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The Bragg-like curve for the directional detection of dark matter

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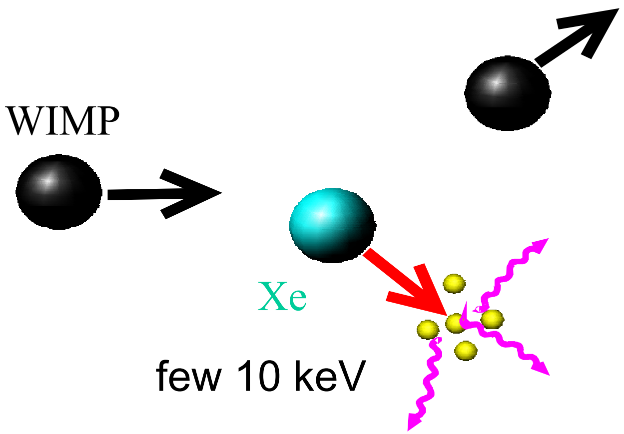
Head-tail discrimination
Treatment for $Z_1 \neq Z_2$
    Pb in α-decay, Ar-gas mixture
Binary gases CS_2

Simulating WIMP signals in a lab.



Neutron scattering experiments

$$E_{rec} = 2E_n \frac{A_n A_{Xe}}{(A_n + A_{Xe})^2}(1-\cos\theta)$$

$$A_{xe} = 131$$

$$E_{rec}^{max} = 0.030 E_n \qquad \text{at } \theta = \pi$$

F. Arneodo et al. / Nuclear Instruments and Methods in Physics Research A 449 (2000) 147–157

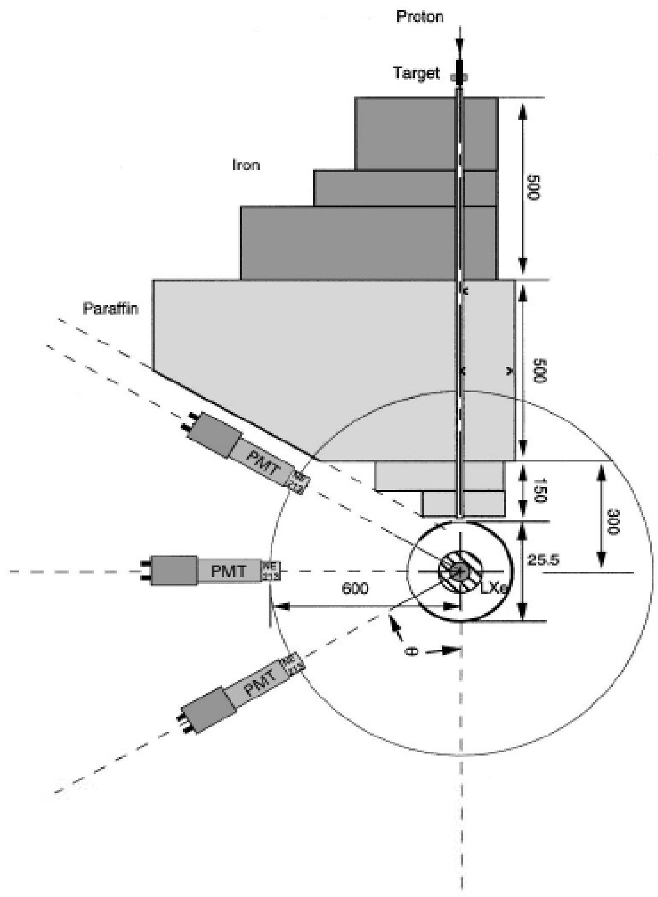


Fig. 3. Top view of the experimental setup at the LNL neutron beam line.

Head-tail discrimination

The Bragg-like curve for head-tail detection

The distribution of the electronic energy η deposited in the detector gas as a function of the ion depth, i.e. projected range R_{PRJ}.

$\Delta\eta/\Delta R_{\mathrm{PRJ}}$

It is an averaged one dimensional presentation. For slow ions, it is not given by the electronic stopping power, $S_{\mathrm{e}} = (\mathrm{d}E/\mathrm{d}x)_{\mathrm{e}}$.

One needs the Lindhard factor $q_{\mathrm{nc}} = \eta/E$.

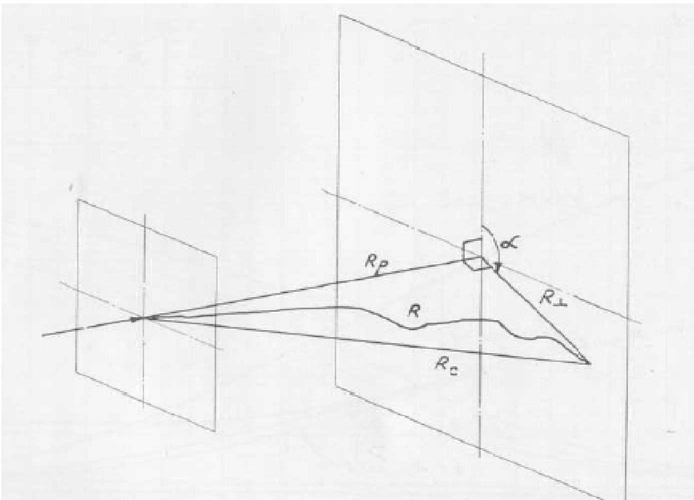


Fig. 10. Sketch illustrating definition of range concepts R, R_p, R_c and $R_\perp$.

Stopping Power Lindhard

Low energy $v < v_0 = e^2/\hbar$

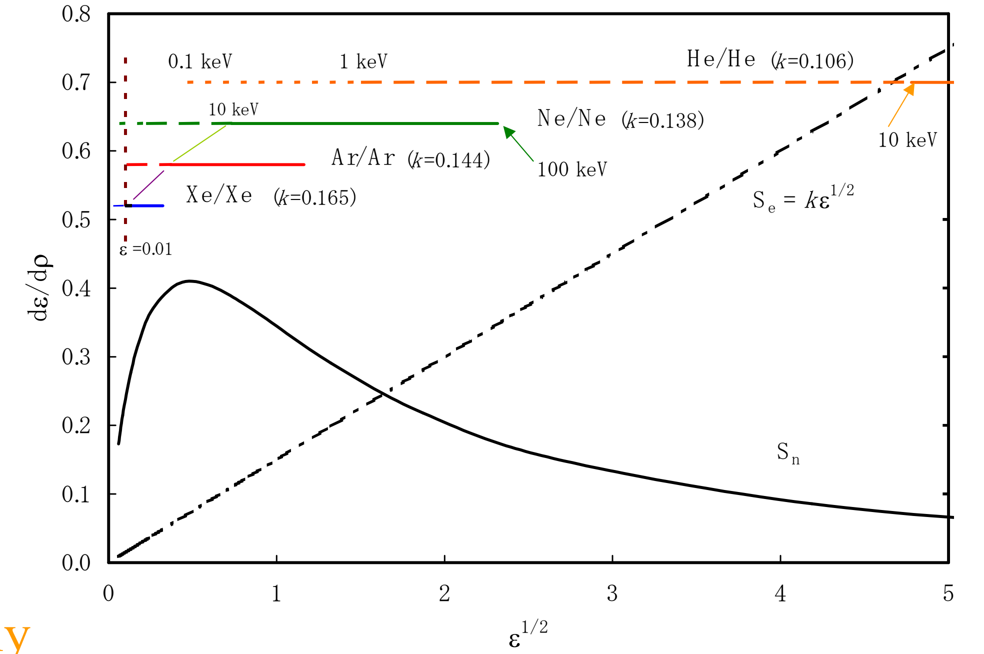
The generalized range and energy

$$\rho = RNA_2 \cdot 4\pi a^2 \frac{A_1}{(A_1 + A_2)^2}$$

$$\varepsilon = E \frac{aA_2}{Z_1 Z_2 e^2 (A_1 + A_2)}$$

The nuclear and electronic collisions are treated separately

$\varepsilon > 0.01$



Nuclear S_n and electronic S_e stopping powers as a function of energy ε for k=0.15.

Nuclear Stopping Power

Interaction potential

$$U(r) = (Z_1 Z_2 e^2 / r) \cdot \phi(r/a)$$

$\phi(r/a)$: Fermi function

$$a = 0.8853 \cdot a_0 \left(Z_1^{2/3} + Z_2^{2/3}\right)^{-1/2}$$

A universal differential cross section

$$d\sigma = \pi a^2 \frac{dt}{2t^{3/2}} f(t^{1/2})$$

$$t = \varepsilon^2 \cdot (T / T_m) = \varepsilon^2 \sin^2 \frac{\theta}{2}$$

The nuclear stopping power

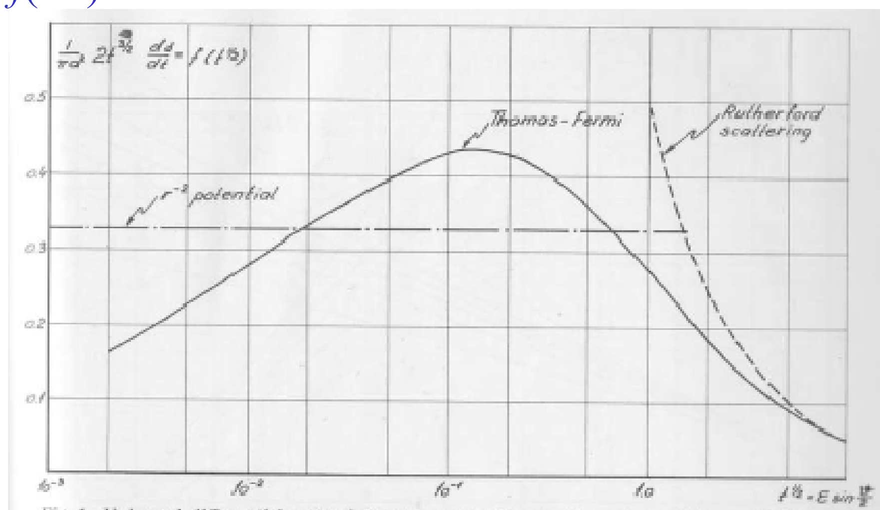
$$\left(\frac{d\varepsilon}{d\rho}\right)_n = \int_0^{\varepsilon} dx \frac{f(x)}{\varepsilon}$$

A screened Rutherford scattering

[⇒ exp($-r/a_B$): Bohr]

a: Thomas-Fermi type screening radius

$f(t^{1/2})$



$t^{1/2} = \varepsilon \sin(\theta/2)$

Stopping Powers

The nuclear stopping power S_n

$$S_n(E) = \frac{< T(E) >}{\lambda(E)} = N\sigma < T > = N\sigma \int_0^\infty T(E,\theta(p)) \frac{2\pi p dp}{\sigma} \quad = \pi N \int_0^\infty T d(p^2)$$

$< T(E) >$ is the mean energy transferred in an elastic collision, and $\lambda(E) = 1/N\sigma$

$$T(E,\theta) = \frac{4A_1A_2}{(A_1 + A_2)^2} E \sin^2 \frac{\theta}{2}$$

S_n can be expressed by the analytical expression [Birsack 19

$$S_n = -\frac{dE}{ds} = \frac{4\pi a N A_1 Z_1 Z_2 e^2}{A_1 + A_2} \cdot \frac{\ln \varepsilon}{2\varepsilon(1-\varepsilon^{-3/2})}$$

for all Z_1, Z_2

The electronic stopping power S_e

An atom moving though an electron gas of constant density.

U t, [Lindhard &

S

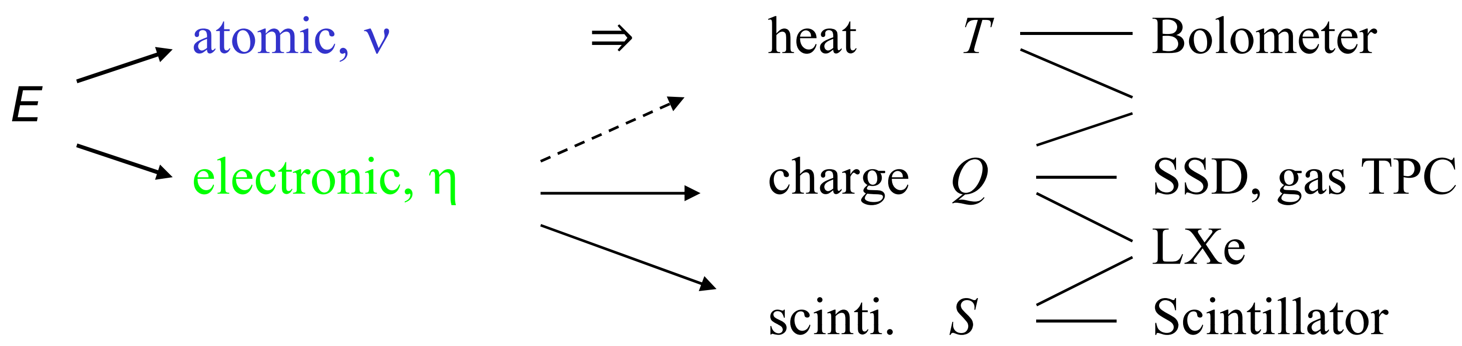
$$S_e = \xi_e \times 8\pi e^2 a_0 \cdot \frac{Z_1 Z_2}{(Z_1^{2/3} + Z_2^{2/3})^{3/2}} \cdot \frac{v}{v_0} \qquad \xi_e \approx Z_1^{1/6}$$

$$S_e = k\varepsilon^{1/2}$$

for all Z_1, Z_2

Energy shearing in low energy

For slow ions, $v < v_0 = e^2/\hbar$, S_e and S_n are similar in magnitude. The secondaries, recoil atoms and electrons, may again go to the collision process and transfer the energy to new particles and so on. After this cascade process complete, the energy of the incident particle E is given to atomic motion ν and electronic excitation η.



