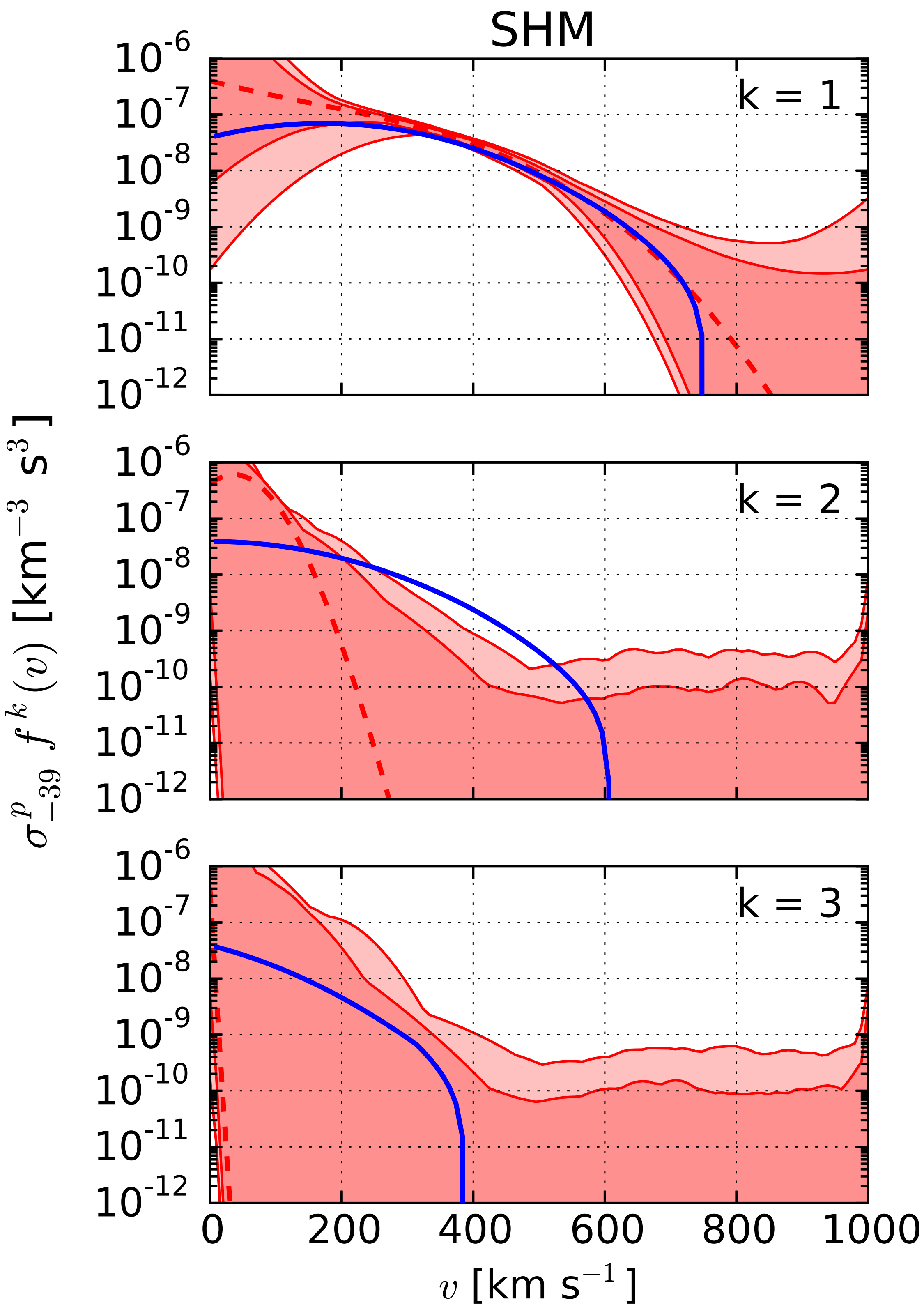
Reconstructing the DM mass



Reconstructing the DM cross section



Reconstructing the velocity distribution



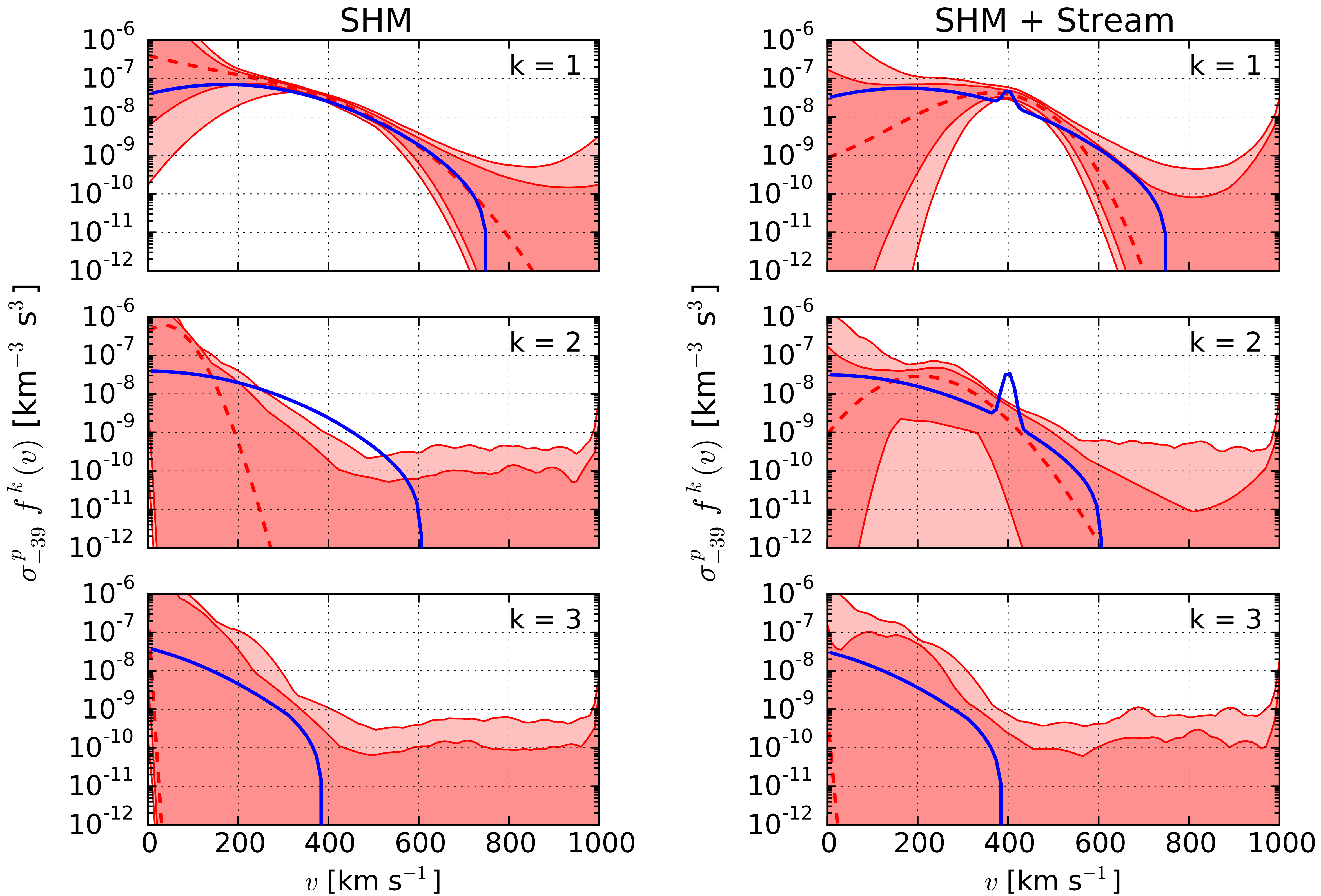
True velocity distribution

Best fit distribution

(+68% and 95% intervals)

