Physics of Massive Neutrinos

1. Theory of neutrino masses
2. Theory of neutrino oscillations
3. Direct neutrino mass measurements
4. Experimental status of neutrino oscillations
5. Conclusions

Theory of neutrino masses

Two ways to add a neutrino mass term to the Standard Model Lagrangian:

1. *Dirac neutrino*

$$-m_D\left(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L\right)$$

just like the other fermion masses.

2. *Majorana neutrino*

$$-\tfrac{1}{2}m_L^M\left(\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L\right) - \tfrac{1}{2}m_R^M\left(\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R\right)$$

where $\nu^c = C\overline{\nu}^T = C\gamma_0\nu^*.$

Majorana neutrinos are their own antiparticles: lepton number violating.

General Lagrangian with both types of mass term:

$$\mathcal{L}_{\text{mass}} = -\tfrac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\mathcal{M}\begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix} + h.c.$$

where the mass matrix $\mathcal{M}$ is

$$\mathcal{M} = \begin{pmatrix} m_L^M & m_D \\ m_D & m_R^M \end{pmatrix}.$$

Assume $$M = m_R^M >> m_D >> m_L^M = \mu,$$

then

$$N \approx \left(\nu_R + \nu_R^c\right) + \frac{m_D}{M}\left(\nu_L + \nu_L^c\right);$$

$$\nu \approx \left(\nu_L - \nu_L^c\right) + \frac{m_D}{M}\left(\nu_R - \nu_R^c\right)$$

with masses

$$m_N \approx M;$$

$$m_\nu \approx \left|\mu - \frac{m_D^2}{M}\right|.$$

Seesaw mechanism:

N is righthanded, sterile, very massive;

ν is lefthanded and light.

Natural explanation of small neutrino masses; expect $m_\nu \propto m_D^2$ if M same for all generations.

Theory of neutrino oscillations[*]

Expect flavour eigenstates $|\nu_\ell\rangle \neq$ mass eigenstates $|\nu_m\rangle$:

$$|\nu_\ell\rangle = \sum_m U_{\ell m} |\nu_m\rangle,$$

where U is a 3×3 unitary mixing matrix analogous to the CKM matrix.

Each mass eigenstate propagates according to

$$|\nu_m(t)\rangle = e^{-i(E_m t - p_m L)} |\nu_m(0)\rangle,$$

L = distance travelled = t if $c = 1$ and $v \approx c$.

If original $|\nu_\ell\rangle$ had a definite energy E, then

$$p_m = \sqrt{E^2 - M_m^2} \approx E - \frac{M_m^2}{2E},$$

giving $$|\nu_m(t)\rangle \approx \exp\left(-i\left(M_m^2/2E\right)L\right)|\nu_m(0)\rangle$$

and $$|\nu_\ell(L)\rangle \approx \sum_m U_{\ell m} \exp\left(-i\left(M_m^2/2E\right)L\right)|\nu_m\rangle.$$

[*] This treatment follows Kayser's article in PDG 2000

Hence $$\left|\nu_\ell(L)\right\rangle \approx \sum_{\ell'}\left(\sum_m U_{\ell m}\exp\left(-i\left(M_m^2/2E\right)L\right)U_{\ell' m}^*\right)\left|\nu_{\ell'}\right\rangle.$$

At time t (or distance L) the original flavour eigenstate $\left|\nu_\ell\right\rangle$ has become a superposition of flavour eigenstates.

Probability of flavour ℓ' at distance L is

$$P\left(\nu_\ell \to \nu_{\ell'};L\right)=\left\langle \nu_{\ell'}\middle|\nu_\ell(L)\right\rangle \approx \left|\sum_m U_{\ell m}\exp\left(-i\left(M_m^2/2E\right)L\right)U_{\ell' m}^*\right|^2.$$

This is the basic equation governing **neutrino oscillations**.

Two-flavour case:

$$U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

and $$P\left(\nu_\ell \to \nu_{\ell'};\, L\right)=\sin^2 2\theta\sin^2\left(\frac{\Delta m^2 L}{4E}\right),$$

where $$\Delta m^2 = m_2^2 - m_1^2,$$

Often written as

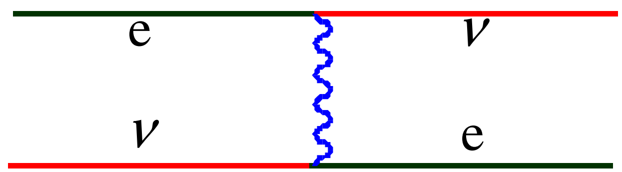
$$\sin^2\left(\pi L/L_V\right)$$

where

$$L_V=\frac{4\pi E\hbar}{\Delta m^2 c^3} = 2.48\frac{E\ (\mathrm{MeV})}{\Delta m^2\ (\mathrm{eV}^2)}\ \mathrm{m}$$

Neutrino oscillations in matter (MSW effect) [*]

CC interaction



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applies to electron neutrino only: modifies mass matrix.

Find $$\Delta_M = \left(\left(\Delta_V \cos 2\theta_V - \sqrt{2} G_\mathrm{F} n_\mathrm{e} \right)^2 + \left(\Delta_V \sin 2\theta_V \right)^2 \right)^{1/2}$$

and $$\tan 2\theta_M = \frac{\tan 2\theta_V}{1 \pm (L_V / L_\mathrm{e}) \sec 2\theta_V},$$

where $\Delta_V = \left(m_2^2 - m_1^2 \right) / 2E$, n_e is the electron number density, and

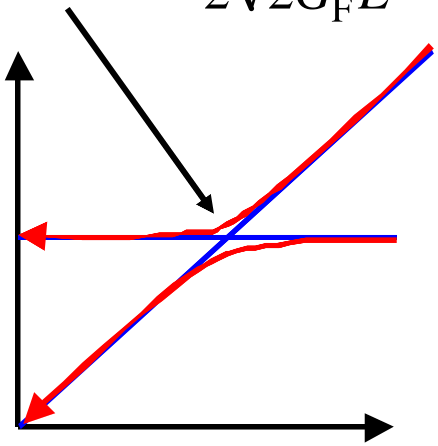
$$L_\mathrm{e} = \frac{\sqrt{2}\pi\hbar c}{G_\mathrm{F} n_\mathrm{e}}.$$

[*] following John Bahcall, *Neutrino Astrophysics*

The mixing angle in matter is maximised at the **MSW resonance density**

$$n_{\mathrm{e,res}} = \frac{|\Delta m^2| \cos 2\theta_V}{2\sqrt{2} G_{\mathrm{F}} E},$$

If ν_{e} generated at $n > n_{\mathrm{e,\,res}}$, it is the heavy eigenstate ν_2.



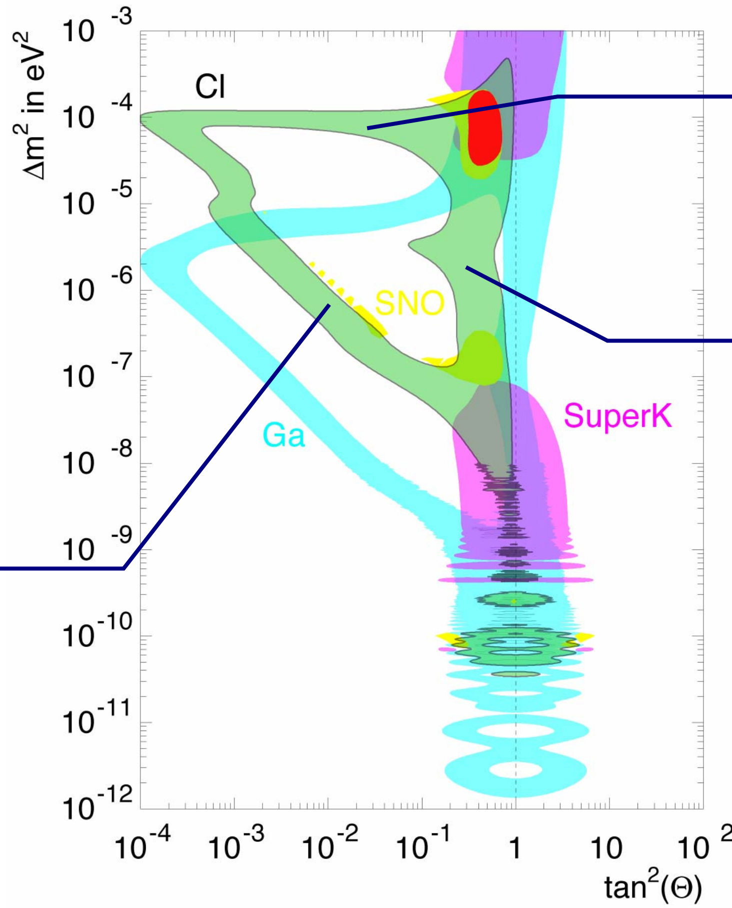
If it then propagates through gradually decreasing n_{e}, it will stay as ν_2—and hence be transformed into ν_x.