Nuclear quenching factor (Lindhard factor) $q_{nc} = \dfrac{\text{Electronic energy}}{\text{Ion energy}} = \eta / E$

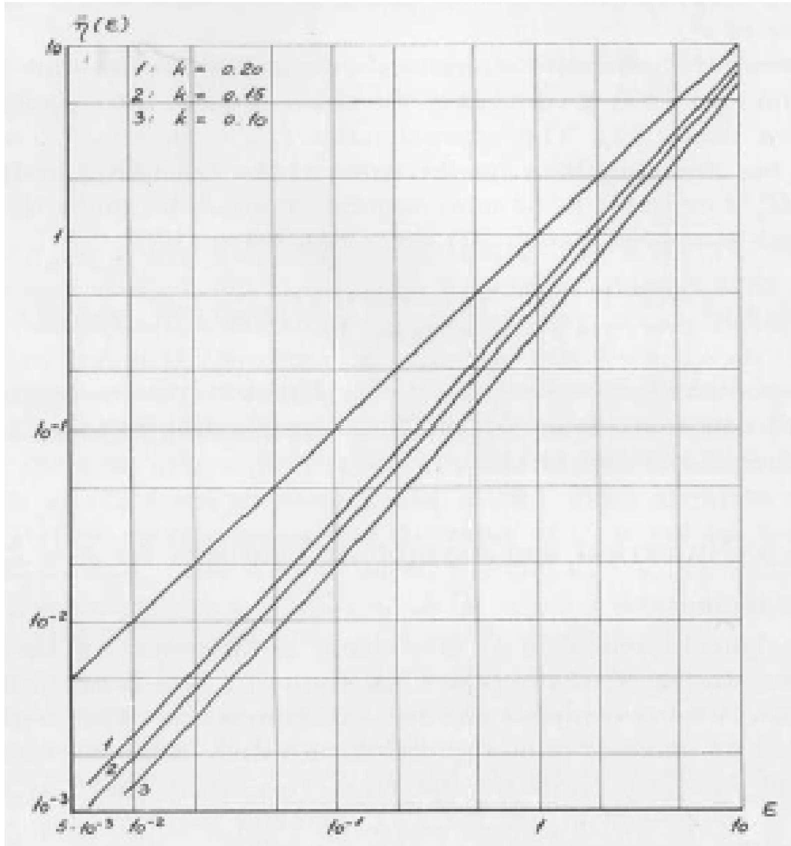
integrated for individual recoil created in the cascade,
the energy that went to electronic excitation.

Lindhard factor   η/ε

The stopping powers contain only a part of the necessary information to obtain the quenching factor, q_{nc} = η/ε ratio. The differential cross section in nuclear collisions is needed for the integral equations.   For $Z_1 = Z_2$,

$$\left(\frac{d\varepsilon}{d\rho}\right)_e \cdot \nu'(\varepsilon) = \int_0^{\varepsilon^2} \frac{dt}{2t^{3/2}} \cdot f(t^{1/2}) \left\{ \nu\left(\varepsilon - \frac{t}{\varepsilon}\right) - \nu(\varepsilon) + \nu\left(\frac{t}{\varepsilon}\right) \right\}$$

$$\left(\frac{d\varepsilon}{d\rho}\right)_e = k\varepsilon^{1/2}$$



η as a fn of ε for *k* = 0.2, 0.15, 0.1

$\varepsilon = \eta + \nu$

Asymptotic equation

$\eta = \varepsilon - \nu \qquad Z_1 = Z_2, \quad 0.1 < k < 0.2$

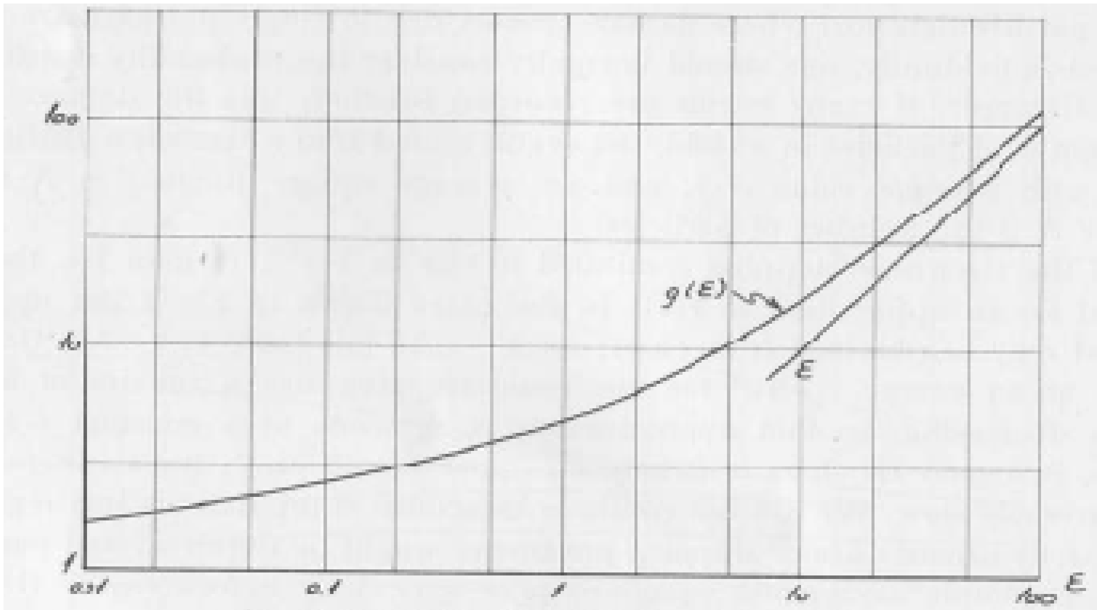
large ε: $\nu \sim g_1(\varepsilon) \cdot k^{-1}$

ε<1: $\nu \approx \varepsilon \qquad \nu \sim \varepsilon - k \cdot g_2(\varepsilon)$

$$\nu(\varepsilon) = \frac{\varepsilon}{1 + k \cdot g(\varepsilon)}$$

$g(\varepsilon)$ is parameterized by
Lewin & Smith (ApP 1996)

$g(\varepsilon) = 3\varepsilon^{0.15} + 0.7\varepsilon^{0.6} + \varepsilon$



Quenching factor in rare gases

$Z_1 = Z_2$

Lindhard factor q_{nc}

Numerical Calc. k =0.1, 0.15, 0.2

Asymptotic form k = 0.1~0.2

$\varepsilon > 0.01$ Xe > 10 keV,

He & Ne

satisfies $\varepsilon > 0.01$

large W-values ⇒ small N_i

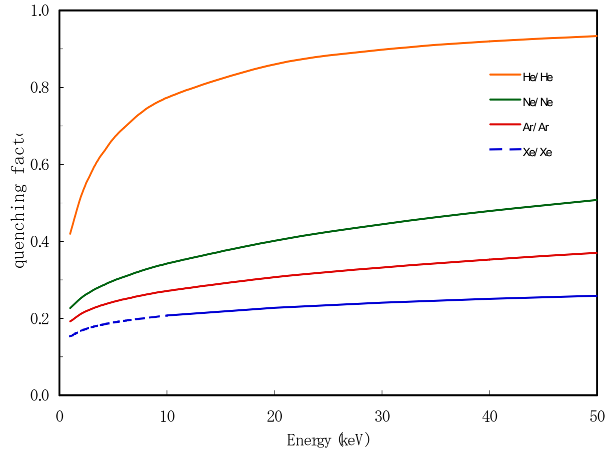
change in the energy balance

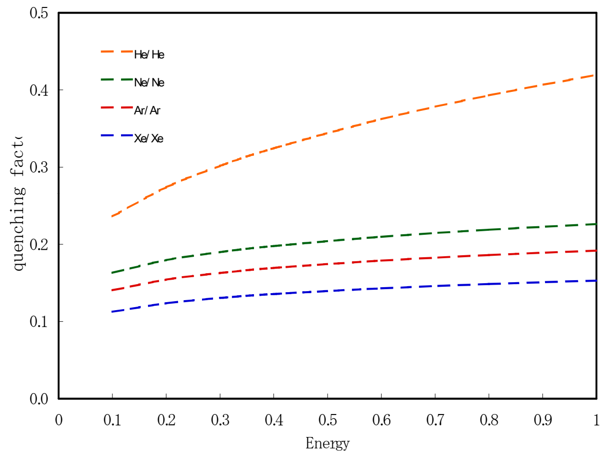
$$W = E_i + (N_{ex}/N_i)E_{ex} + E_{sb}$$

RN/γ ratio in gas

$$q_{nc} \approx RN/\gamma$$

Energy dependence in W_γ and W_{RC}.





Dashed curves are not reliable

Linear Energy Transfer (LET)

LET: The energy deposited per unit length

LET ≡ $-dE/dx$ for fast ions $\qquad S_T \approx S_e$

The electronic LET $LET_{el} = -d\eta/dx$ $\qquad S_T = S_e + S_n$

should be introduced for slow ions

The ionization density ⟶ The quenching calc., S/T ratio etc.

is given by LET_{el} in liquid

[not by the electronic SP, $(dE/dx)_{el}$]

The Bragg-like curve for TPC ⟶ The direction of recoil ions

practical & macroscopic

Electronic Linear Energy Transfer (LET_{el})

$LET_{el} \equiv -d\eta/dR = -\Delta\eta/\Delta R$ R: the range

$= -(\eta_1-\eta_0)/(R_1-R_0)$

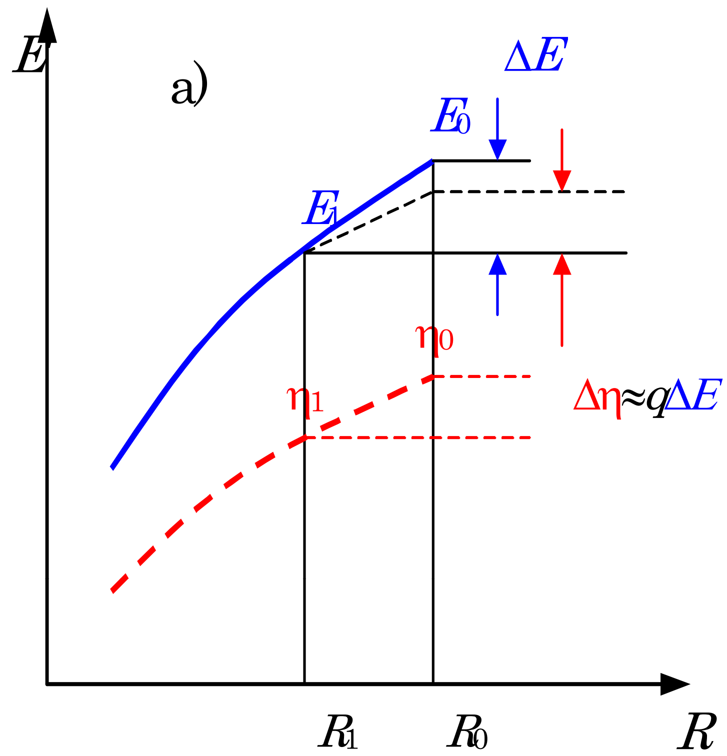
for quenching calc. etc.