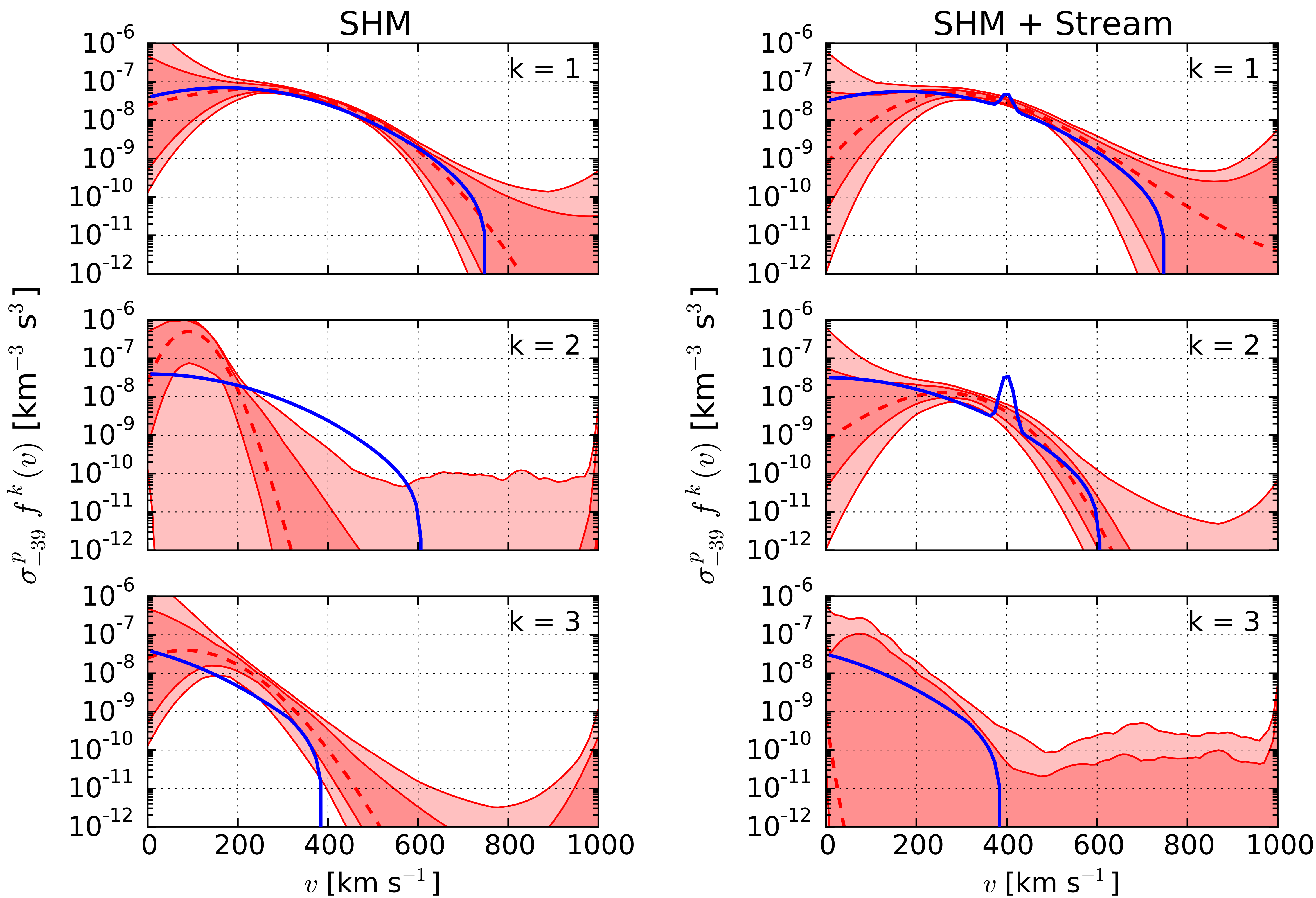
Directional F and non-directional Xe

Reconstructing the velocity distribution



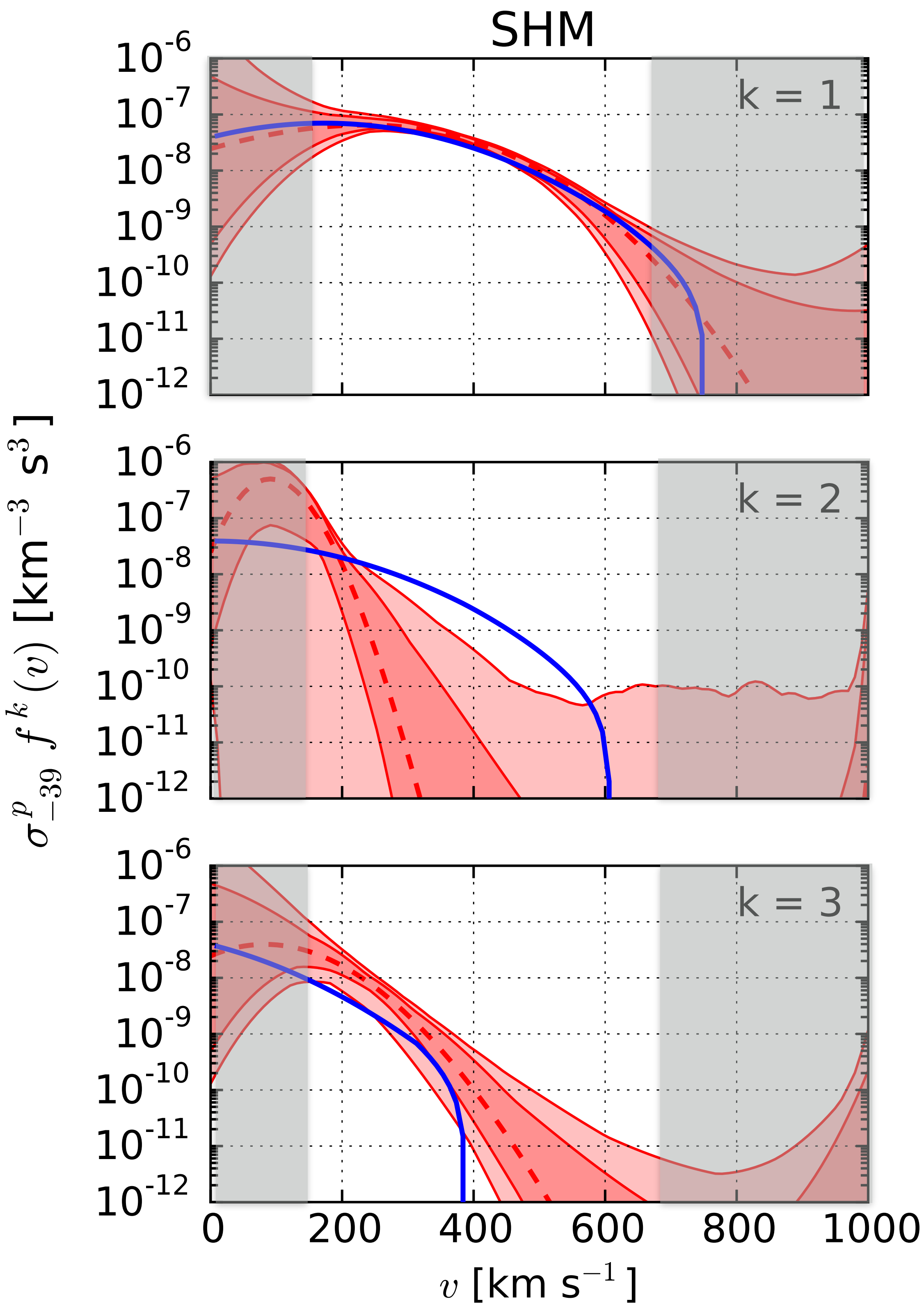
Directional F and non-directional Xe

Reconstructing the velocity distribution



Directional F and Directional Xe

Caveats



Only sensitive to speeds inside the energy window of the detector

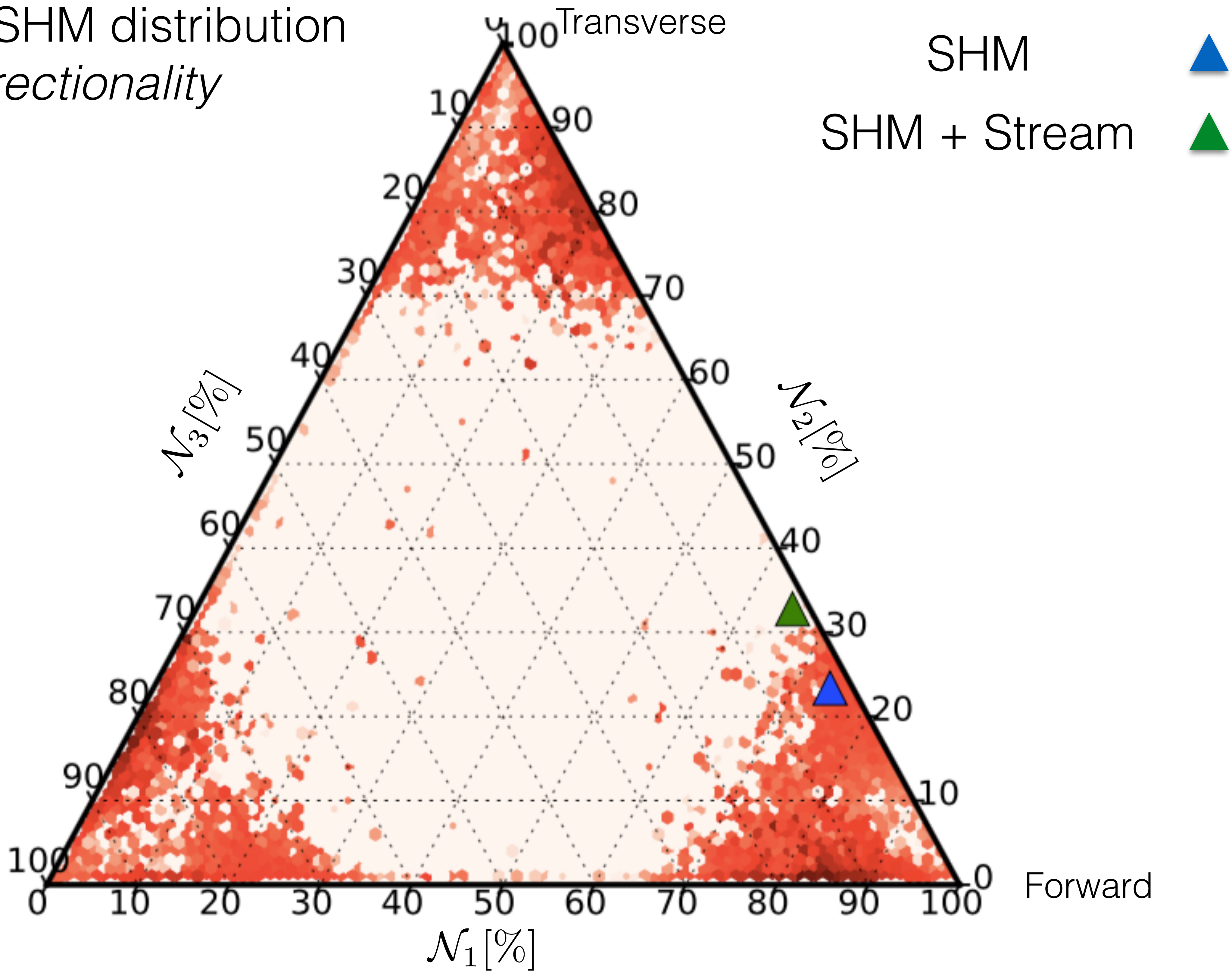
We don't know the true cross section (or local DM density) in advance, difficult to compare with a given velocity distribution

Fraction of DM particles in each angular bin is less sensitive to changes in overall normalisation