Survival probability: $$\left|\langle \nu_{\mathrm{e}} | \nu_{\mathrm{e}} \rangle_t\right|^2 = \tfrac{1}{2} + \left(\tfrac{1}{2} - P_{\mathrm{jump}}\right) \cos 2\theta_M \cos 2\theta_V,$$

where θ_M is evaluated at the production point and P_{jump} is the probability of crossing between mass eigenstates,

$$P_{\mathrm{jump}} = \exp\left(-\frac{\pi \Delta m^2 \sin^2 2\theta_V}{4E\cos 2\theta_V} \left(\frac{n_{\mathrm{e}}}{|\mathrm{d}n_{\mathrm{e}}/\mathrm{d}r|} \right)_{\mathrm{res}} \right) \qquad \left(n_{\mathrm{e}} > n_{\mathrm{e,\,res}}\right).$$

The MSW effect is usually applied to the solar neutrino problem. Analysing rates in terms of the MSW survival probability produces triangular contours in the mass vs mixing angle plot:



produced above resonance

ν_e content of heavy eigenstate

non-adiabatic: set by P_{jump}

Three-flavour oscillations

We need a 3×3 mixing matrix analogous to the CKM matrix; it's known as the MNS or PMNS matrix. The most instructive parametrisation is

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\beta} & 0 & 0 \\ 0 & e^{i\beta'} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric neutrinos — reactor/T2K/NuMI OA — solar neutrinos

The Dirac phase δ is observable in neutrino oscillations but not in $\beta\beta$; the Majorana phases β and β' are observable in $\beta\beta$ but not in oscillations. So far we do not have a good measurement of θ_{13}, although it is known to be less than either θ_{12} or θ_{23} (both of which are near-maximal). Note that if $\theta_{13} = 0$ one cannot measure the phase δ.

Direct neutrino mass measurements

Neutrino masses can be directly measured in three ways:

1. measure missing energy: β decay for electron, $\tau \rightarrow 5h^{\pm}\nu_{\tau}$;
2. time of flight from an astrophysical source (SN 1987A);
3. neutrinoless ββ decay (${}^{A}_{Z}\mathrm{X} \rightarrow {}_{Z+2}^{\;\;A}\mathrm{X}' + 2\mathrm{e}^{-}$; Majorana neutrinos only).

They can also be inferred from cosmological data, which are sensitive to the total Hot Dark Matter content of the Universe.

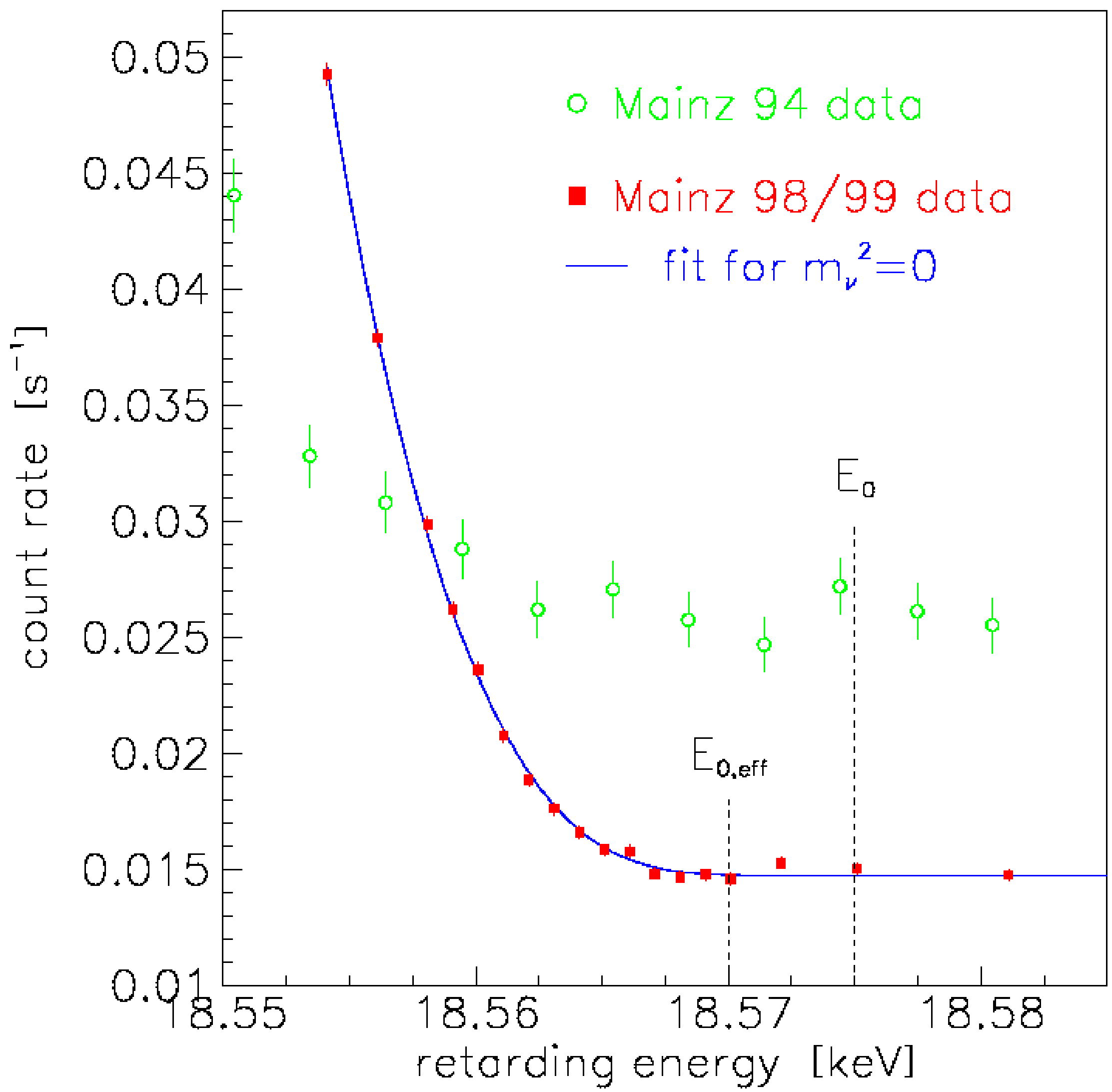
Tritium β decay

Principle is straightforward: measure endpoint of electron energy spectrum, look for difference from $m_\nu = 0$ prediction.

Main problem: evil tendency to give negative values – traced to problems with the source (tritium film on graphite substrate).

Best current limit <2.2 eV (95% CL) from Mainz experiment.

Future prospect: KATRIN experiment (gaseous tritium): prospective sensitivity ~0.2 eV.