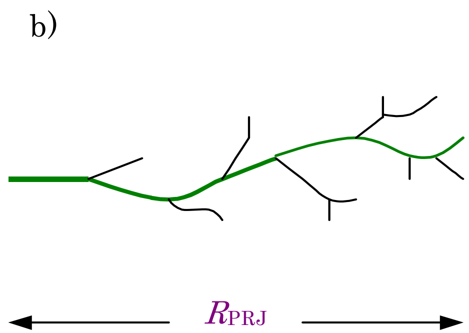
The true range R is given by the total stopping power

$$R_T = \int (dE/dx)_{total}^{-1}\, dE$$

The Bragg-like curve for TPC

The projected range, R_{PRJ}, may be used (depth)





Stopping Power and LET

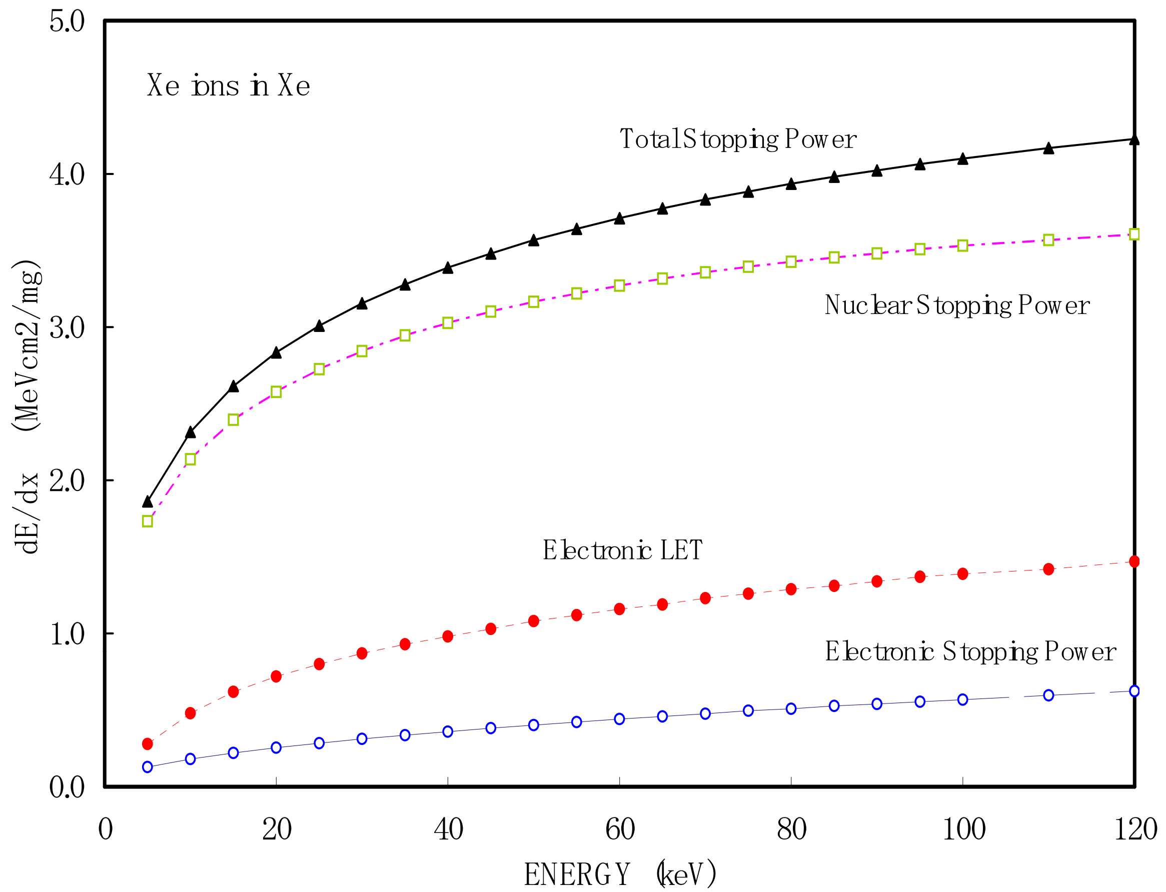
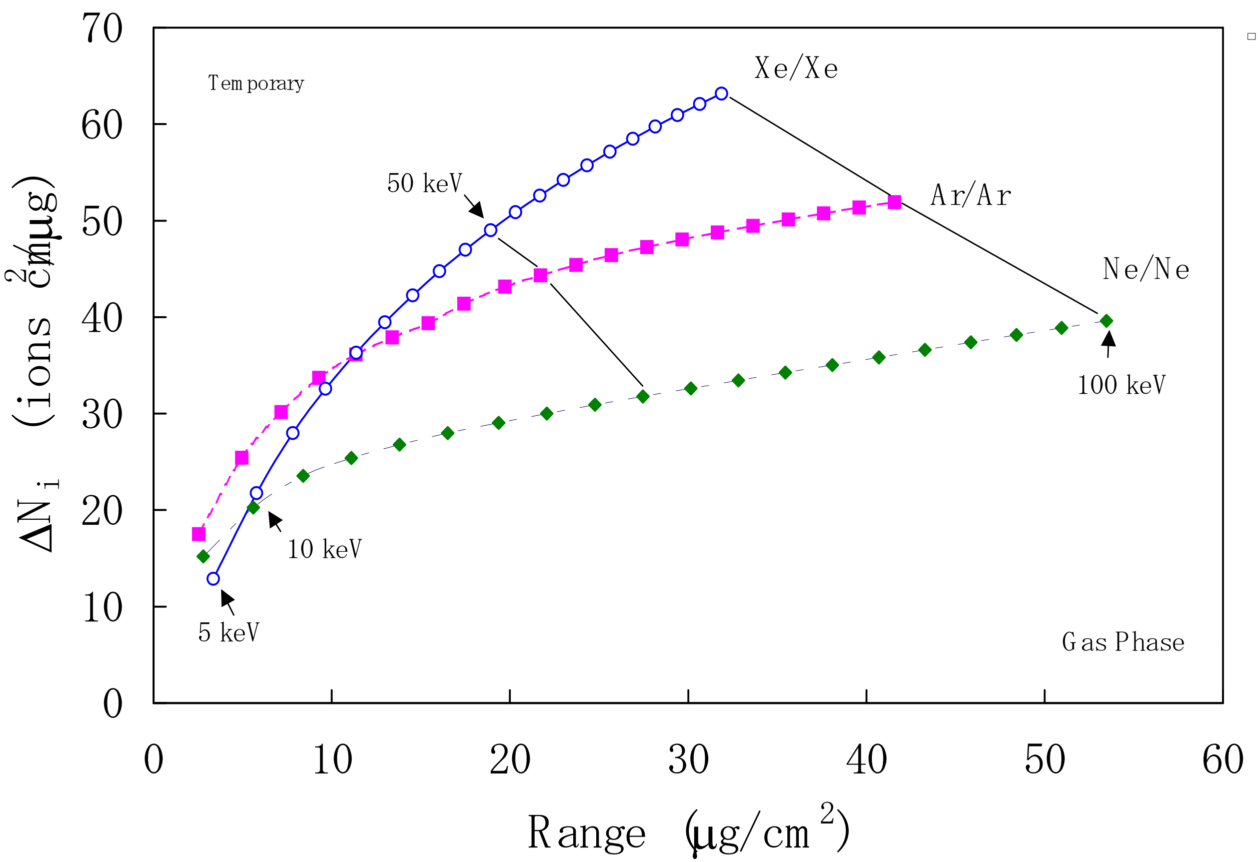


Fig. 1 The stopping power and the electronic LET as a function of the recoil energy for Xe in Xe.

HMI & Lindhard

electronic LET



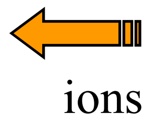


Fig. The electronic LET, $d\eta/dR_T$ for recoil ions in rare gases.

The ions enter from the right hand side. Points are plotted at every 5 keV.

Used for the electronic quenching calc in condensed media.

A. Hitachi, Astropt. Phys. 24, 247 (2005)
Gwin (1962)

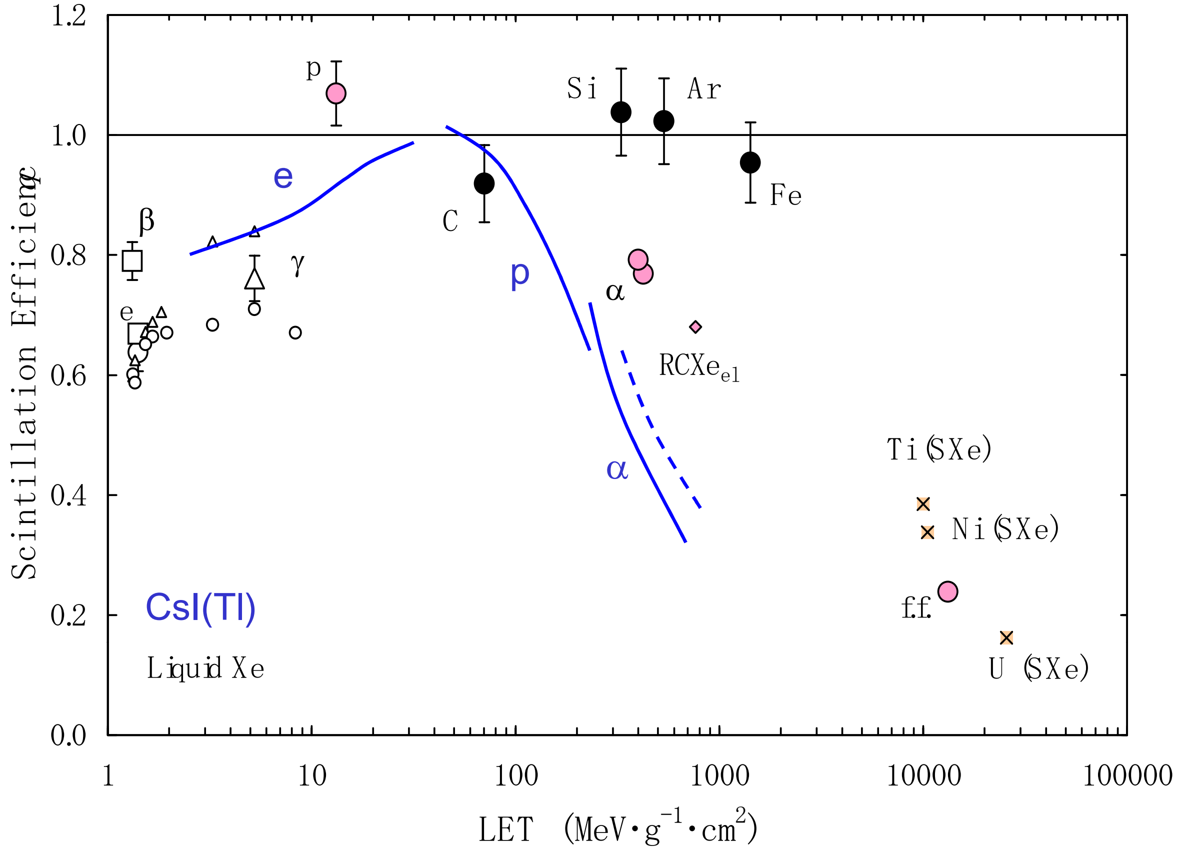


Fig. 2 Scintillation efficiency for various ions as a function of the electronic LET for liquid Xe and CsI(Tl).

A. Hitachi, Astropart. Phys. 24, 247 (2005)

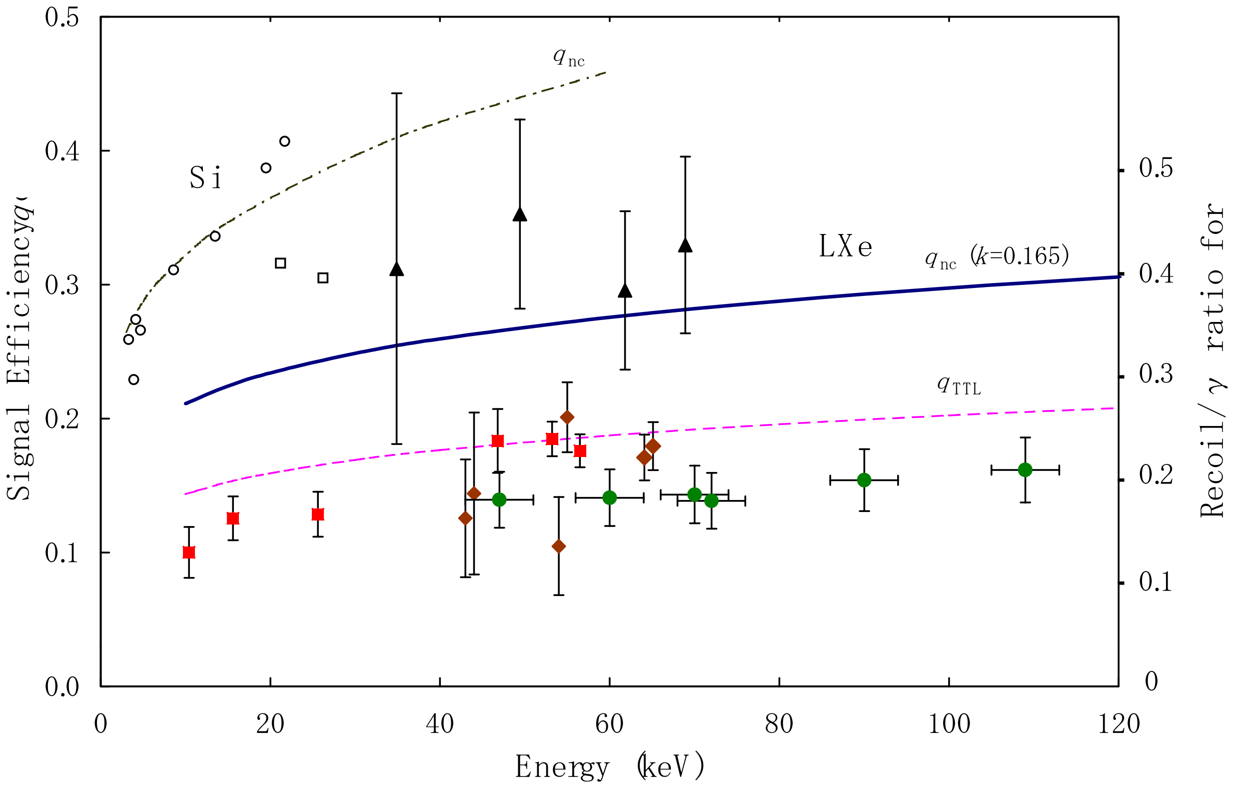


Fig. 4 Scintillation efficiency for recoil ions in LXe as a function of recoil ion energy. The nuclear (q_{nc}: Lindhard) and the total ($q_{TTL}=q_{nc}\times q_{el}$) quenching factors are shown. The results for Si are also shown for comparison. q_{el}: exiton-exciton collision model

Difference between theory and expt. is due to mainly uncertainty in γ efficiency (α/γ ratio).