Use directionality of f(v) as a discriminator between different distributions

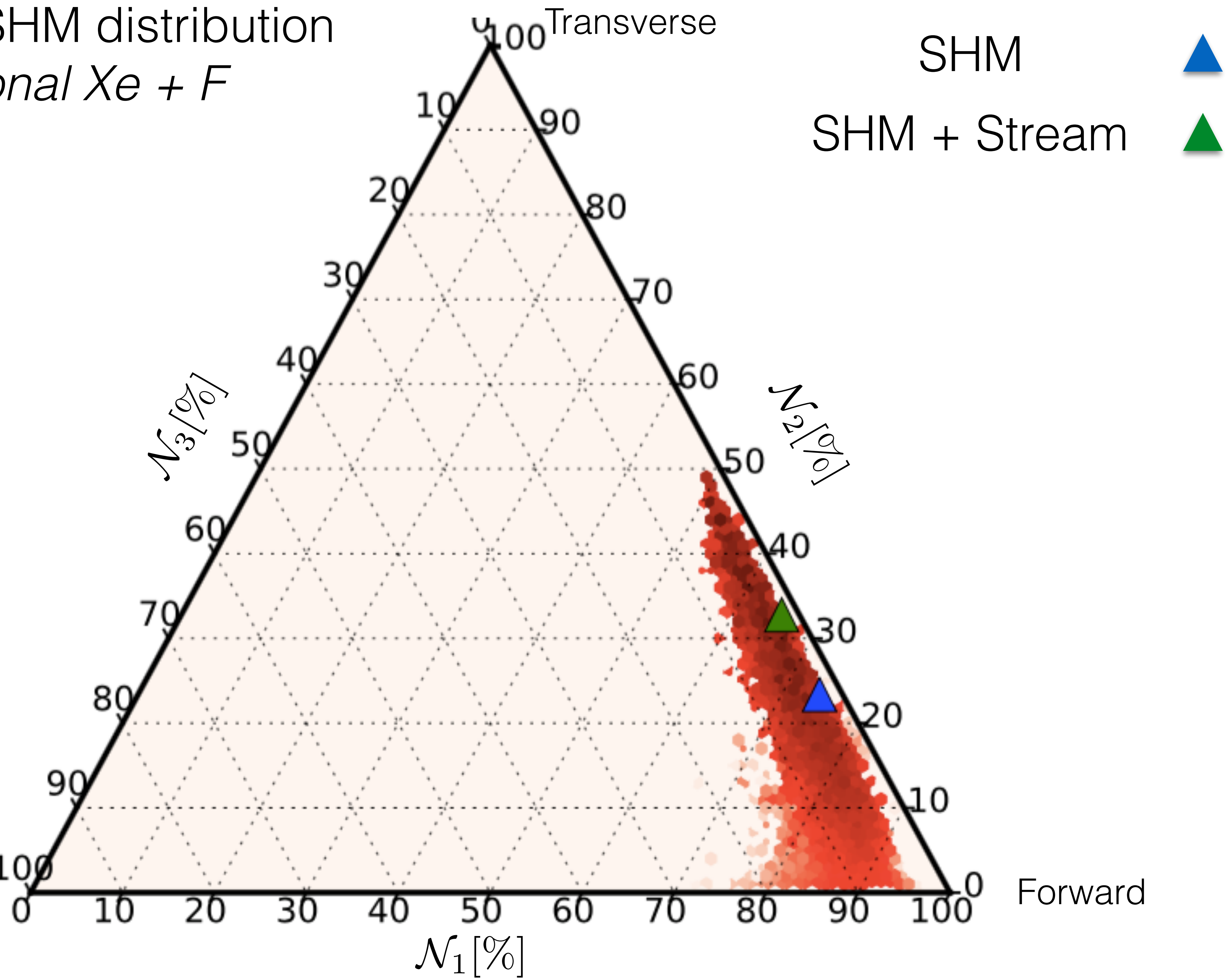
Distinguishing distributions

Underlying SHM distribution
No directionality



Distinguishing distributions

Underlying SHM distribution
Directional Xe + F



The strategy

In case of signal break glass

Perform parameter estimation using two methods:
'known' functional form vs. empirical parametrisation

Compare reconstructed parameters

Estimate fraction of DM particles in each angular bin

Check for consistency with SHM

In case of inconsistency, look at reconstructed shape of f(v)

Hint towards unexpected structure?