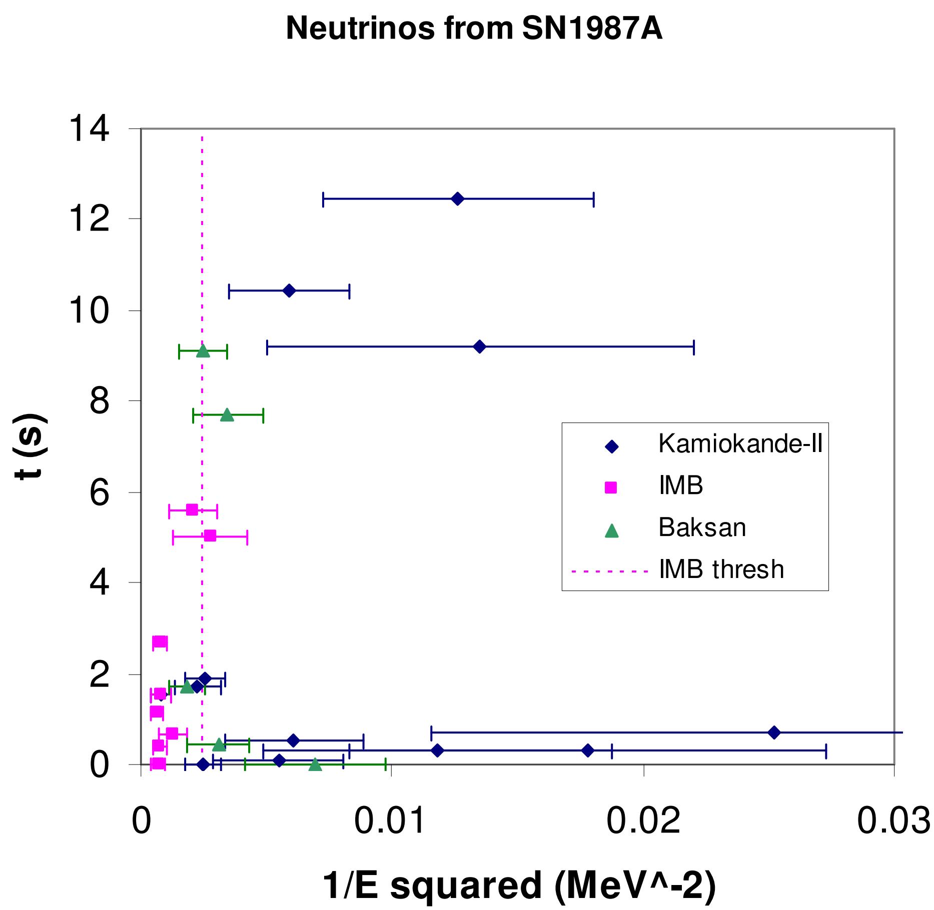
Time of flight

In units where $c = 1$, $E^2 = p^2 + m^2$ and $\gamma = E/m$, so for relativistic neutrinos

$$v = \frac{p}{\gamma m} = \frac{p}{E} \approx 1 - \frac{m^2}{2E^2}.$$

Therefore if a supernova produces an instantaneous burst of neutrinos, we should find that the arrival time is correlated with the energy,

$$t = \frac{D}{v} \approx D\left(1 + \frac{m^2}{2E^2}\right).$$

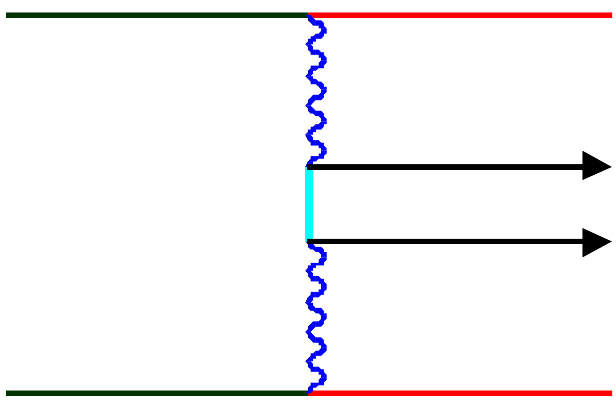


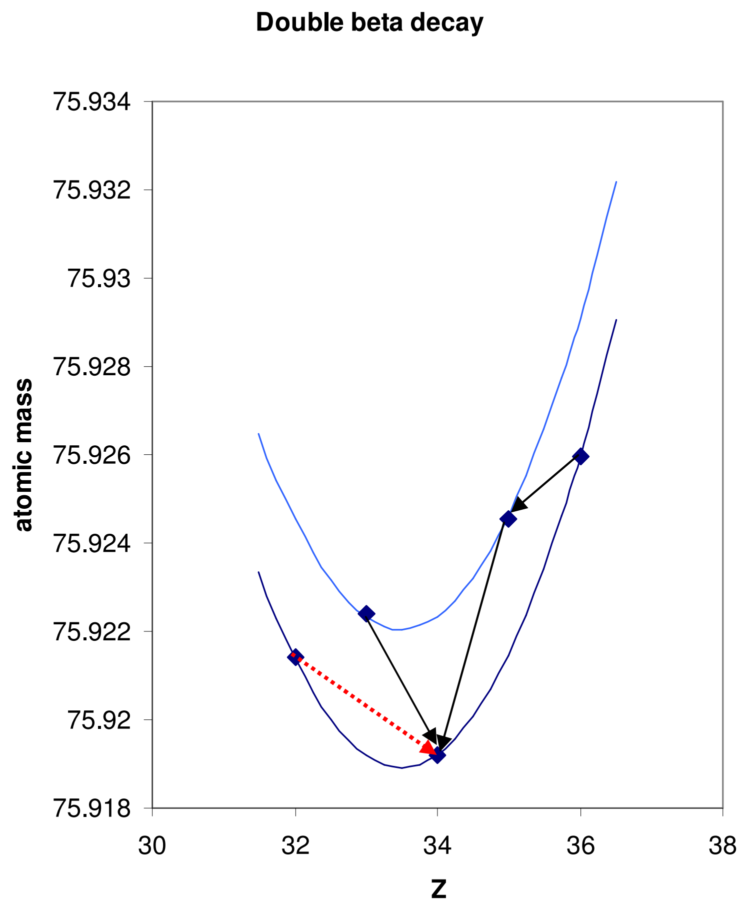
Unfortunately a supernova does *not* produce an instantaneous burst of neutrinos, so this doesn't work well. Best limit is from a Bayesian analysis by Loredo and Lamb (astro-ph/0107260) which gives $m_\nu < 5.7$ eV for electron antineutrinos.

Double beta decay

For even isobars, the pairing term means that even-even nuclei are more tightly bound than odd-odd. So it may not be possible to reach the most stable isobar through single beta decays.

These isobars can decay with *very* long lifetime through double beta decay ($\beta\beta 2\nu$). But with Majorana neutrinos they can also decay through neutrinoless double beta decay ($\beta\beta 0\nu$).





The signal is a spike at the energy endpoint. The limit depends on the nuclear matrix element: best current limit ~0.35 eV.

Cosmological bounds

Big Bang Nucleosynthesis

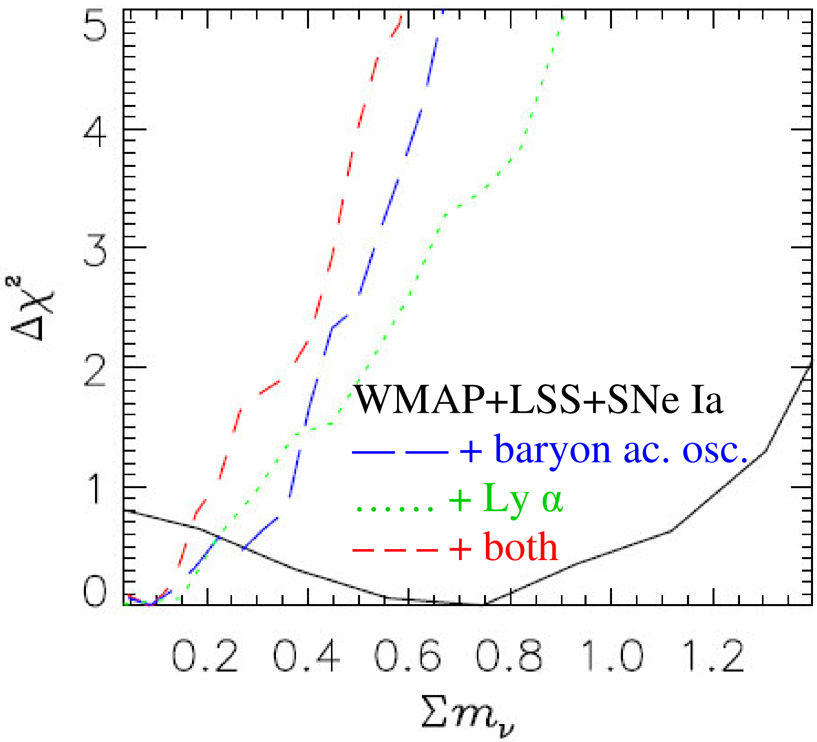
Constrains the effective number of neutrino species, *including* sterile neutrinos, to <3.4 (95%)

(Pierce & Murayama, hep-ph/0302131)

Large Scale Structure

Neutrinos decouple before structure forms, then free-stream. Result is loss of power at small angular scales.