Park, NIMA 491, 460 (2002)
Pécourt, Astropt. Phys. 11, 457 (1999)

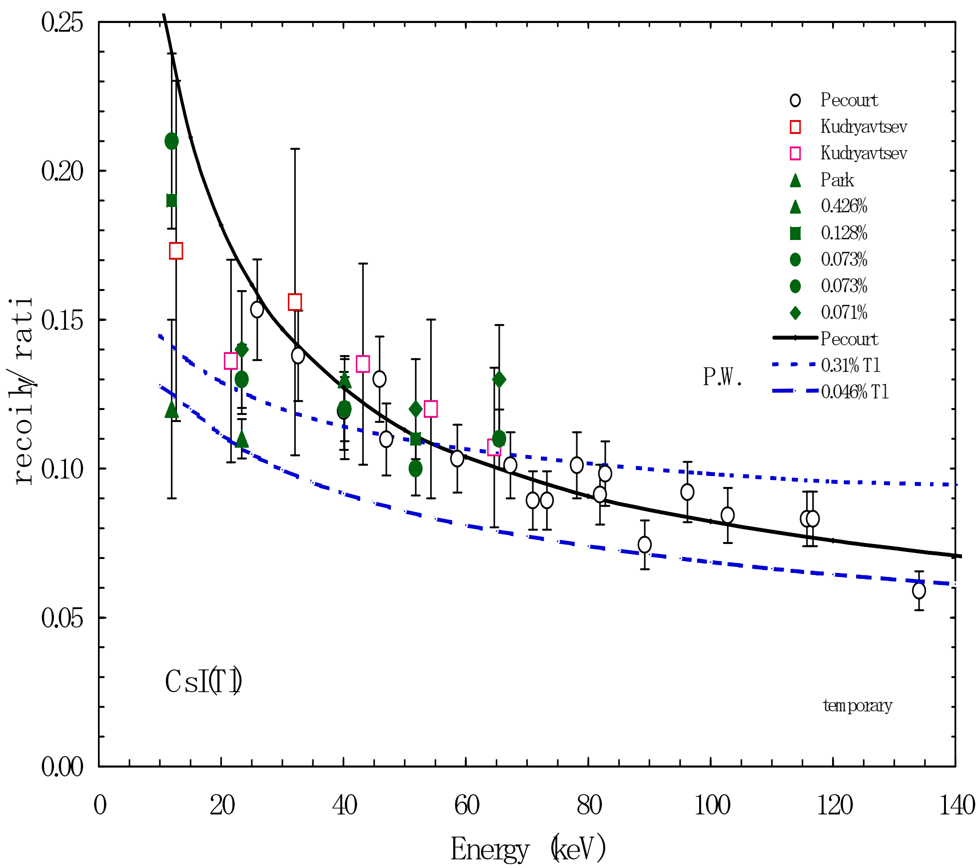
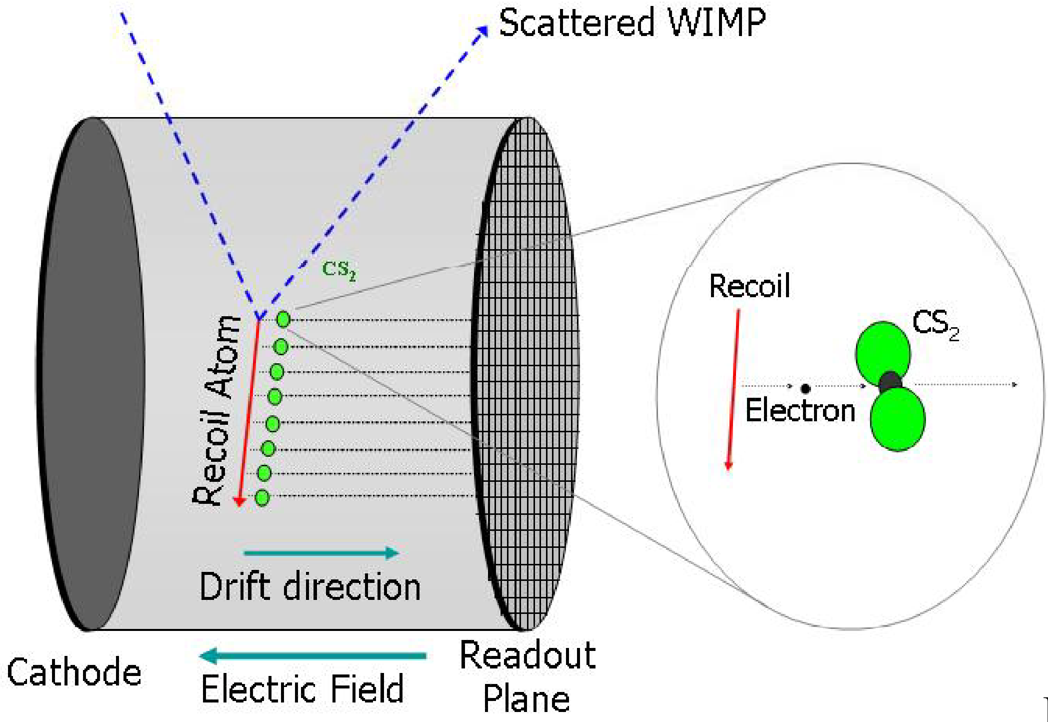


Fig. 3 The recoil ion to γ ratio in CsI(T) as a function of recoil ion energy. The broken lines are present estimates. The solid line is fitting to the Birks-Lindhard model by Pecourt.

Time Projection Chamber



DRIFT

gas TPC (Time Projection Chamber)
electronegative gas such as CS_2 , CF_4 , low diffusion
The Bragg-like curve

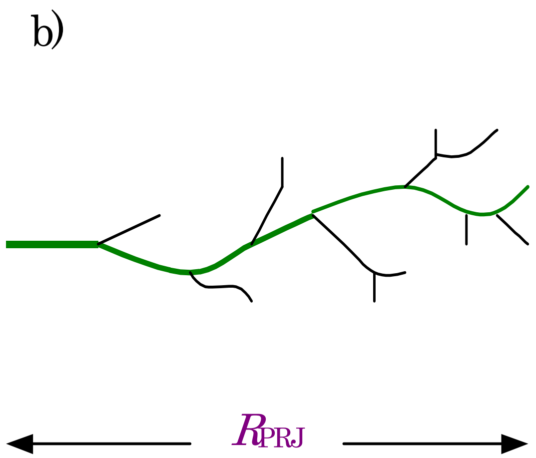
The Bragg-like curve

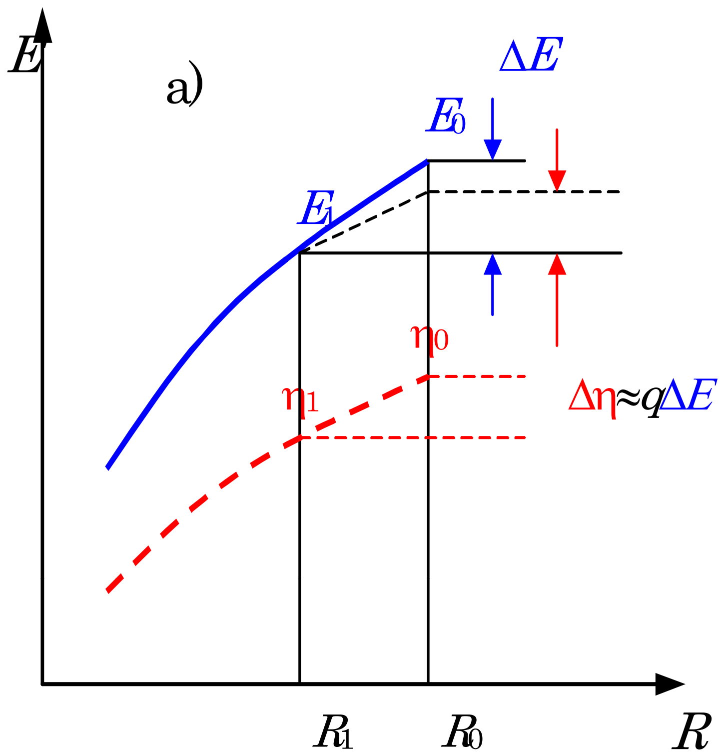
$LET_{el} \equiv -d\eta/dR = -\Delta\eta/\Delta R$

R: the range

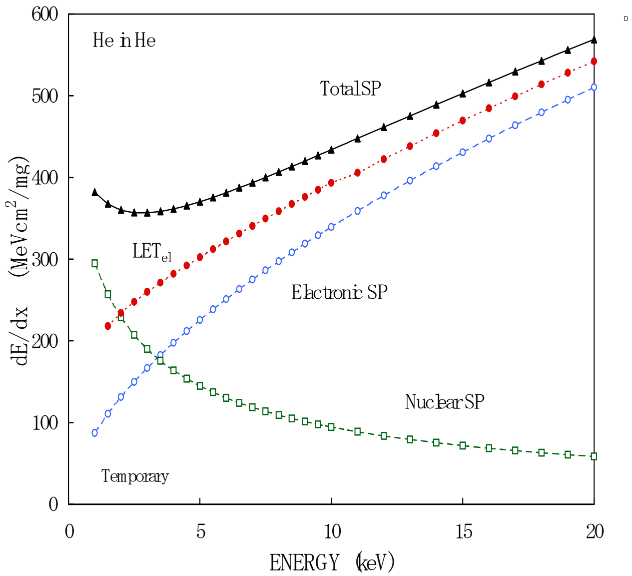
The Bragg-like curve for TPC

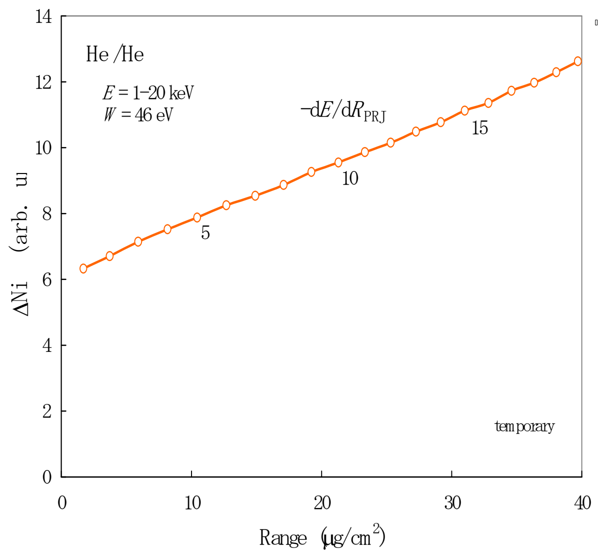
The projected range (depth), R_{PRJ}, may be used





Stopping Power and Bragg-like curve for He/He





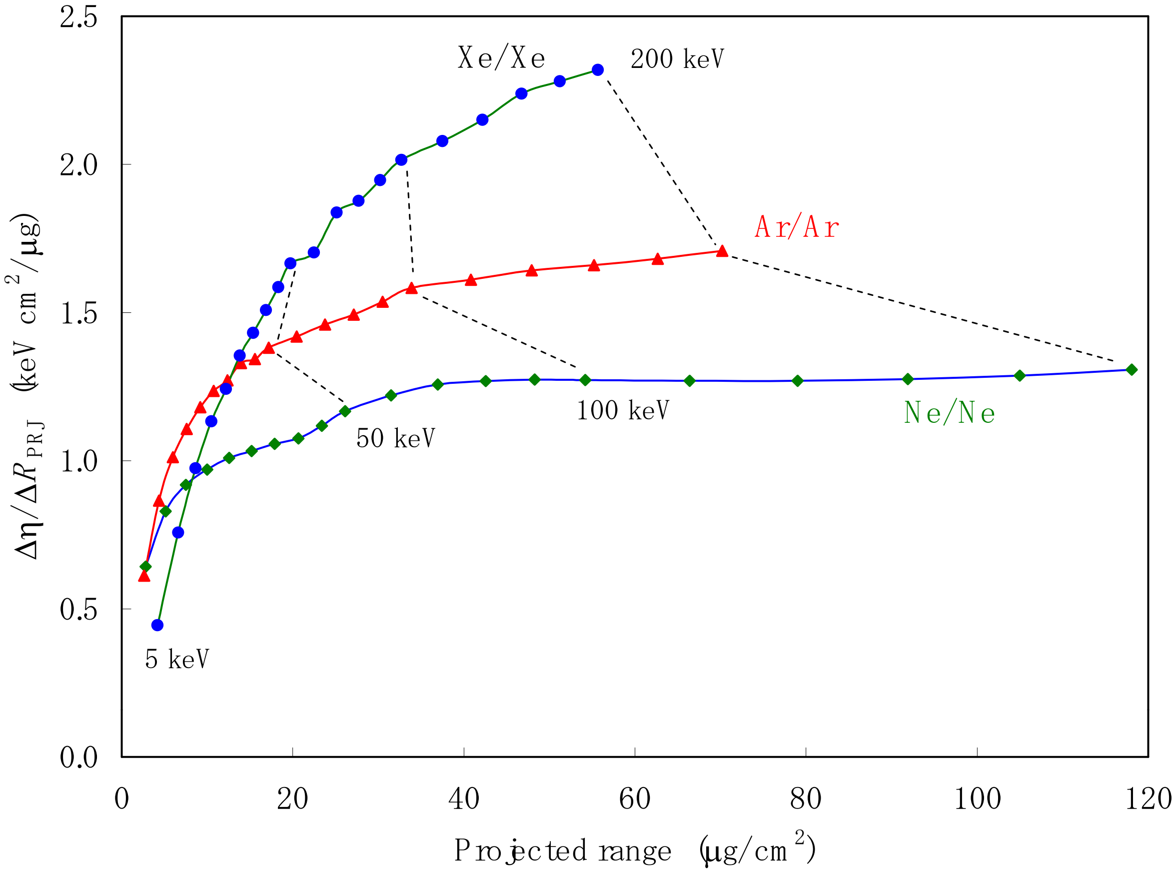
For quenching calc.
$LET_{el} = -d\eta/dR_T$

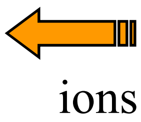
$d\eta/dR_T > (dE/dx)_{el}$

For TPC, the Bragg-like curve
$LET_{el} = -d\eta/dR_{PRJ}$

$d\eta/dR_{PRJ}$ can be lager than $(dE/dx)_T$

Bragg-like curve





Bragg-like curves, $d\eta/dR_{PRJ}$ for recoil ions in rare gases.