Conclusions

Astrophysical uncertainties are a problem for parameter estimation in direct detection

Use halo-independent, general parametrisation

This can be extended to directional detection (with angular binning)

Naturally account for angular resolution

Doesn't spoil the reconstruction of the DM mass

But lose information about cross section

May allow us to distinguish different velocity distributions (and tell us something about the Milky Way)

Much harder to do without directionality

Conclusions

Astrophysical uncertainties are a problem for parameter estimation in direct detection

→ Use halo-independent, general parametrisation

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Doesn't spoil the reconstruction of the DM mass

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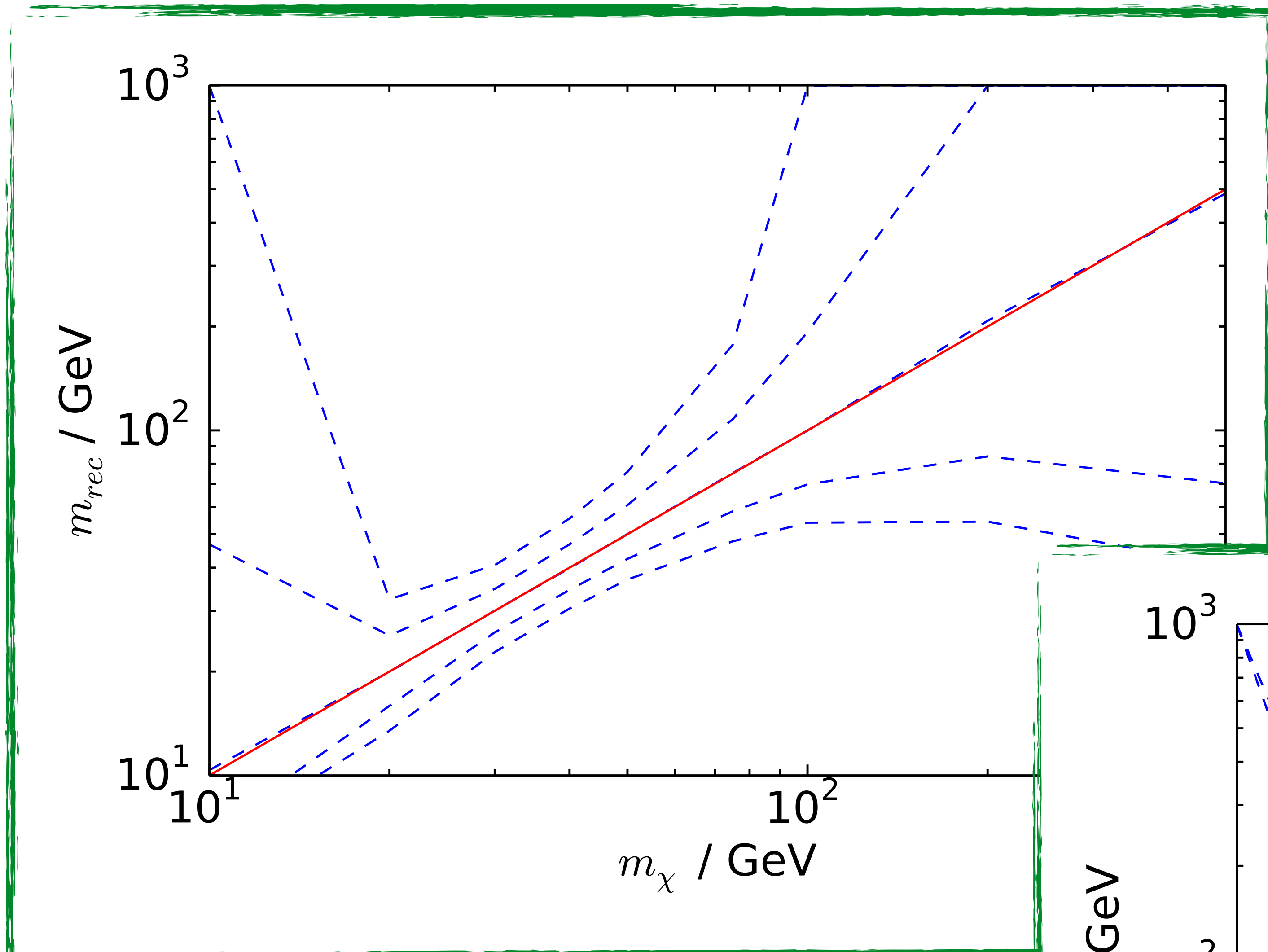
→ Much harder to do without directionality