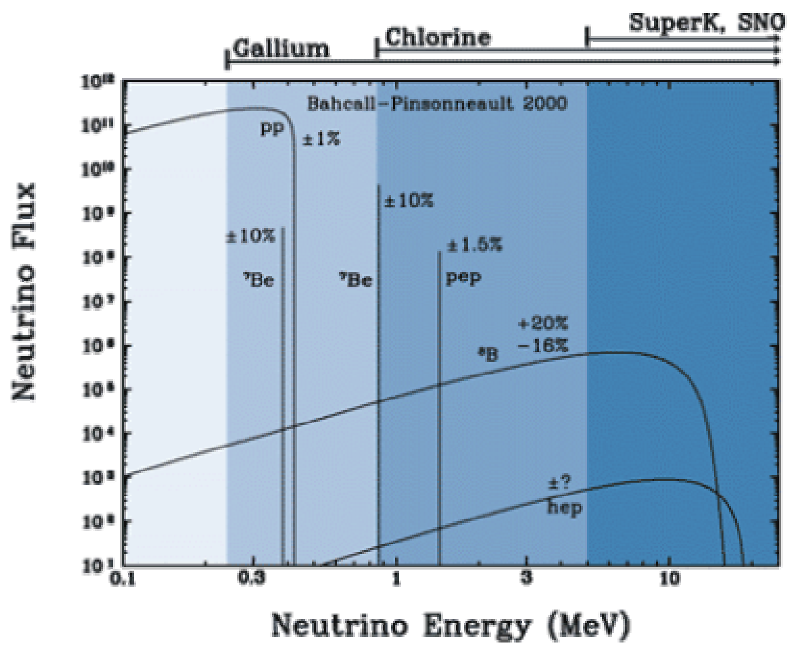
Limits depend on exactly which data you use and what priors you assume. Goobar et al. (astro-ph/0602155) get $\sum_{i=1}^{3} m_{\nu i} < 0.62\,\mathrm{eV}$ (95% CL) for a very general model, and $\sum_{i=1}^{3} m_{\nu i} < 0.48\,\mathrm{eV}$ assuming 3 neutrinos and a cosmological constant equation of state for dark energy.

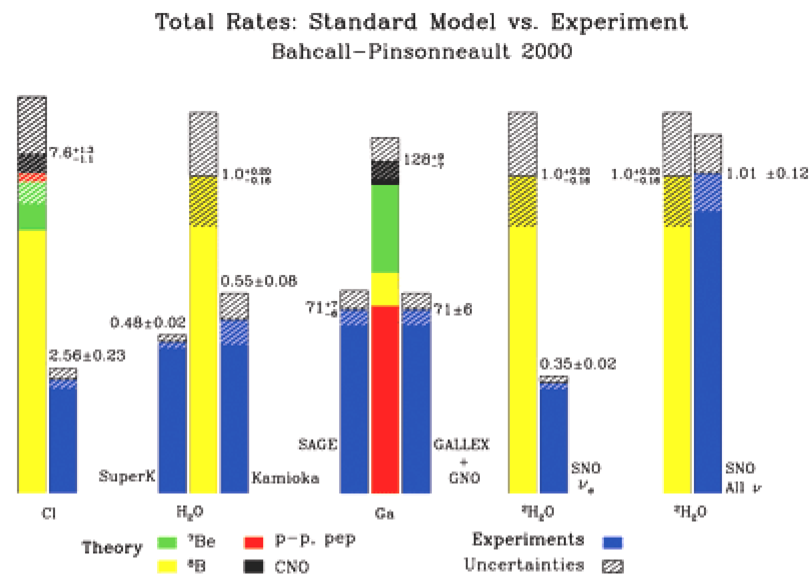


Goobar et al, astro-ph/0602155

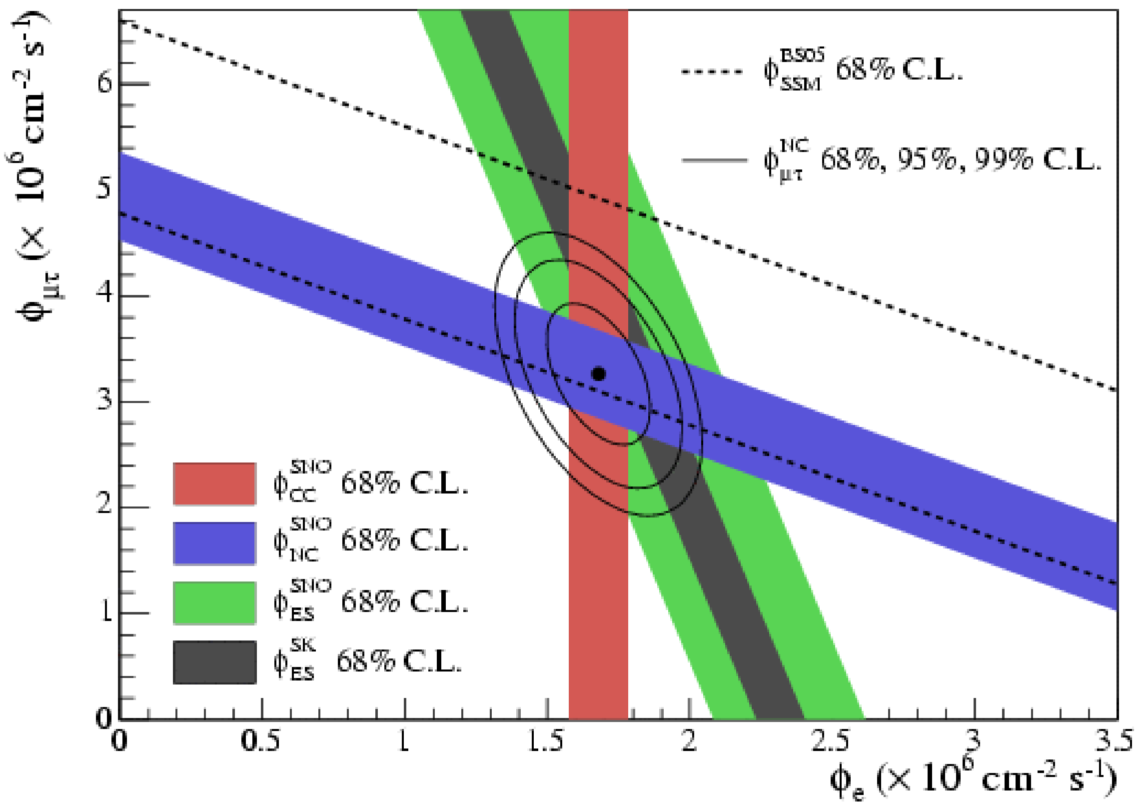
Experimental status of neutrino oscillations

Solar neutrinos (Pictures from John Bahcall)