The ions enter from the right hand side. Points are plotted at every 5, 10 or 20 keV

The projected ranges are taken from SRIM.

Head and tail detection of WIMPs

$\Delta\eta/\Delta R_{PRJ}$ for recoil ions in TPC gases

Rare gases

Rare gas – molecule mixture

Ar – CO_2, CH_4, CF_4, CS_2 (low concentration)

S_T in pure Ar or use the Bragg rule

q_{nc} for recoil ions are obtained as each ions in pure Ar

the $W - v_0/v$ plot (empirical)

Binary gases

CS_2, CF_4

S_T the Bragg rule + compound correction

q_{nc} no general methods

Lindhard factor for $Z_1 \neq Z_2$

The formation of accurate general solutions become quite complicated.
The integral equation for the ion Z_1 in the matter Z_2 is,

$$\nu_1'(E) \cdot S_{1e} = \int d\sigma_1 \{\nu_1(E-T) - \nu_1(E) + \nu(T)\}$$

$d\sigma_1$: the diff. cross section for an elastic colli. between Z_1 and Z_2.
$T < T_m = \gamma E$; $\gamma = 4A_1A_2/(A_1+A_2)^2$

Characteristic energies:

$$E_{1c} \cong A_1^3 (A_1 + A_2)^{-2} Z^{4/3} Z_1^{-1/3} \cdot 500 \text{ eV}, \; E_{2c} \cong (A_1 + A_2)^2 A_1^{-1} Z_2 \cdot 125 \text{ eV}, \quad \text{where } Z^{2/3} = Z_1^{2/3} + Z_2^{2/3}$$

The power law approximation for very low energy,

$$\eta = CE^{3/2} \qquad C = \frac{2}{3}\left\{E_{1c}^{-1/2} + \frac{1}{2}\gamma^{1/2} E_c^{-1/2}\right\},$$ for $E < E_{1c}, E_{2c}$,

The straggling:

$$\Omega^2 / \eta^2 = \frac{1}{14}\gamma\left\{\left(\frac{\gamma^{1/2}}{CE_c^{1/2}} - \frac{7}{4}\right) + \frac{7}{16}\right\}$$

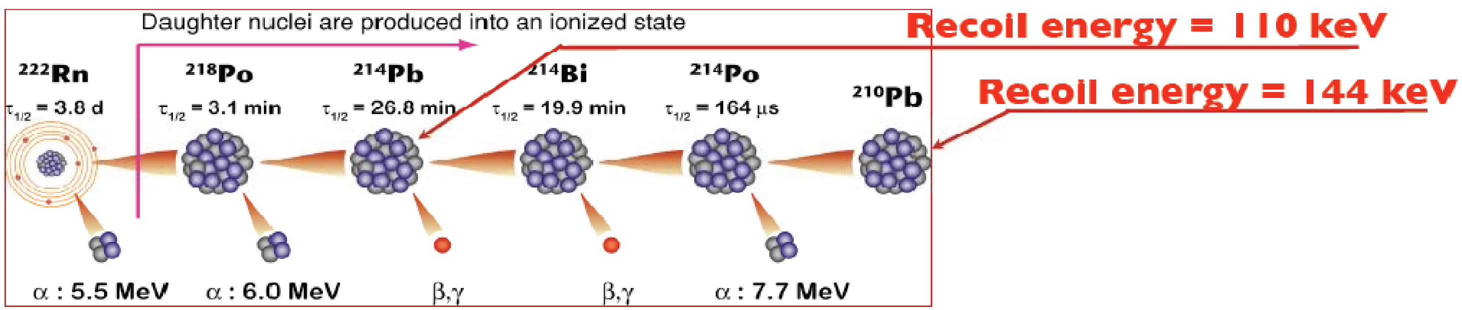
Good for heavy ions such as Pb ions in α-decay

Recoil ions in α-decay - Power Law Approx. -

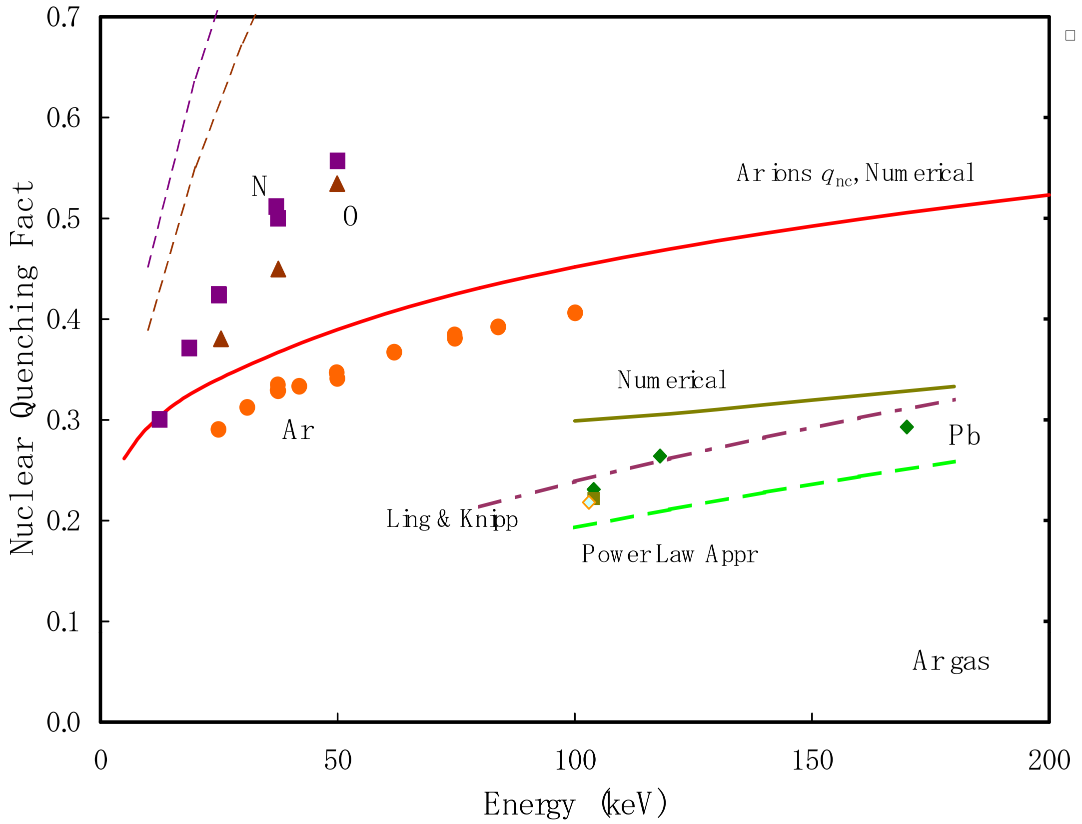
Rare gases

	E in keV	Ω^2/η^2	E_{2c}
He	$\eta = 0.0523 E^{3/2}$	0.006	54 keV
Ne	$\eta = 0.0253 E^{3/2}$	0.016	312 keV
Ar	$\eta = 0.0193 E^{3/2}$	0.020	660 keV
Xe	$\eta = 0.0122 E^{3/2}$	0.041	3.7 MeV

The asymptotic equation does not work



Quenching factor in Ar

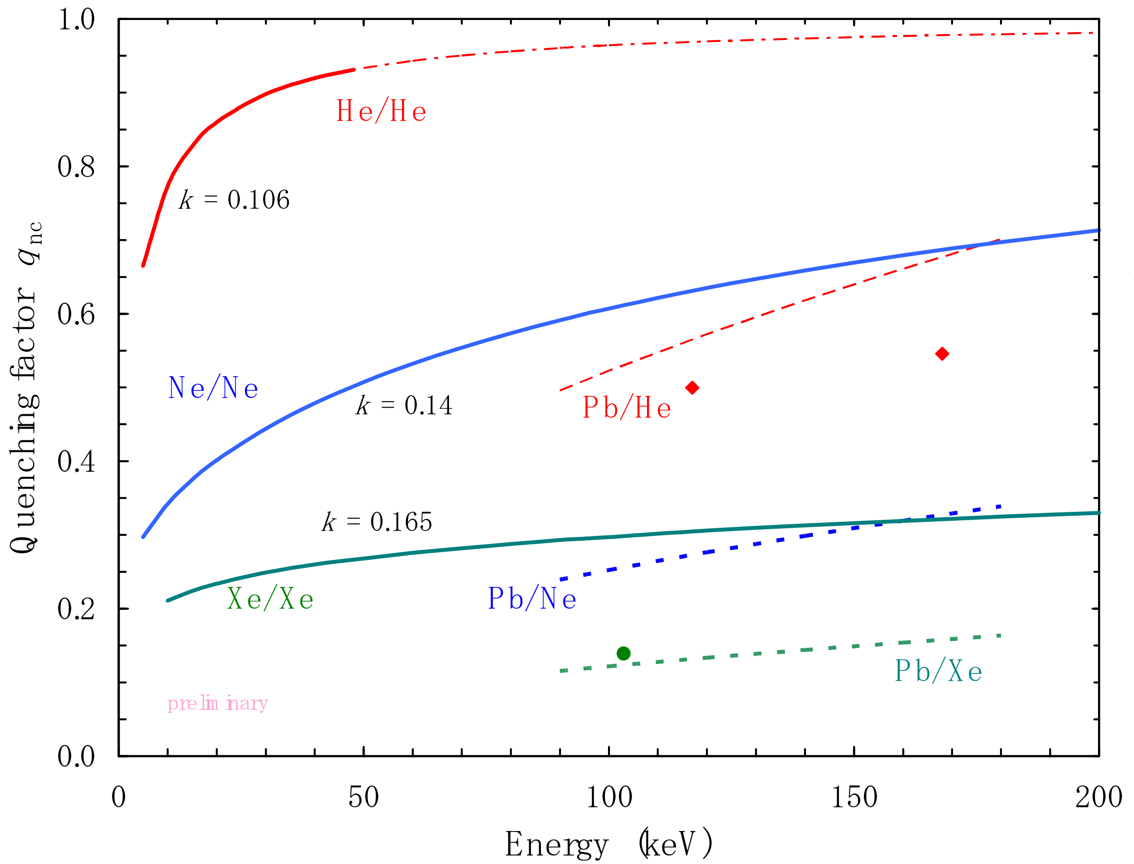


Experimental results in gas $W(\alpha)/W(\mathrm{RC}) \sim q_{nc}$

N, O and Ar; Phipps et al [1964]

Pb; Madsen [1945], Jesse & Sadauskis [1956]

Lindhard factors in rare gases



Energy Straggling

Recoil Pb ions in α-decay in LAr

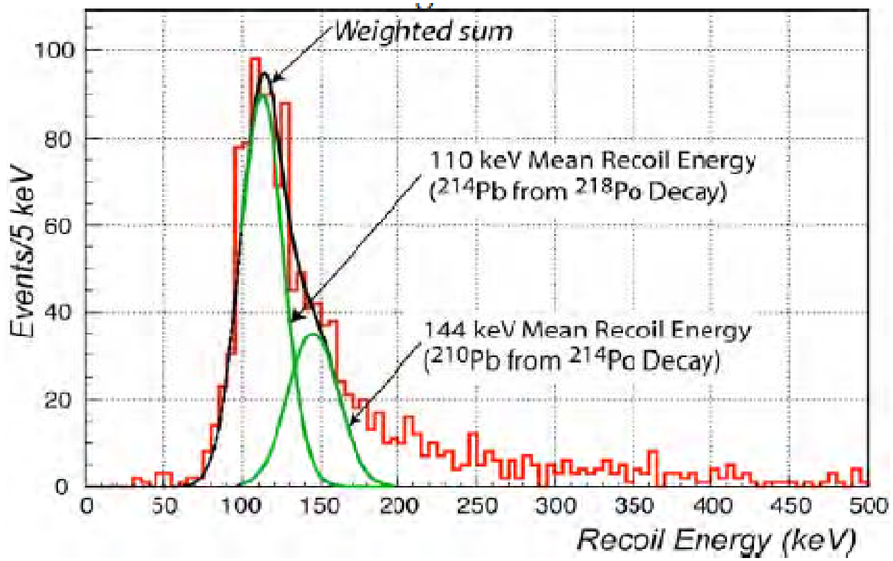
110 keV σ ~ 12%
144 keV σ ~ 11%

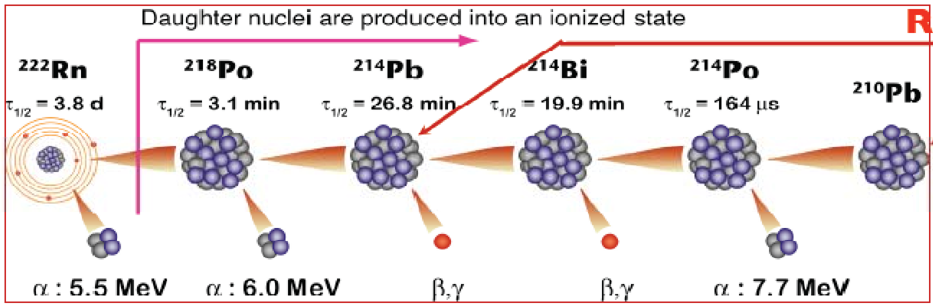
Power law approximation (Lindhard)

$$\sigma^2 = \Omega^2/\eta^2 = \frac{1}{14}\gamma\left\{\left(\frac{\gamma^{1/2}}{CE_c^{1/2}} - \frac{7}{4}\right) + \frac{7}{16}\right\}$$

σ ~ 14%

WARP 2007





Recoil energy = 110 keV

Recoil energy = 144 keV

Recoil ions in α-decay - compounds -

No works has been done for η/ε in compounds.