Thank you

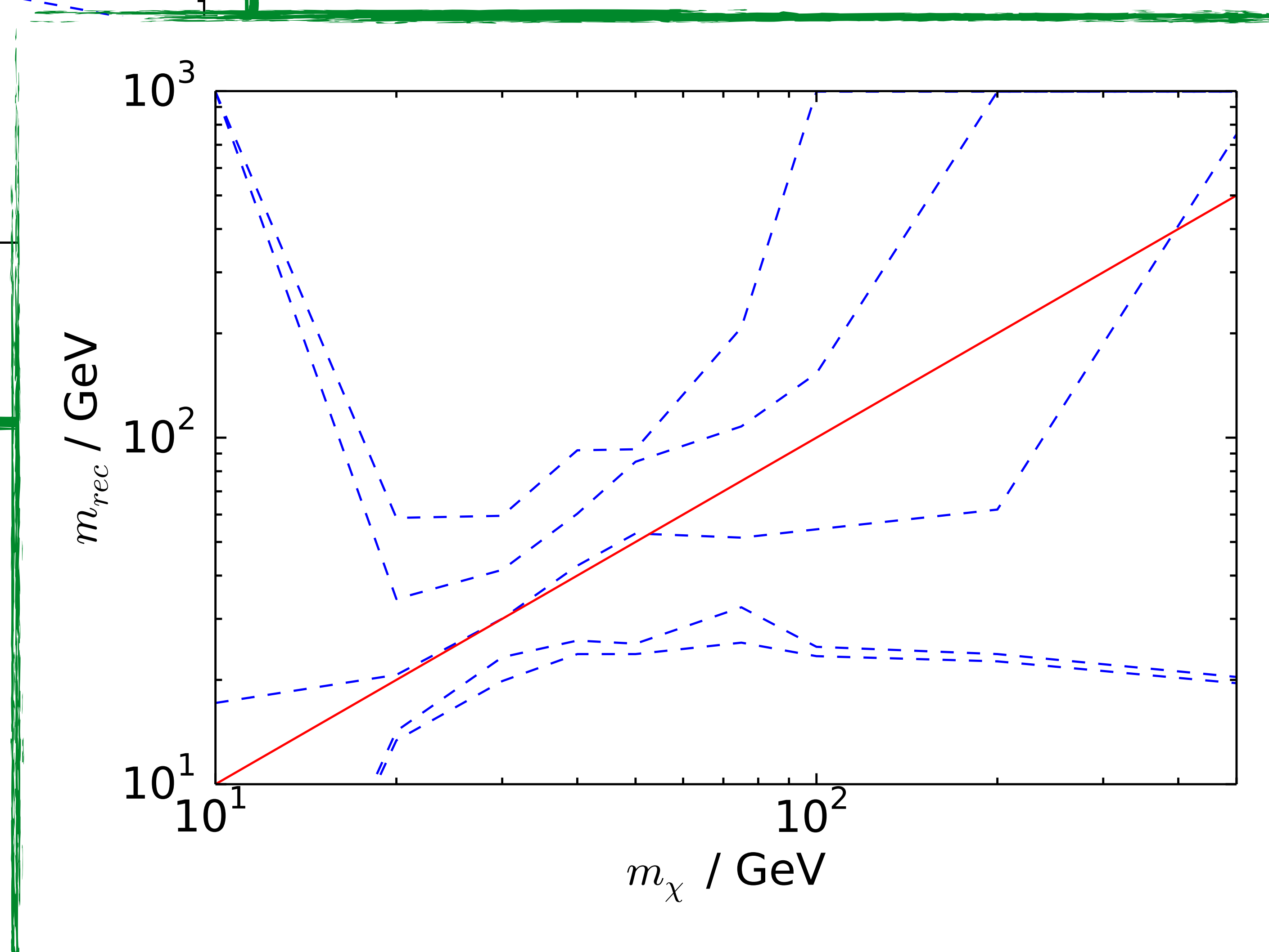
Backup Slides

Reconstructing the mass (1-D)



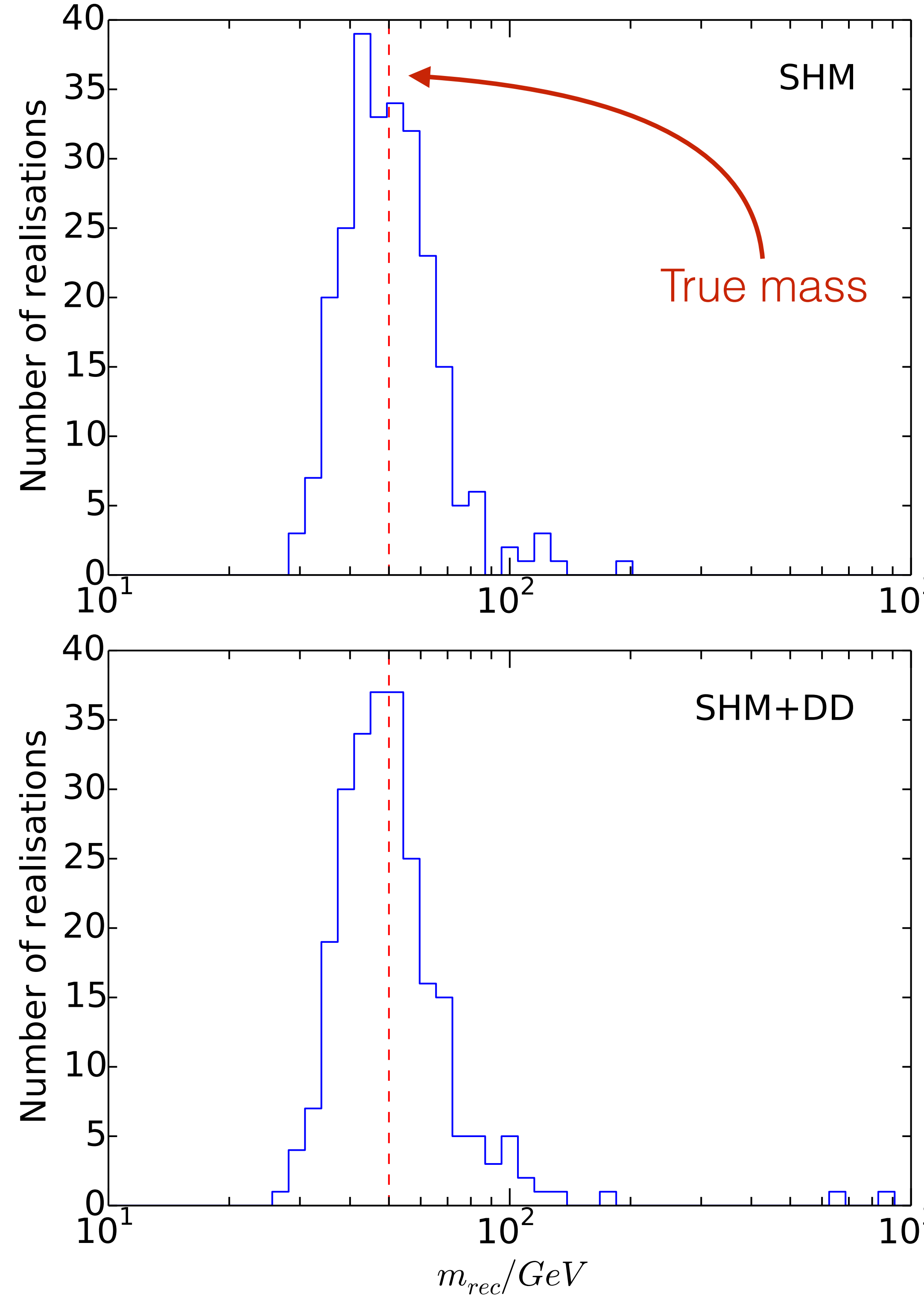
Ideal experiments

'Real' experiments



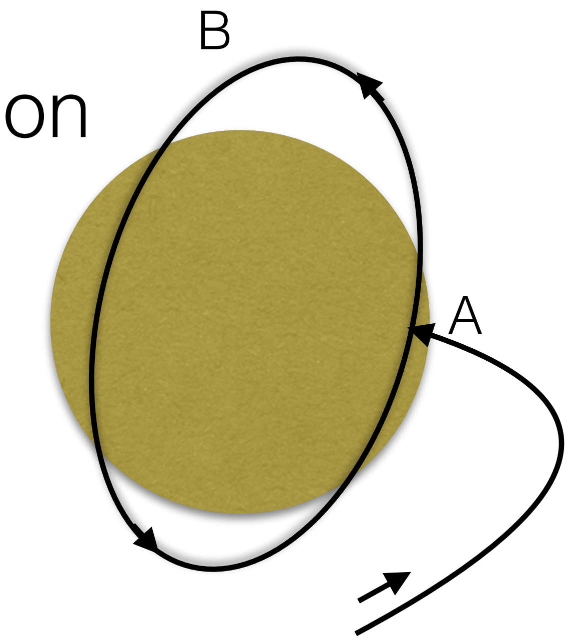
Different speed distributions (1-D)

- Generate 250 mock data sets
- Reconstruct mass and obtain confidence intervals for each data set
- True mass reconstructed well (independent of speed distribution)
- Can also check that 68% intervals *are really 68% intervals*