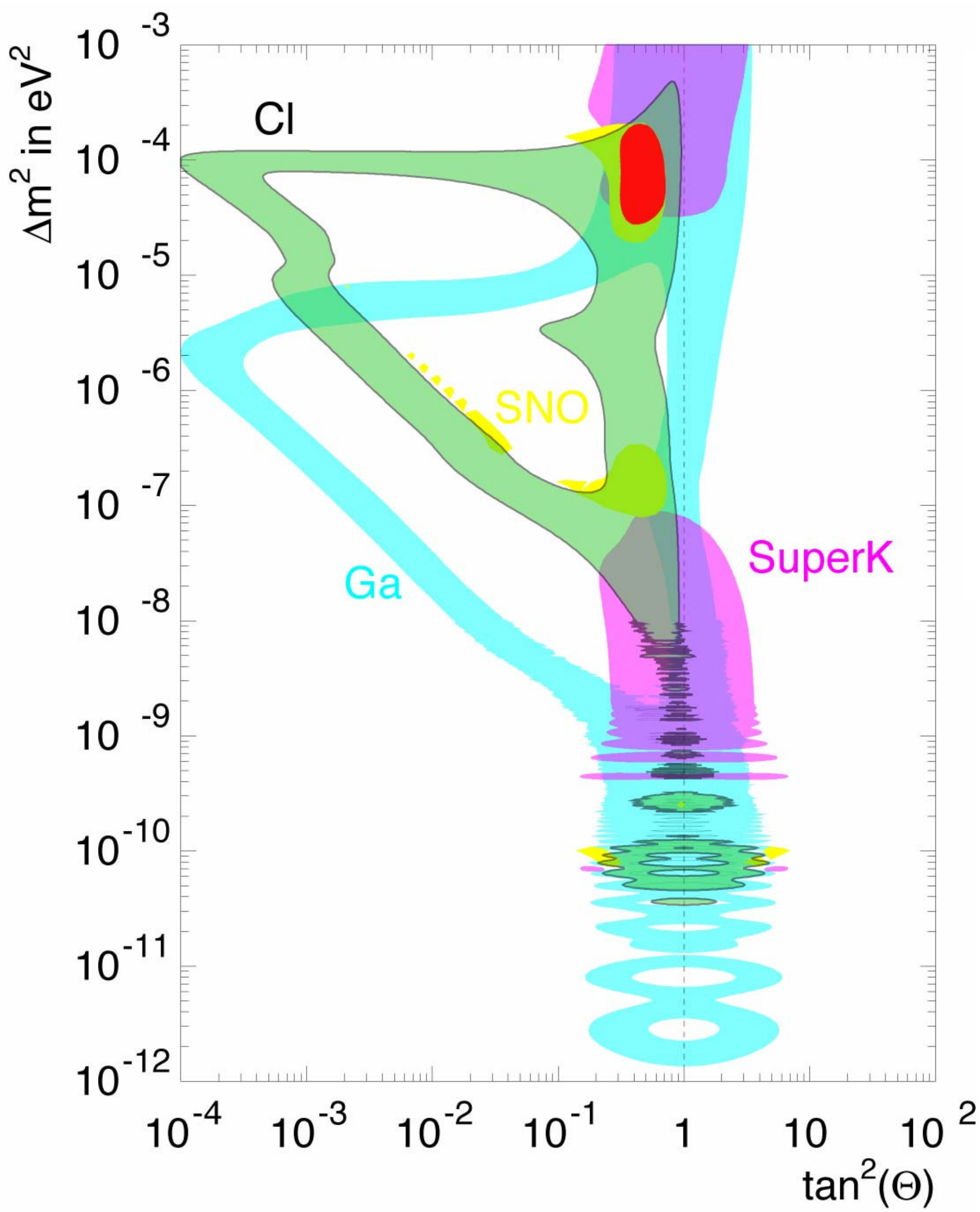




All experiments sensitive only to electron neutrinos see fewer than expected. If we accept their error bars, they also see *different* suppression factors: this implies that the oscillation probability must be energy dependent (MSW, or vacuum oscillations fine-tuned so that $L_V \sim 1$ AU).



SNO neutral current data confirm that there is *no* deficit if all flavours are taken into account—does not support large mixing into sterile neutrino.

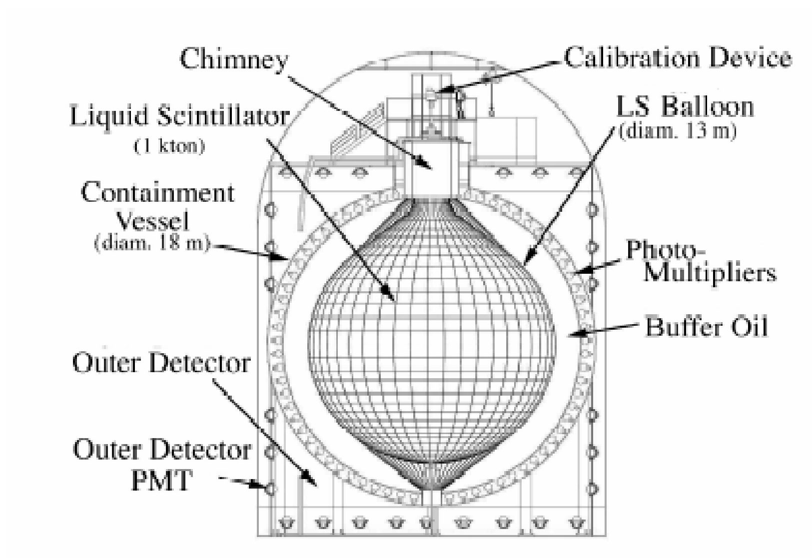


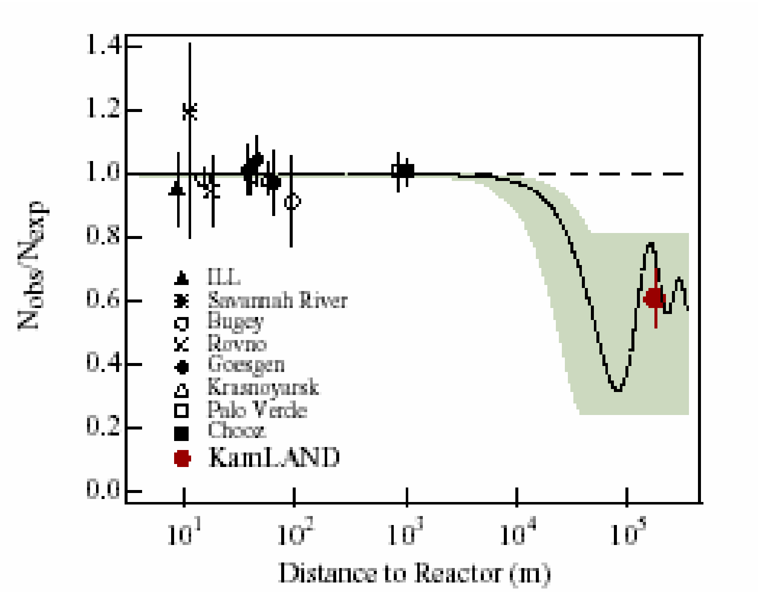
Solar neutrino summary plot from Murayama at http://hitoshi.berkeley.edu/neutrino

Note use of $\tan^2\theta$ instead of $\sin 2\theta$: this is because the MSW effect is *not* symmetric around 45° (it matters which species is the heavy one).

KamLAND experiment

Reactor neutrino experiment sensitive to solar neutrino parameters

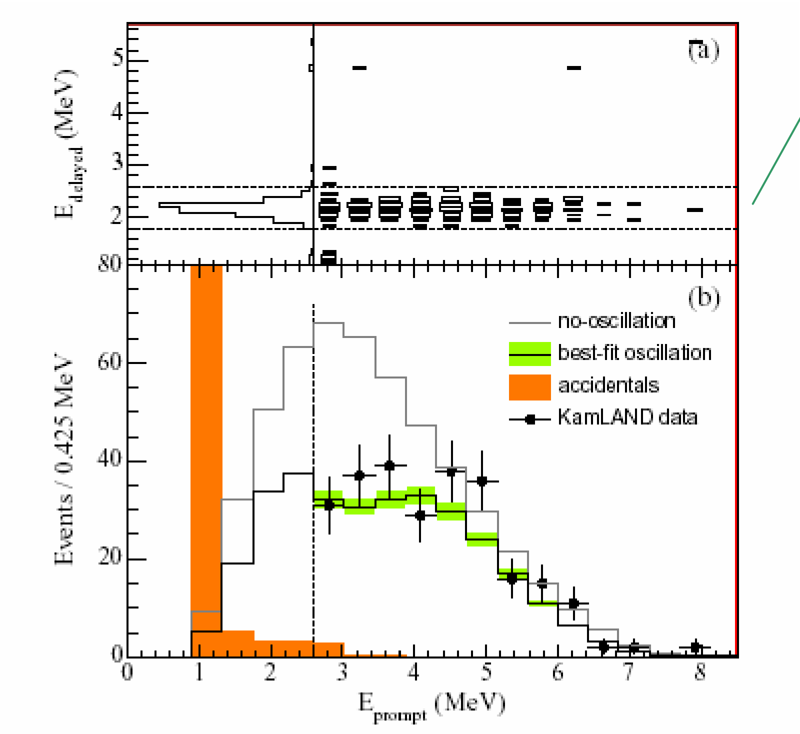


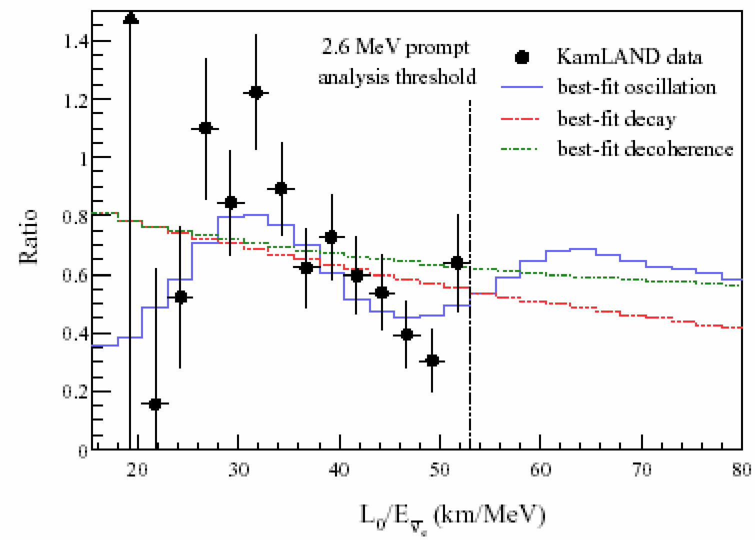


Detection technique:

- Inverse beta decay, $\bar{\nu}_e + p \to e^+ + n$, produces prompt scintillation light plus annihilation γs from positron.
- Neutron capture on to hydrogen, $n + {}^1H \to {}^2H + \gamma$, produces 2.2 MeV γ after ~200 μs delay

band for 2.2 MeV γ from neutron capture on [1]H

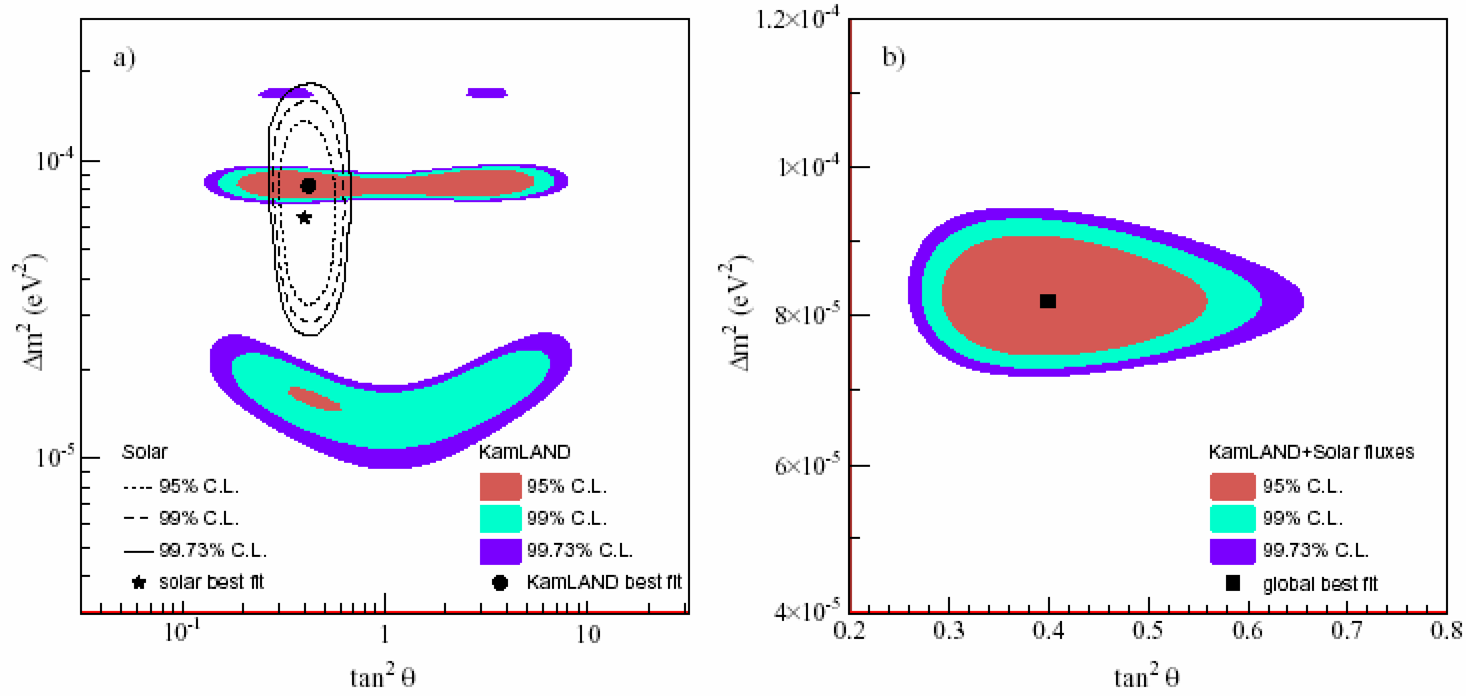




KamLAND data show clear deficit and spectral distortion consistent with oscillation model.

KamLAND 2004: T. Araki et al., hep-ex/0406035

Comparison with solar neutrino data



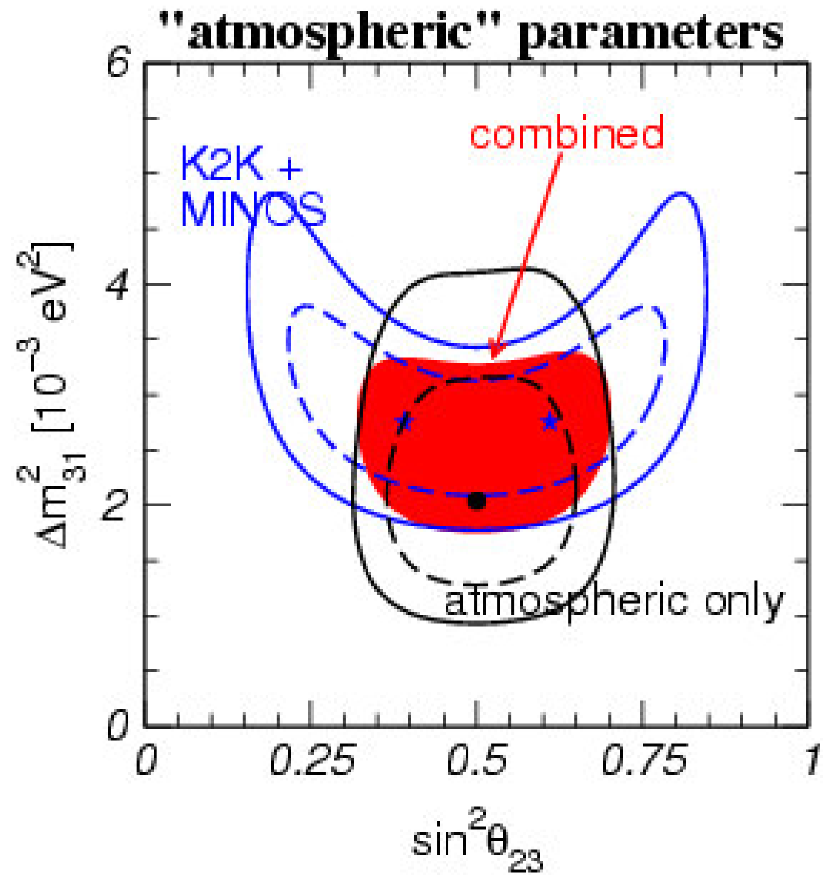
KamLAND result consistent with LMA solution to solar neutrino deficit.

Combination very effective at restricting parameter space, because of different alignment of error ellipse (vacuum oscillations vs MSW effect).

Atmospheric neutrinos

Atmospheric neutrinos are generated by the interaction of cosmic ray protons with the upper atmosphere, producing pions whose decays yield muon and electron neutrinos in the ratio 2:1.
SuperKamiokande finds a deficit of muon neutrinos in the upgoing direction, corresponding to vacuum oscillation in the Earth. The preferred solution is $\nu_\mu \to \nu_\tau$.

The result is confirmed by long-baseline experiments (K2K, MINOS).



Thomas Schwetz, hep-ph/0606060

Note that this is again near-maximal mixing, in contrast to the near-diagonal CKM matrix.