A) Power low approx. $q_{nc} = \eta/E$

I) replace target CS_2 with single element (Al): 112 keV

Pb-214

Pb in Al $\eta = 0.022 E^{3/2}$ (E: keV) 0.234

II) Divide the target elements and calculate separately

Pb in C and Pb in S, (Pb in C)/3 + (Pb in S)·2/3

Pb/C $\eta = 0.031 E^{3/2}$ (E: keV) 0.331

Pb/S $\eta = 0.020 E^{3/2}$ (E: keV) 0.213

Pb/C/3 + Pb/S*2/3 0.253

B) Asymptotic equation $Z_1 = Z_2$

103keV Pb in He, C, N, O, S $k = 0.10\text{-}0.16$ 0.20-0.21

168keV Pb 0.22-0.23

q_{nc} values are practically the same therefore not agree with expt.

does not include the effect of the secondary ions

Lindhard factor for recoil ions in α-decay

Recoil ion	206Pb		208Tl		208Pb		*E*2c
Energy keV	103		117		168		
gas	expt	calc	expt	calc	expt	calc	keV
Ar	0.221a	0.196	0.263	0.205	0.294	0.249	665
Xe	0.139 a	0.124		0.132		0.158	3730
H2		0.73	0.457	0.78	0.544	0.93	26
He		0.53	0.500b	0.56	0.546 b	0.68	53
CH4	0.250		0.265		0.307		
C2H4	0.236		0.269		0.321		
C3H6			0.272		0.281		
CO2			0.336		0.347		
C + 2O		0.297		0.316		0.378	174
C		0.323		0.344		0.411	174
O		0.284		0.302		0.361	241
N2	0.319	0.302		0.320		0.384	207
Dry air	0.296						
4N + O		0.298		0.317		0.379	207
CS2							
C + 2S		0.246		0.262		0.314	174
S	0.208						550
Al (for CS2)	0.228						428

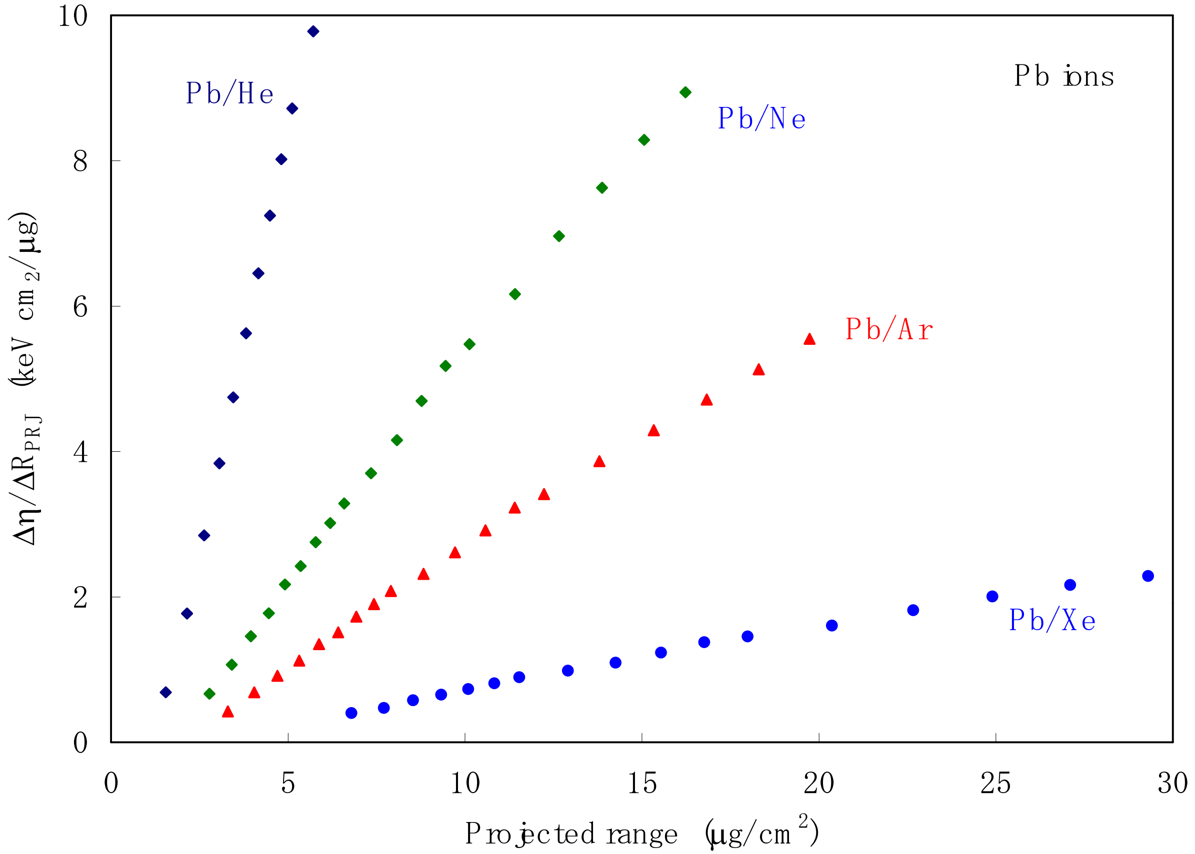
Recoil ion	214Pb		210Pb			
Energy keV	112	calc	147	calc		
CS2		0.346		0.397	SRIM by P.J.	
C + 2S		0.253		0.292		174
Al (for CS2)		0.237		0.272		430

calc : the power law approximation by Lindhard which is only good at E<E2c.

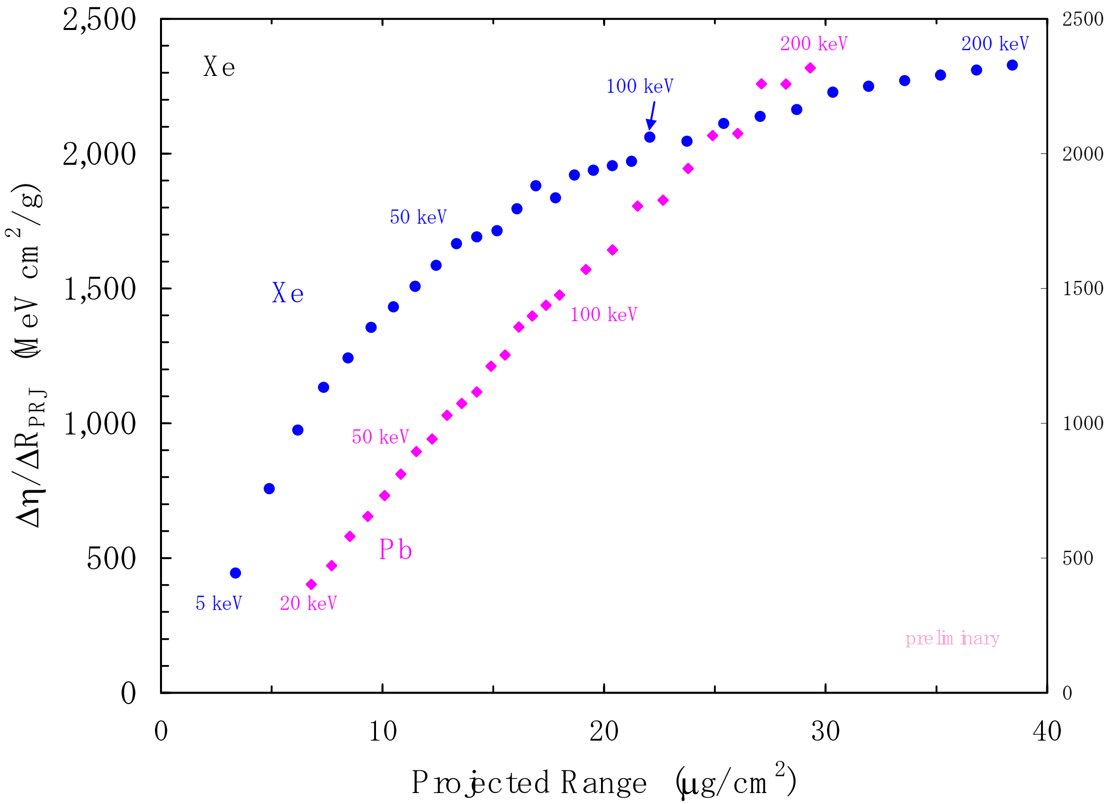
expt : W(alpha)/W(recoil) from:

[a] G.L.Cano, Phys. Rev. 169, 227 (1968). [b] W.G. Stone & L.W. Cochran, Phys. Rev. 107, 702 (1957).

Bragg-like curves for Pb ions in rare gases

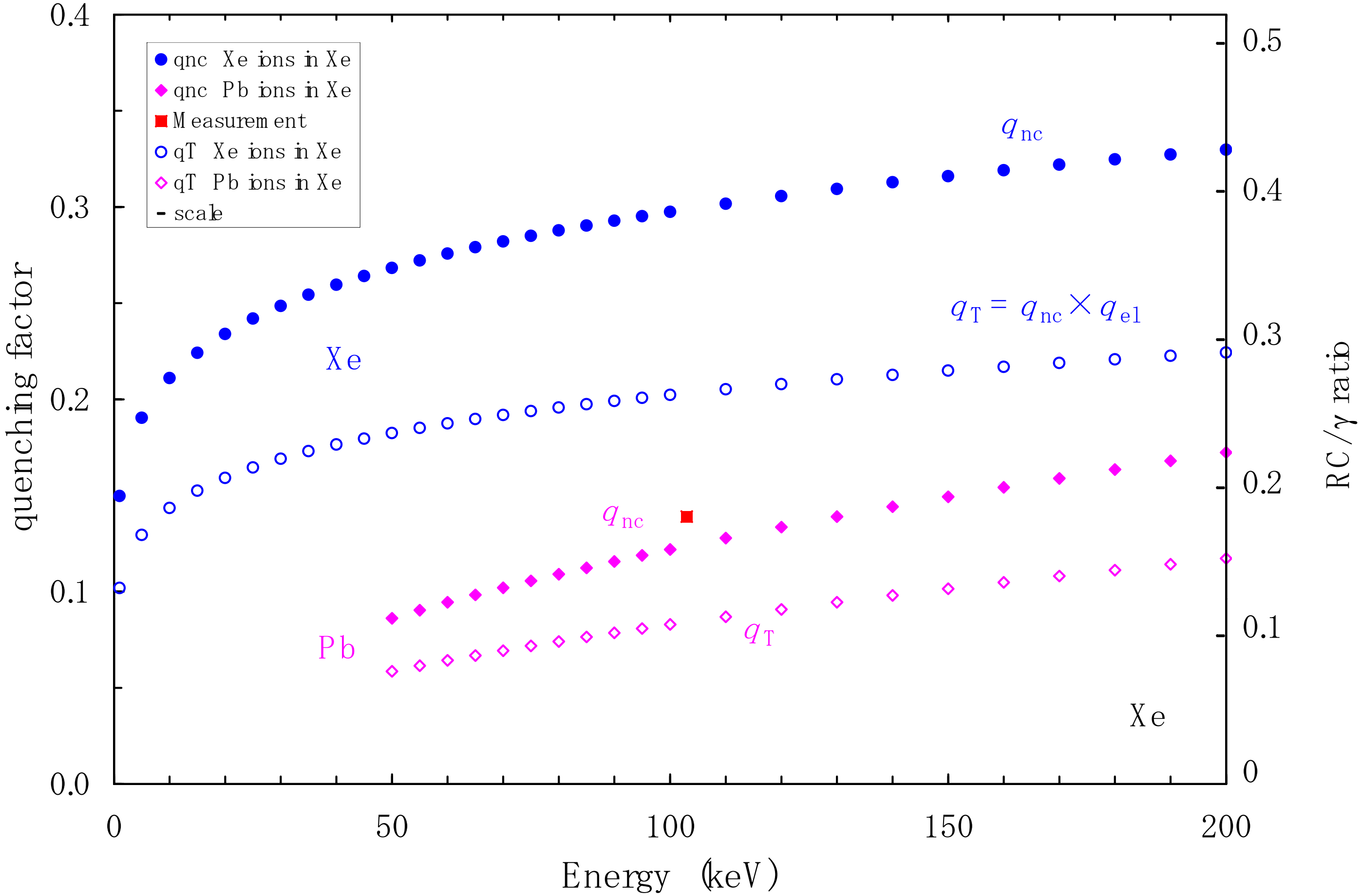


Bragg-like curves for Xe and Pb ions in Xe



The LET_{el} for Pb ions in a-decay and 40-100 keV Xe ions are also quite similar therefore q_{el} for those ions in liquid Xe are expected to be close.