Incorporating IceCube

IceCube can detect neutrinos from DM annihilation in the Sun

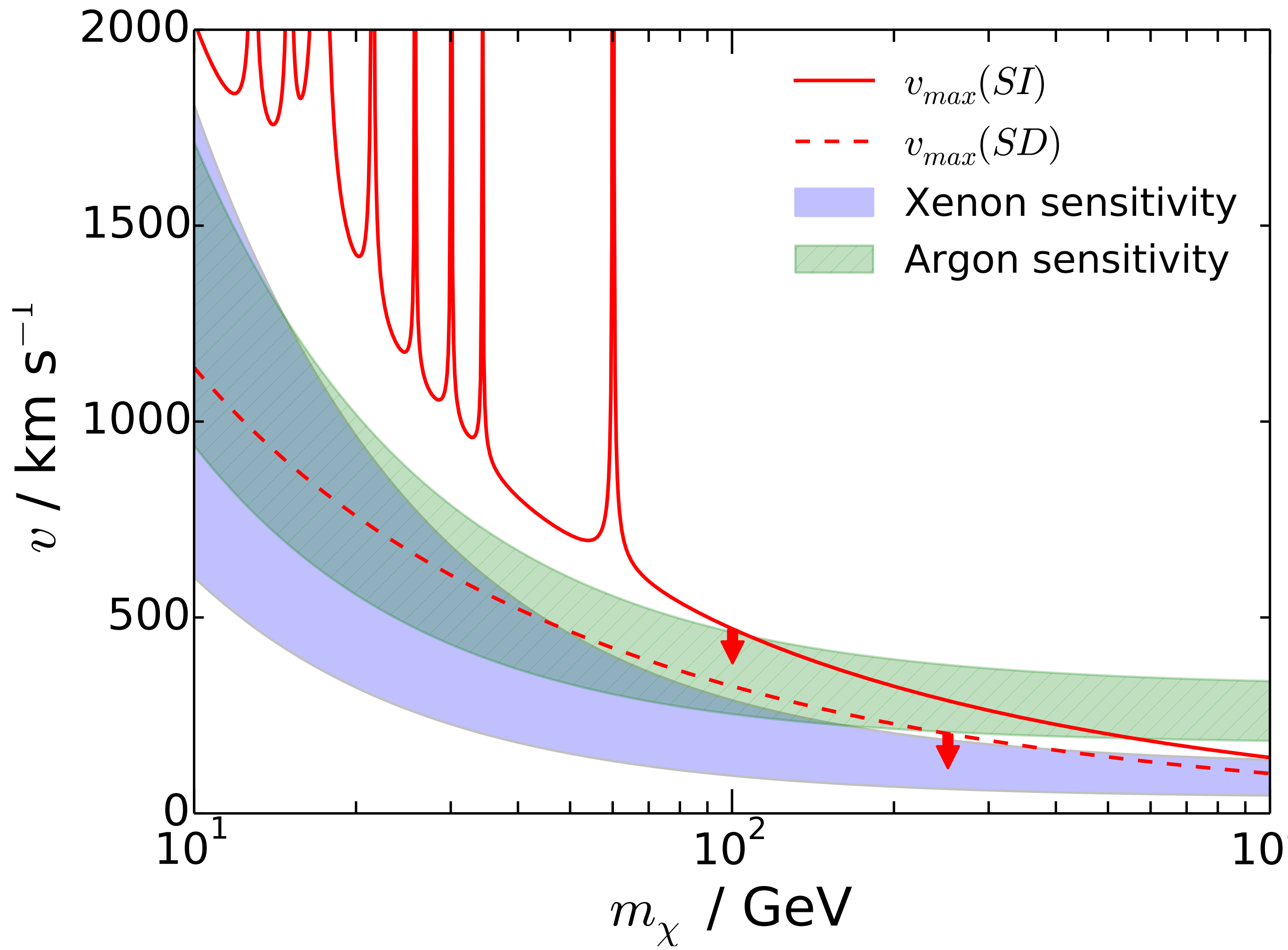


Rate driven by solar capture of DM, which depends on the DM-nucleus scattering cross section

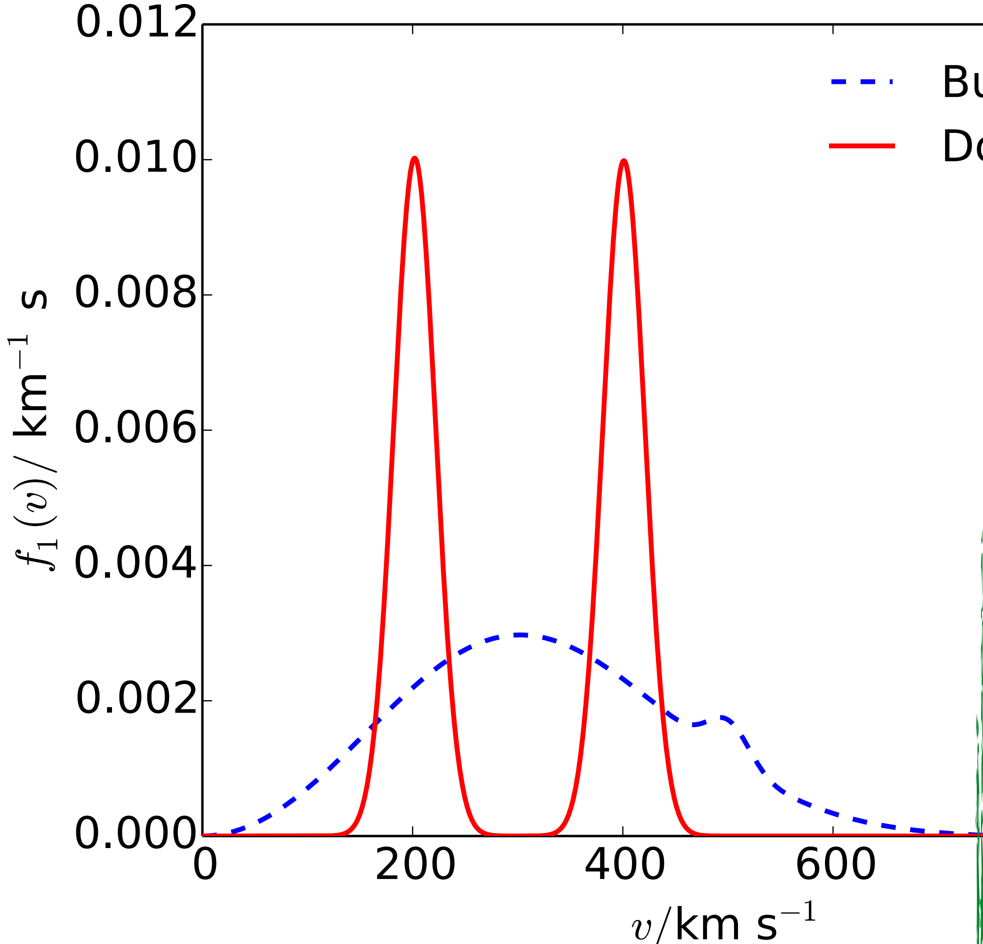
Crucially, only low energy DM particles are captured:

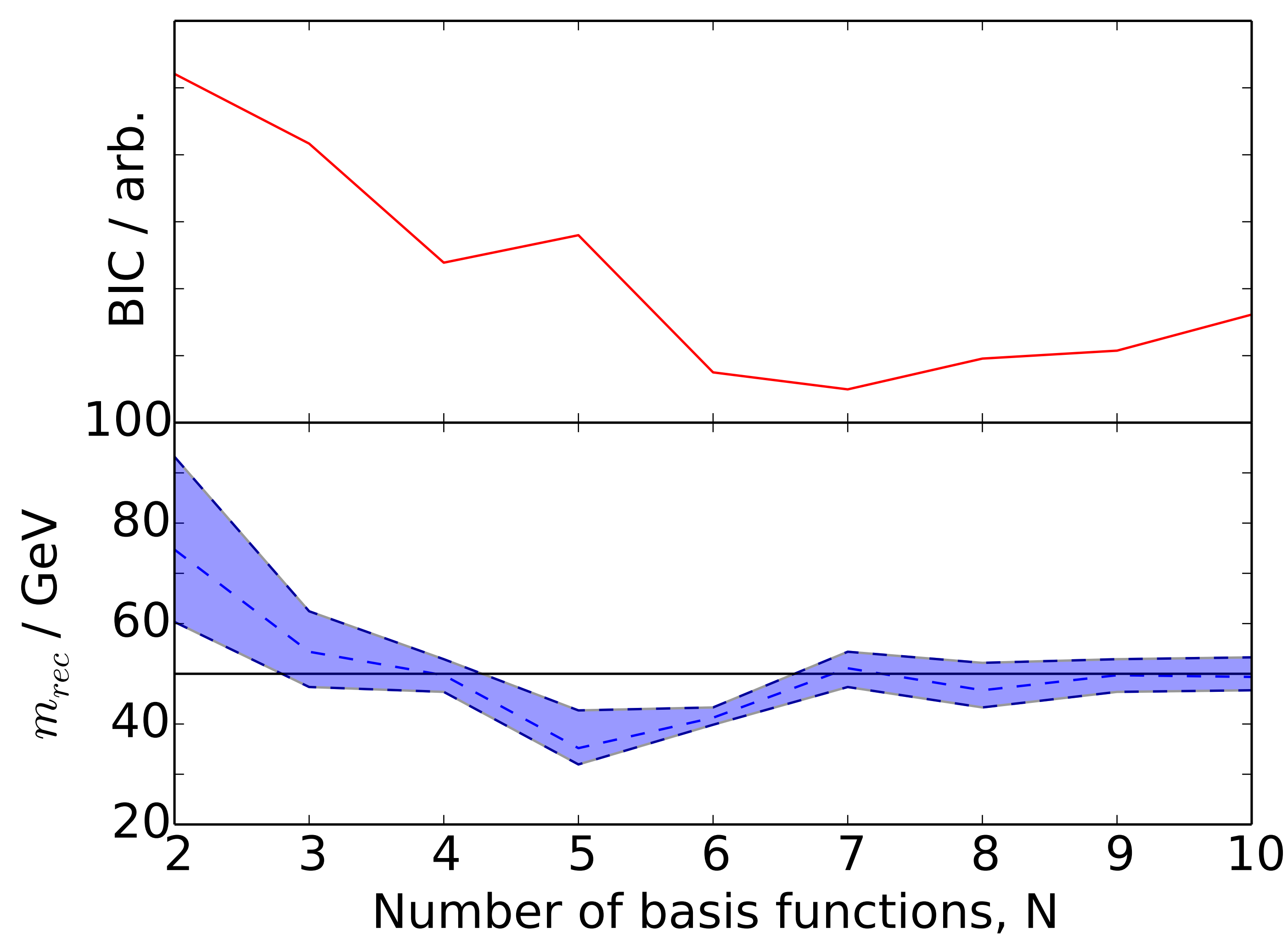
$$\frac{\mathrm{d}C}{\mathrm{d}V} \sim \sigma \int_0^{v_{\max}} \frac{f_1(v)}{v} \, \mathrm{d}v$$

But Sun is mainly spin-1/2 Hydrogen - so we need to include SD interactions…



How many terms do we need?