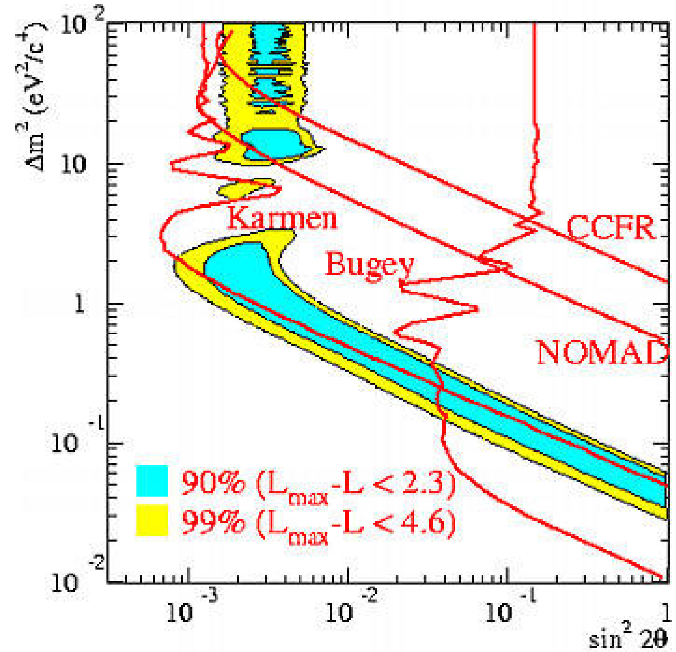
LSND

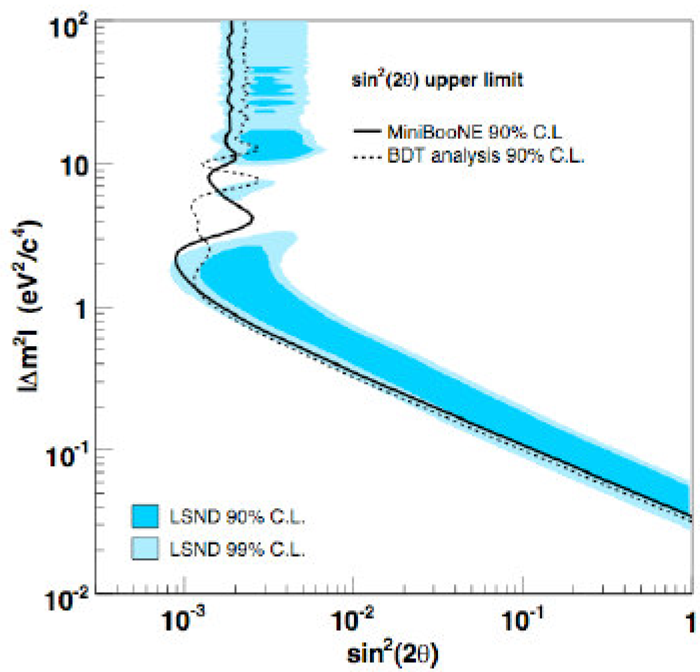
This was a short-baseline (30 m) accelerator experiment studying $\overline{\nu_\mu} \to \overline{\nu_e}$ and $\nu_\mu \to \nu_e$ by looking for inverse β decay—i.e. it was an appearance experiment. It reported a positive signal, shown by coloured contours.

Some, but not all, of the preferred region is excluded by the KARMEN experiment at RAL. The inferred mass difference is ~0.5 eV, inconsistent with solar and atmospheric results – a 4th neutrino??

Not a popular result! Recently checked by MiniBooNE experiment at Fermilab: excluded at 98% CL.

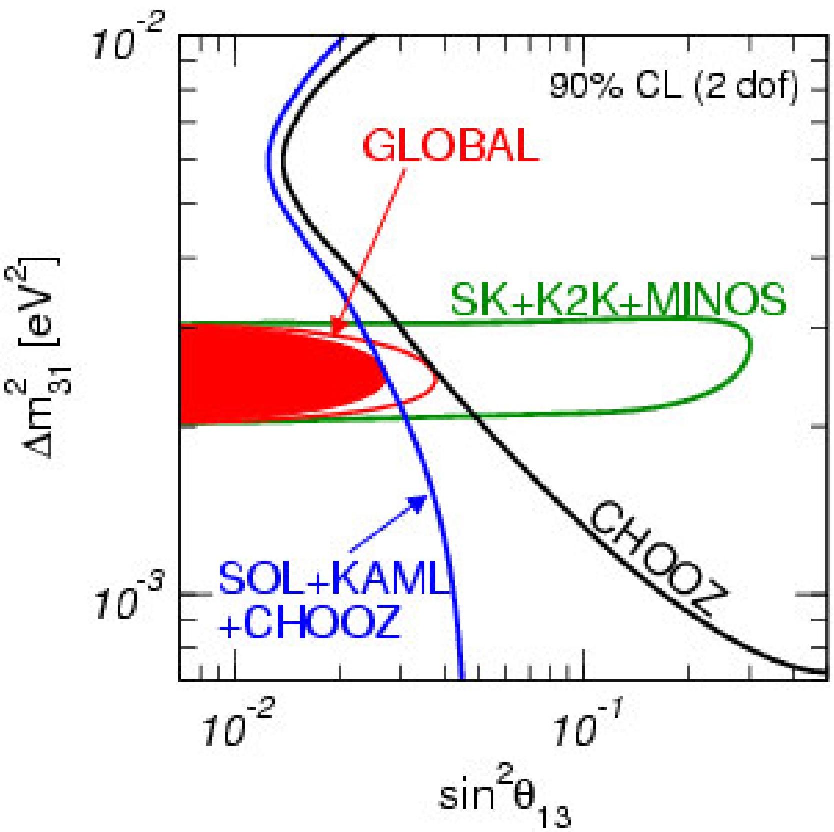
Ignoring LSND allows us to retain "simple" picture of three massive neutrinos with different weak and mass eigenstates.





T2K

Current experiments have focused on ν_1-ν_2 (solar/KamLAND, small mass difference) and ν_2-ν_3 (atmospheric, large mass difference) mixing. To study CP violation effects need θ_{13} as well; currently there are only upper limits, dominated by the reactor neutrino experiment at Chooz. This is a sub-leading effect: need to use full three-neutrino mixing formula…



Thomas Schwetz, hep-ph/0606060

…which is truly hideous…

$$P(\nu_\mu \to \nu_e) = 4C_{13}^2 S_{13}^2 S_{23}^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} \times \left(1 + \frac{2a}{\Delta m_{31}^2} \ (1 - 2S_{13}^2)\right)$$
$$+8C_{13}^2 S_{12} S_{13} S_{23} (C_{12} C_{23} \cos\delta - S_{12} S_{13} S_{23}) \cos \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{21}^2 L}{4E}$$
$$-8C_{13}^2 C_{12} C_{23} S_{12} S_{13} S_{23} \sin\delta \sin \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{21}^2 L}{4E}$$
$$+4S_{12}^2 C_{13}^2 \left\{ C_{12}^2 C_{23}^2 + S_{12}^2 S_{23}^2 S_{13}^2 - 2C_{12} C_{23} S_{12} S_{23} S_{13} \cos\delta \right\} \sin^2 \frac{\Delta m_{21}^2 L}{4E}$$
$$-8C_{13}^2 S_{13}^2 S_{23}^2 \cos \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \frac{aL}{4E} \ (1 - 2S_{13}^2)$$

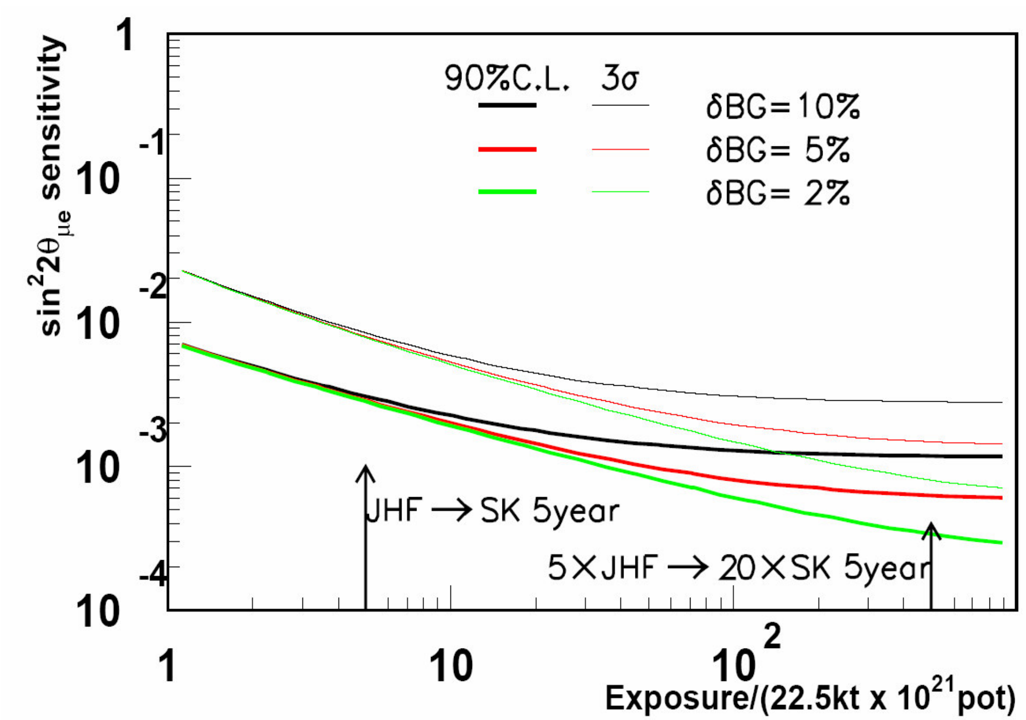
…or a slightly friendlier approximation…