Quenching factor in LXe



Quenching factor in Ar – light ions

Empirical formula in Ar

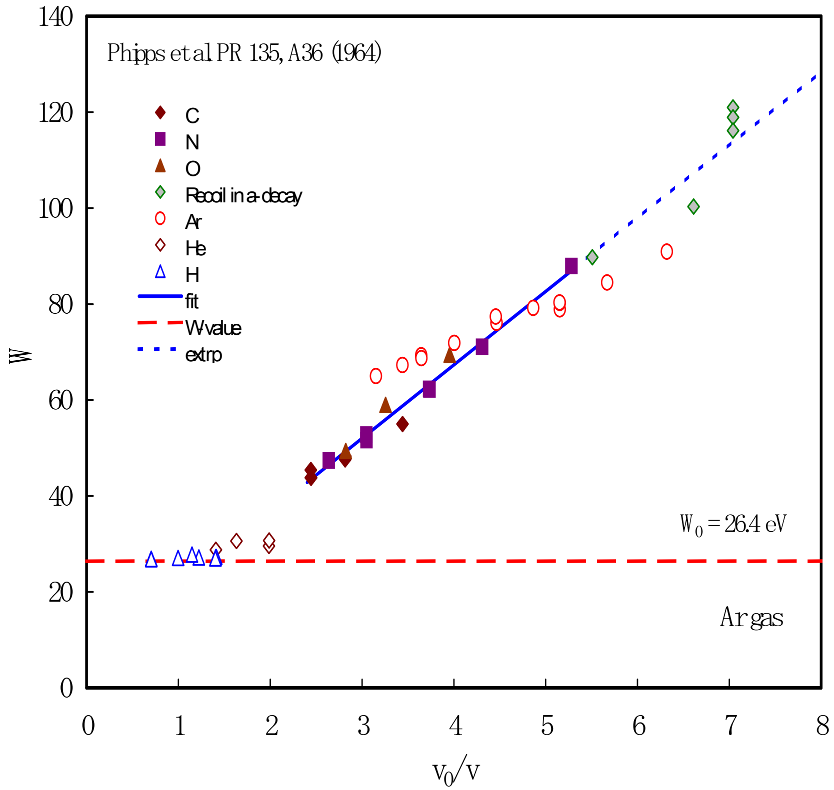
$Z_1 < Z_2$, $v_0/v > 2.5$

$W_{RN}\,(\mathrm{eV}) = 15.3(v_0/v) + 6.1$

$q_{nc} = W_\alpha \,/\, W_{RN}$

$= 26.4/[15.3\,(v_0/v) + 6.1]$

empirical



Rare gas – molecule mixture

Ar (75%) + CS_2 (25%) mixture.
The recoil atom ionize mostly the host Ar, then the electron is immediately attached to CS_2.

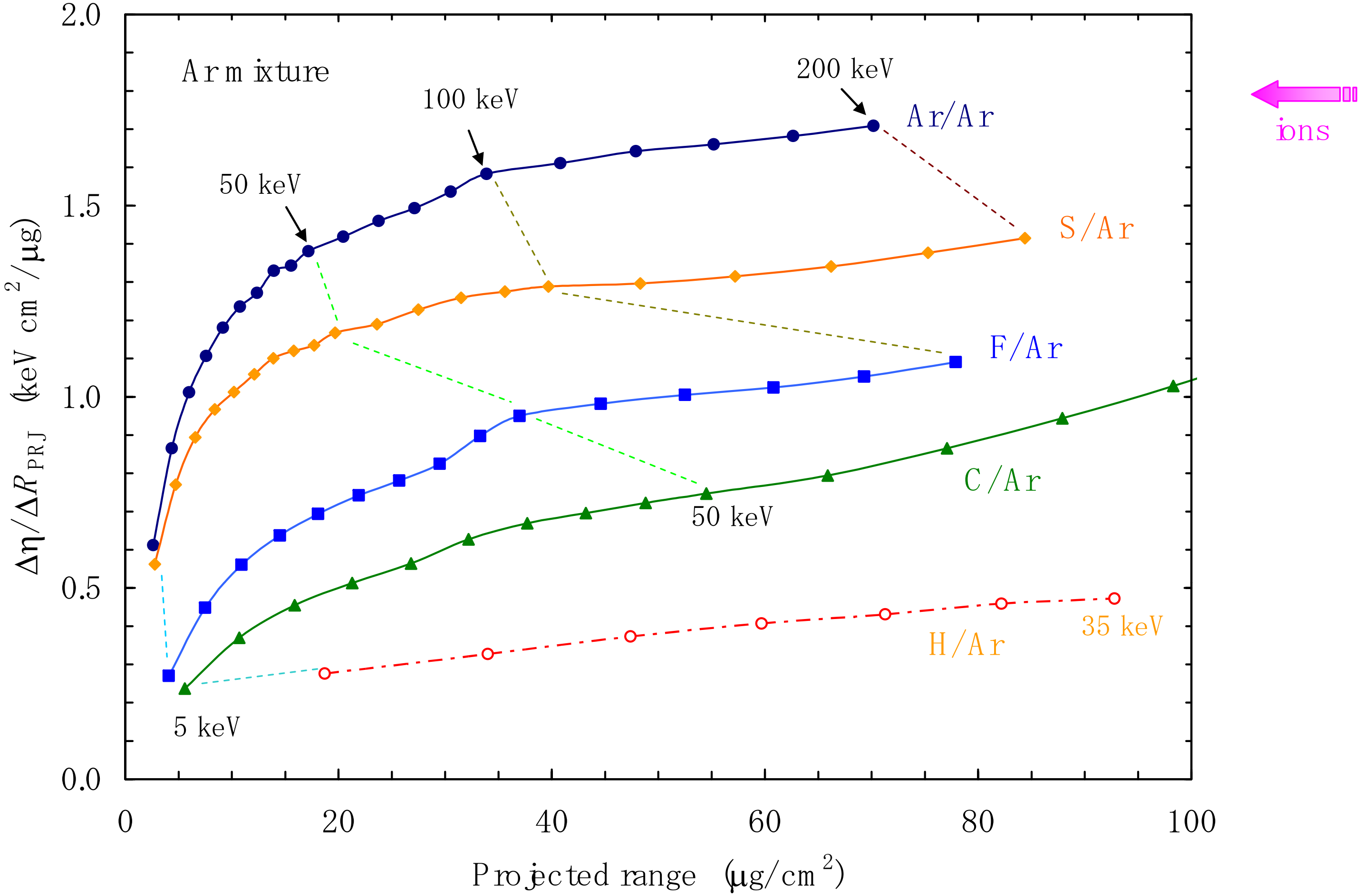
$$Ar \rightarrow Ar^+ + e^-$$

$$e^- + CS_2 \rightarrow CS_2^-$$

The CS_2^- ions drift to the proportional region and give the signal. It is necessary to obtain q_{nc} and R_{PRJ} to obtain the Bragg-like curve for TPC.

S atom is close to Ar atom in the periodic table. It is fairly safe to consider S ion as Ar ion with the same velocity in the calculation for q_{nc}. Then, the mixture may be regarded as Ar (92%) + C (8%).
The value of q_{nc} for C ions in Ar can be estimated by the W-v_0/v plot.

Bragg-like curves for light ions in Ar



The values for R_{PRJ} are obtained by SRIM in pure Ar

The *W*-values can be different in pure and mixture gases.

Binary gases

Binary gases such as CS_2 , CF_4

S_T the Bragg rule + compound correction (e.g. SRIM)

R_{PRJ}

The Lindhard factor q_{nc} → no general methods

Approximations

A) Replace the molecule with single element

e.g., C in CS_2 ⇒ C in Al

then use 1) the power law approximation (at the low energy)

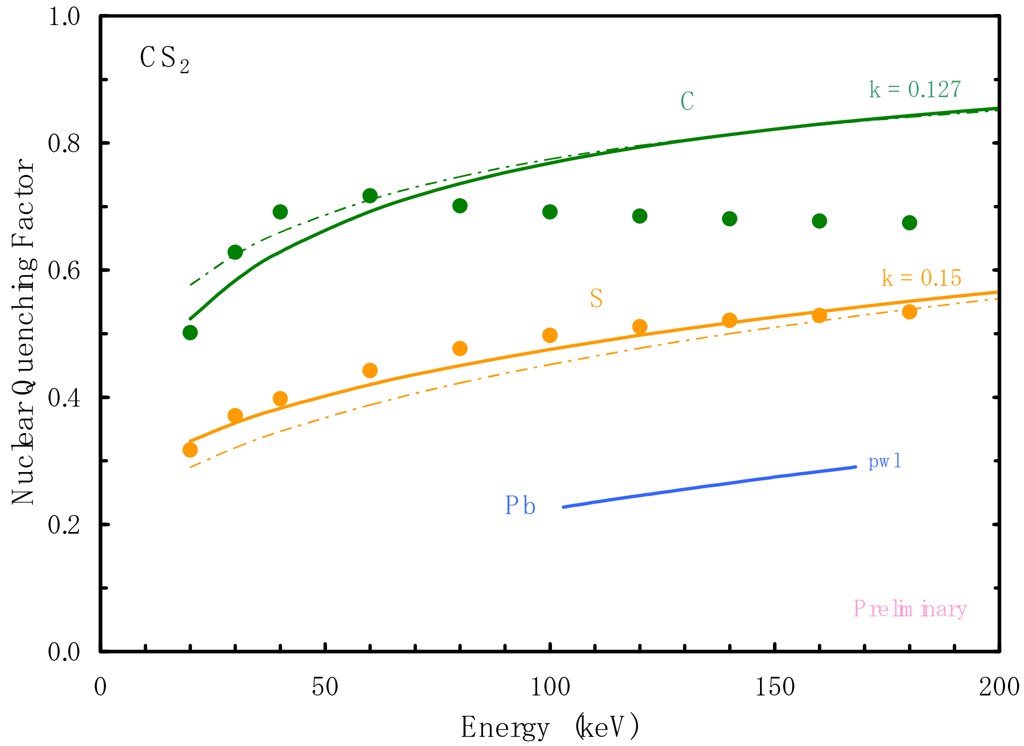
not appropriate for the light ions.

2) the asymptotic equation ($Z_1 = Z_2$, $k = 0.1 \sim 0.2$)

B) Consider only one element at a time.

e.g., C in CS_2 ⇒ C in C, S in CS_2 ⇒ S in S

Lindhard factor for binary gases – CS_2

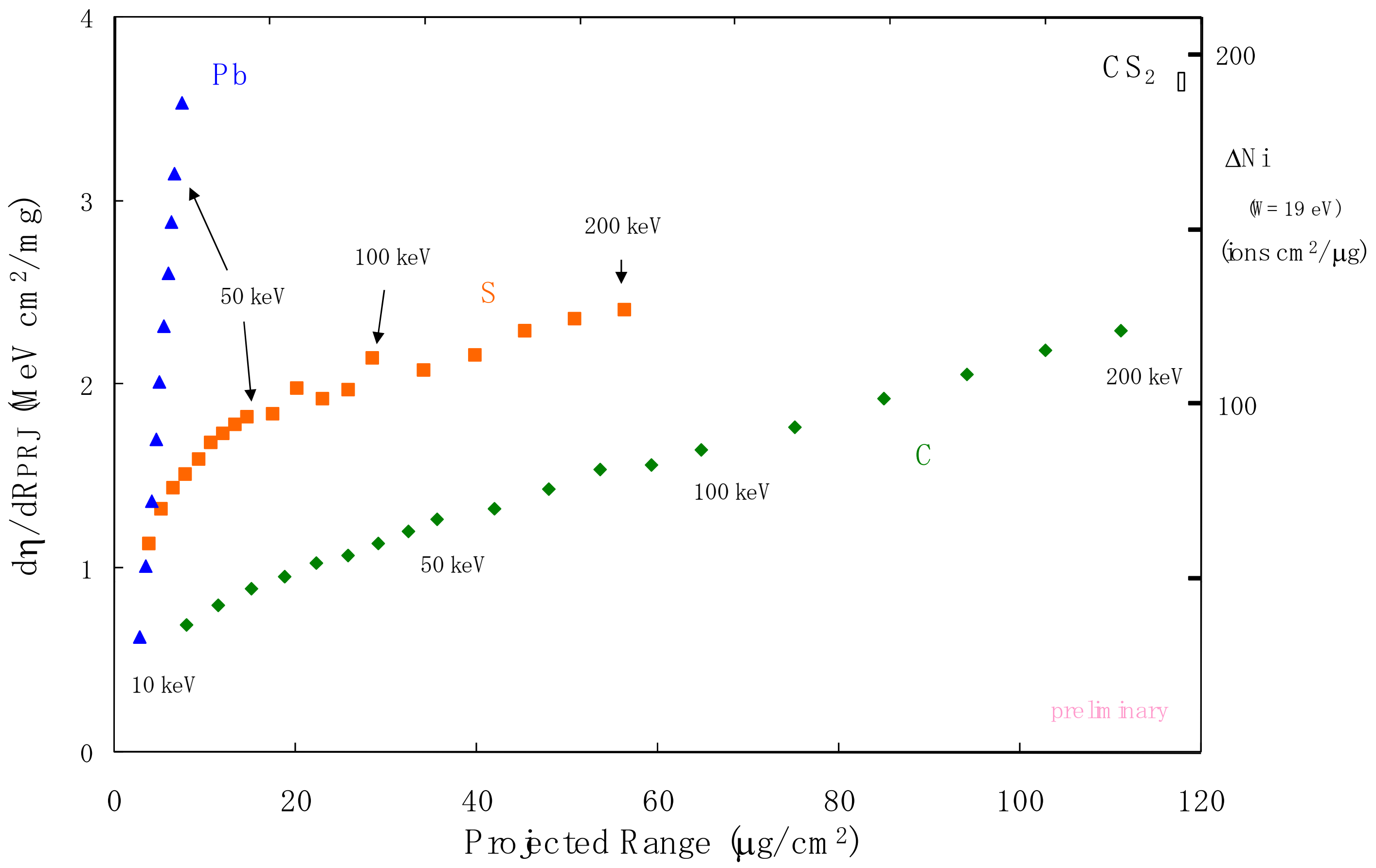


solid : C ions in C, S ions in S
dashed : Asymptotic eq. C and S in Al

Power law approximation
1/3(Pb in C)+2/3(Pb in S)

•,• : Snowdon-Ifft expt.