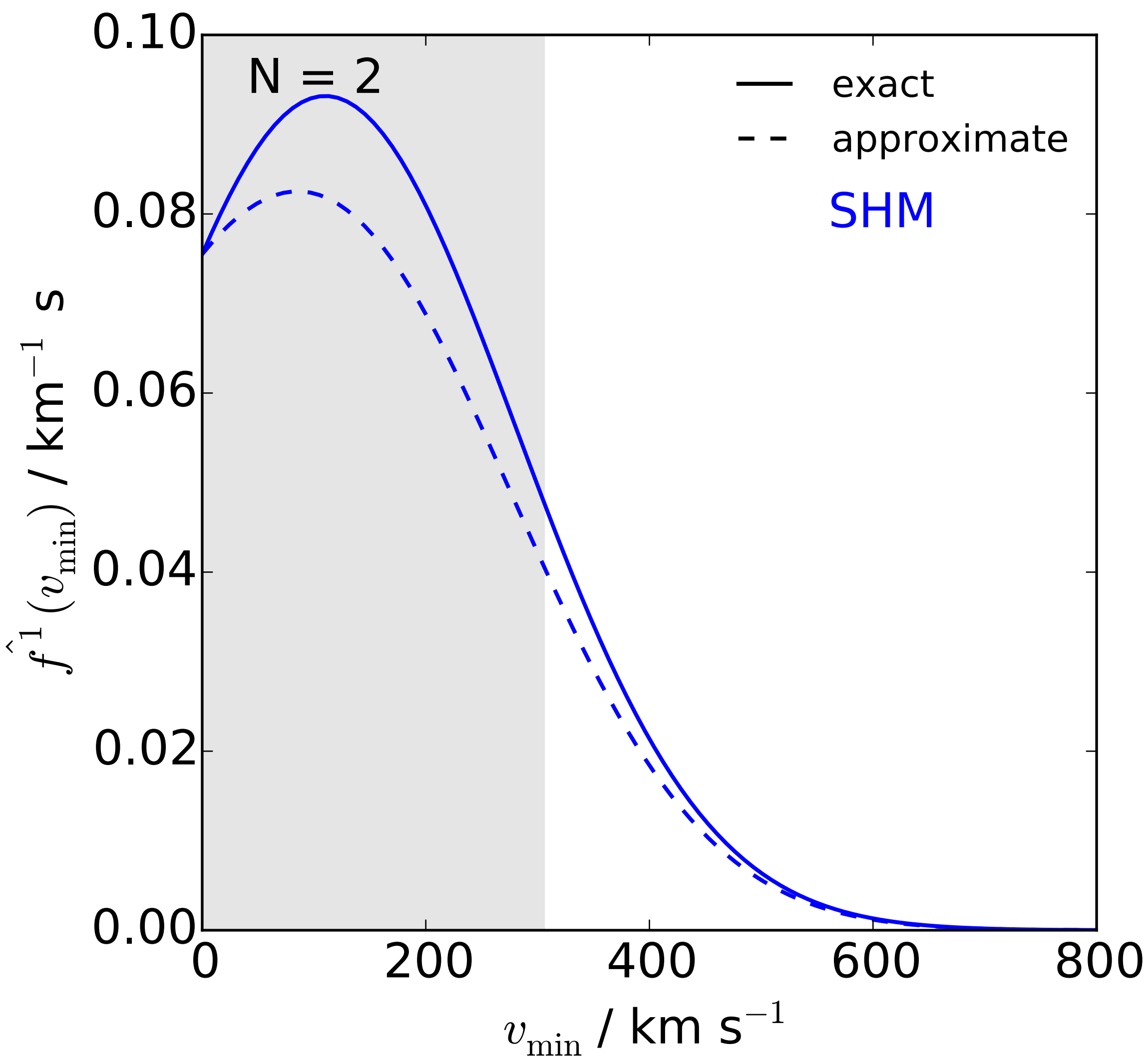


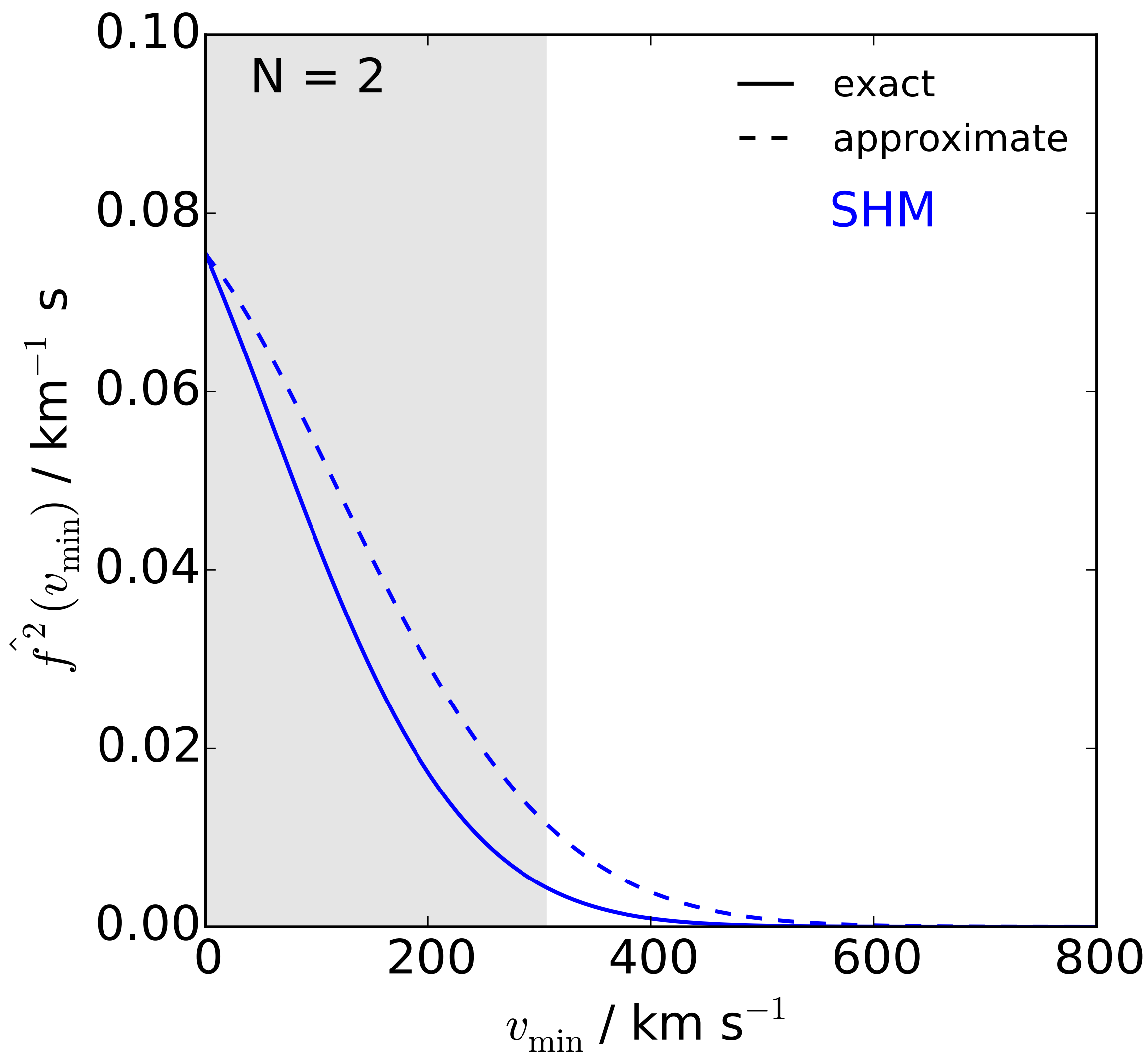
N = 2

Compare:

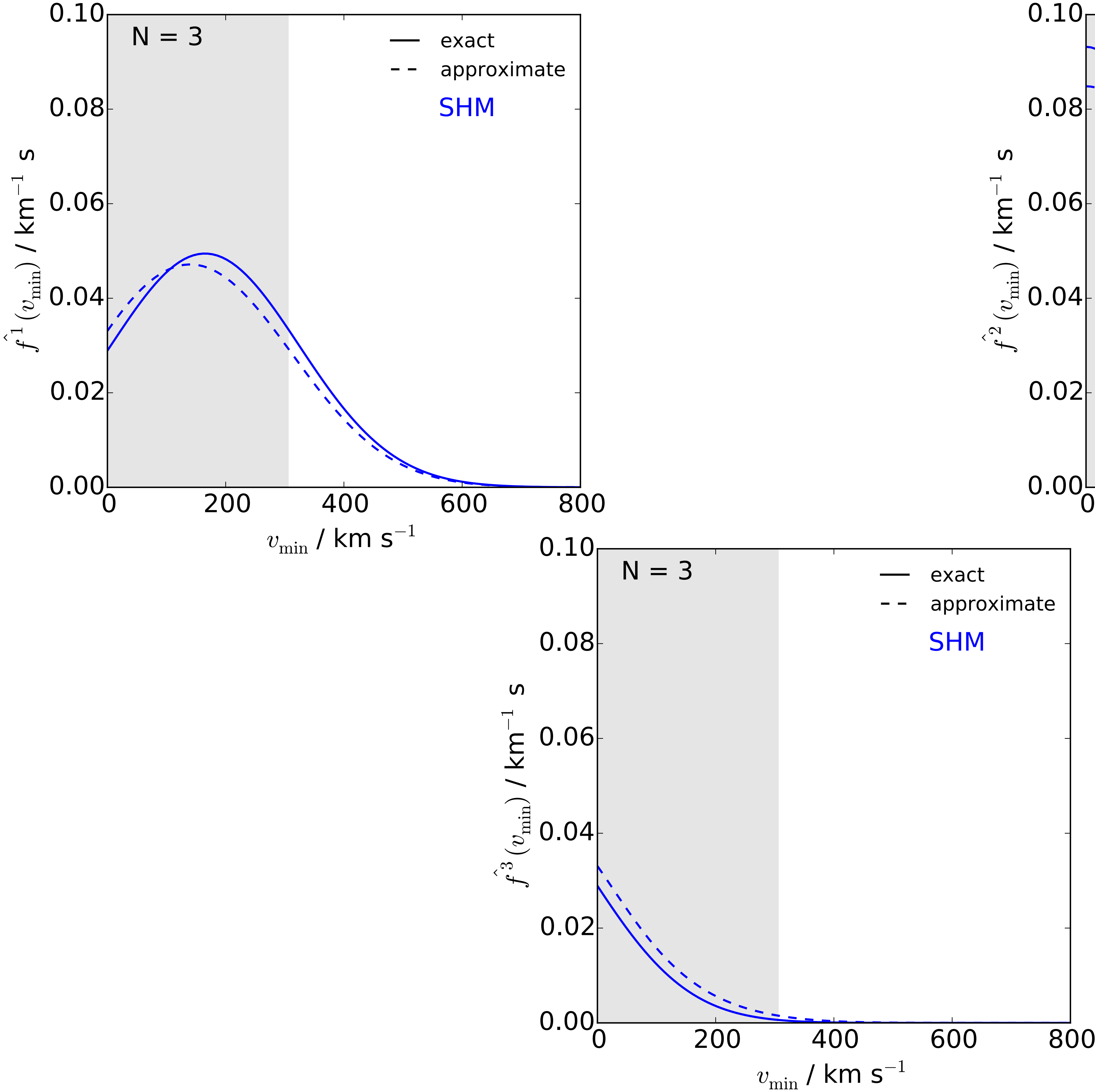
Exact IRT - calculated from the true, full distribution

Approx. IRT - calculated from discretised distribution





N = 3



N = 3
exact
approximate
SHM
$\hat{f}^1(v_{\rm min})$ / km^{-1} s
$\hat{f}^2(v_{\rm min})$ / km^{-1} s
$\hat{f}^3(v_{\rm min})$ / km^{-1} s
$v_{\rm min}$ / km s^{-1}
0.00
0.02
0.04
0.06
0.08
0.10
0
200
400
600
800

Number of angular bins