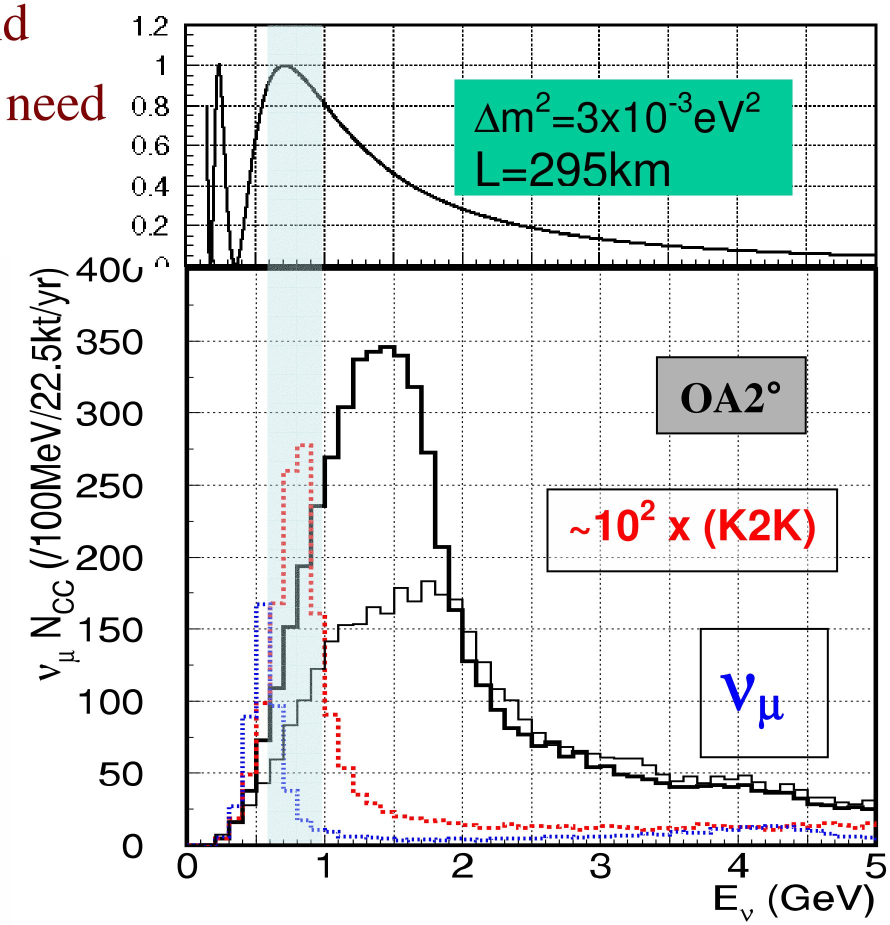
$$P_{e\mu} \cong \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 \Delta$$
$$\mp \alpha \sin 2\theta_{13} \sin\delta \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin^3 \Delta$$
$$- \alpha \sin 2\theta_{13} \cos\delta \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos\Delta \sin 2\Delta$$
$$+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2 \Delta$$

where $\alpha = \Delta m_{21}{}^2 / \Delta m_{31}{}^2$ and $\Delta = \Delta m_{31}{}^2 L / 4E$

T2K experiment aims to send a muon neutrino beam from a new nuclear physics facility at JPARC to Super-Kamiokande (295 km baseline). The beam is directed slightly off-axis to give better energy resolution.

Critical issue for sensitivity is background control: intrinsic ν_e content of beam, and π^0s produced by inelastic interactions and mistaken for electrons in Super-K. For this, need a near detector (possibly two!).





Conclusions

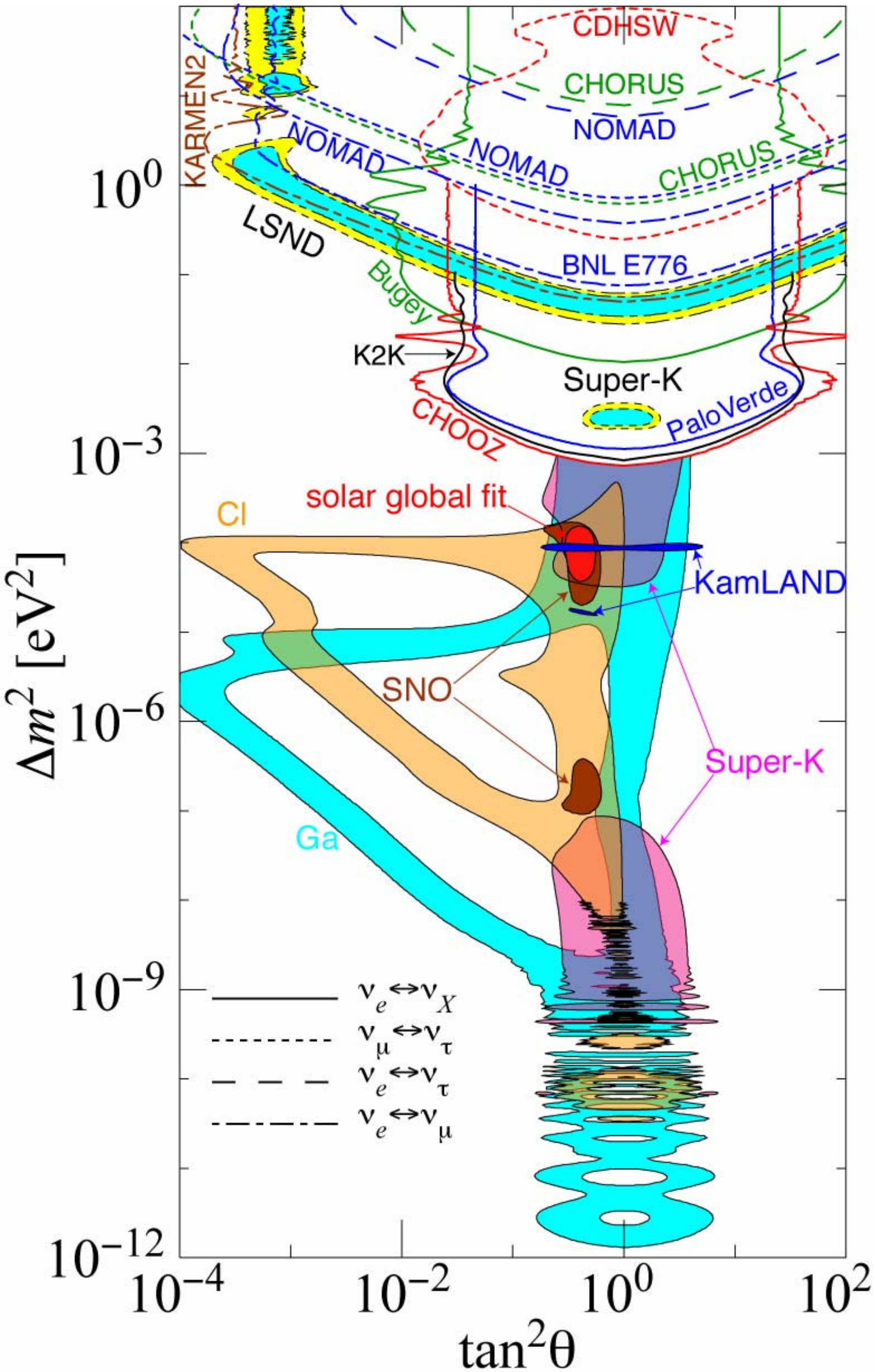
In any case it seems clear that the neutrino mixing matrix is very different in form from the quark CKM matrix! Giunti and Laveder (hep-ex/0310238) give as best fit

$$U = \begin{pmatrix} -0.84 & 0.55 & 0.00 \\ 0.39 & 0.59 & 0.71 \\ 0.39 & 0.59 & -0.71 \end{pmatrix}$$

Zhi-Zhong Xing (hep-ph/0307359) gives

$$|U| = \begin{pmatrix} 0.70-0.84 & 0.50-0.69 & <0.16 \\ 0.21-0.61 & 0.34-0.73 & 0.60-0.80 \\ 0.21-0.63 & 0.36-0.74 & 0.58-0.80 \end{pmatrix}$$

There are various symmetry patterns that could fit this, e.g. "tri-bimaximal mixing" – or it could be random ("anarchy")! One critical test is to measure θ_{13}.



http://hitoshi.berkeley.edu/neutrino