The Bragg-like curves for recoil ions in CS_2



Binary gases – comparisons, Th & Mes.

The approximations for S ions in CS_2 are fairly good. Those for C ions gives smaller q_{nc} values for > 80 keV.
Pb ions is given by the power law approximation.

CF_4 contains only light ions also the structure for CF_4 is the same as CH_4.

SP calc.	compounds corrections?
W-value	particle & energy dependence?
Pb ions	the power law approx. does not work as in hydrocarbons?

NEEDS measurements for q_{nc}

Stopping power for compounds

The stopping power for compounds are obtained using the Bragg-rule: the weighted sum of the elemental matter.

$$S_{mix} = \sum N_i S_i / \sum N_i \qquad [eV \cdot A^2 /atom]$$

The compound correction is needed for some compounds in low energy.

compounds containing mostly H, C, N, O, and F

1) The outer shell electrons have different orbitals in the compound than in element matter. The core and bond approximation

Ziegler & Manoyan, NIM B35, 215 (1988)

2) The triplet states in those molecules are metastable which is not excited by fast particles but can be excited effectively by slow heavy ions.

Summary

The Bragg-like curves, $d\eta/dR_{PRJ}$, are introduced for head-tail detection for WIMPs in gas TPCs. The estimation of Lindhard factor, $q_{nc} = \eta/E$, for the recoil ions is the key.

a) Pure gas: He, Ne, Ar, Xe — Lindhard, $\varepsilon > 0.01$ (Xe: E >10keV)

b) Rare gas-molecule mixture

Ar – CS_2 , CO_2, CF_4 — W-v_0/v plot for light ions (empirical)

c) Binary gas

Some approximations are necessary.

CS_2 — C ions in CS_2 → C ions in C

S ions in CS_2 → S ions in C

The asymptotic equation (?)

d) q_{nc} for very heavy ions in α-decay can be obtained by the power law approx. except for the hydrocarbons.

The asymptotic equation does not work because it does not include the effect of the secondary ions.

Summary – cont'd

The experimental results coming in the R&D process of the TPCs for dark matter detection are quite important also for the low energy stopping theory. The comparison between theory and experiments have been done almost exclusively through the range and straggling. However, the Lindhard factor becomes very important at the low energy.

- *W*-value measurements at a high-enough field

  $q_{nc} \approx W(\alpha)/W(\mathrm{RC})$
- 3D construction of the track

Electronic Stopping Power

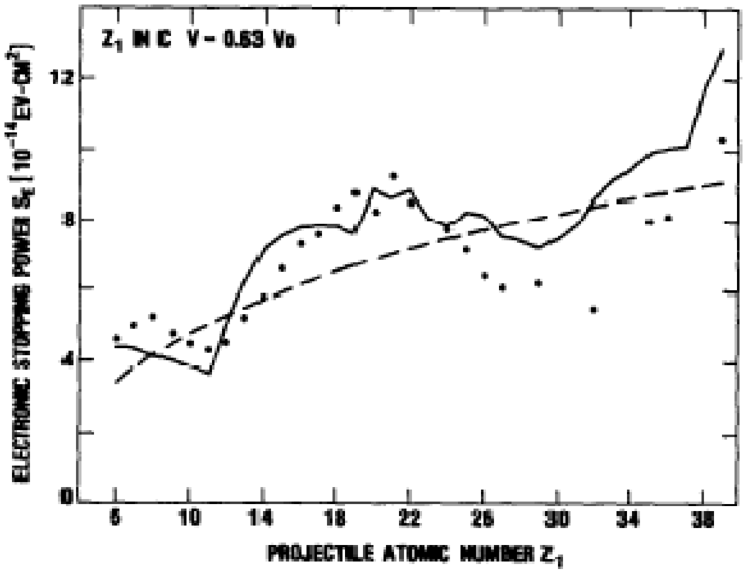


Fig. 1. Electronic stopping power as a function of projectile ion at the velocity $v = 0.63\ v_0$ in a carbon target: —— present values; – – – Lindhard-Scharff values; · · · data from Aarhus[10]

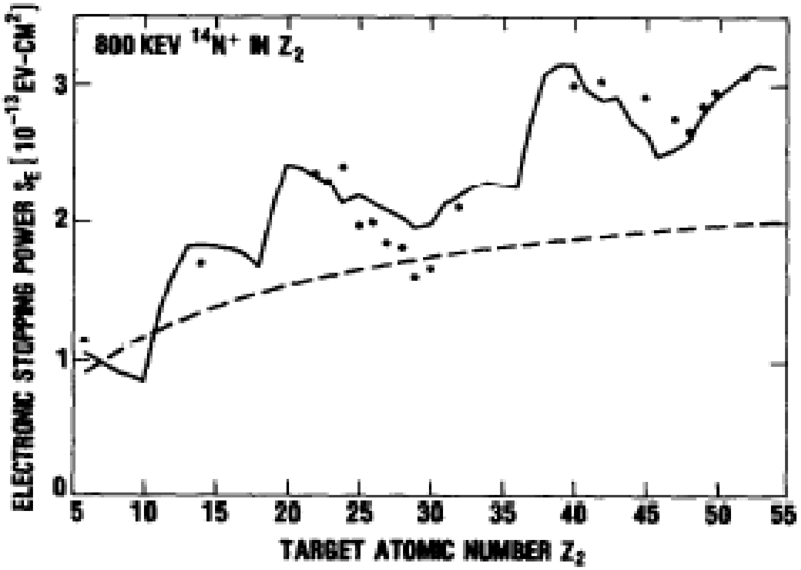


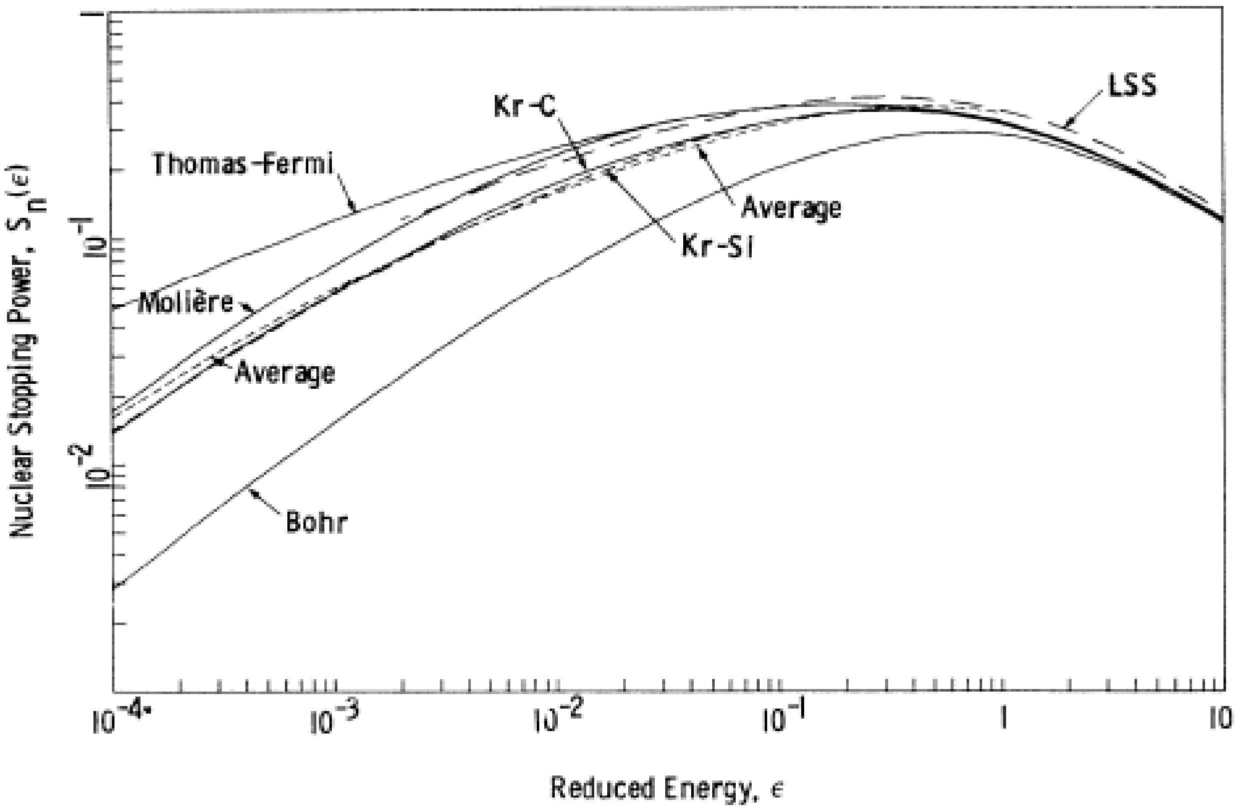
Fig. 2. Electronic stopping power as a function of target atom for incident 800-keV nitrogen ions ($v = 1.51\ v_0$): —— present values; – – – Lindhard-Scharff values; · · · data from NAVSWC[6,7]

● Aarhus expt.

- - - - Lindhard-Scharff

—— Land & Brennan, Atom Data Nucl Data Tables, 22, 235 (1978)

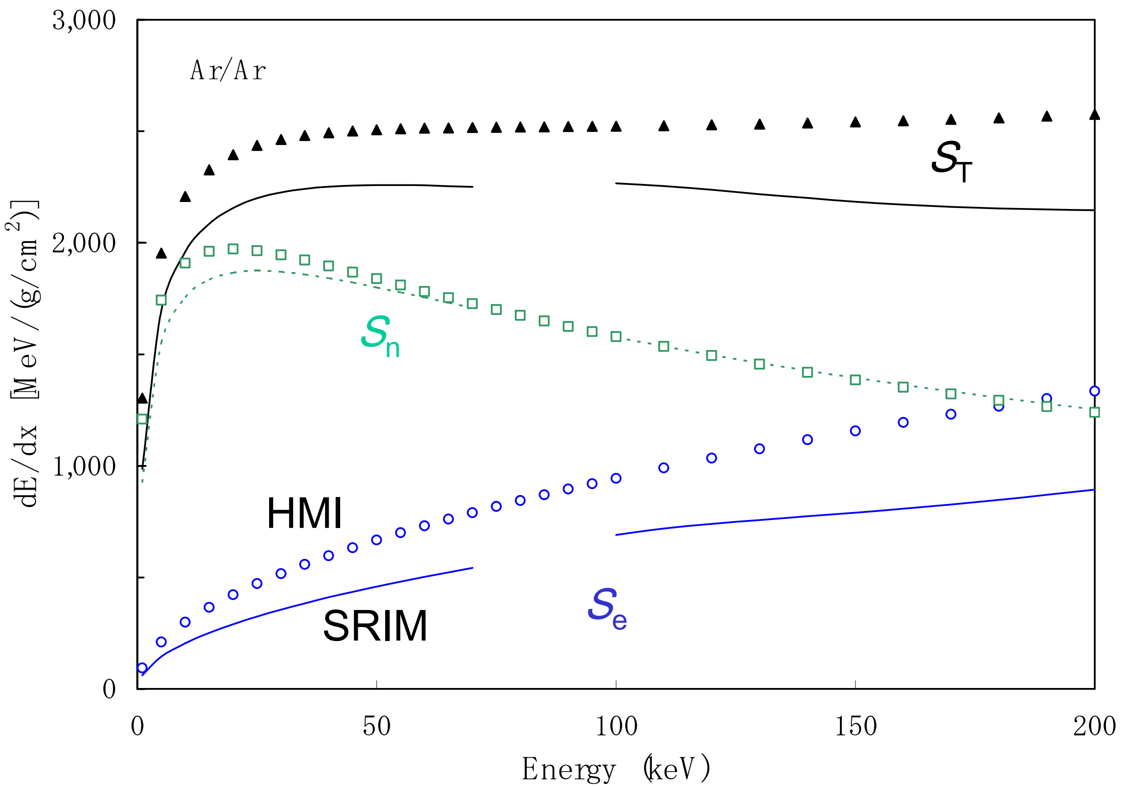
Nuclear Stopping Power



Comparisons of the nuclear stopping powers

D.W. Wilson et al. P.R.B. 15, 2458 (1977).

SP HMI & SRIM



Ar/Ar
3,000
2,000
1,000
0
dE/dx [MeV/(g/cm2)]
0
50
100
150
200
Energy (keV)
ST
Sn
Se
HMI
SRIM