PHY340 Data Analysis Feedback:

Group A04 doing Problem A3

# Data Analysis

Much of the data analysis was quite well done, although not always well explained. For exam­ple, you talk about converting between electron energy and neutrino energy, but it is not obvi­ous where you need this conversion: you refer to “the data”, but there is no difference in the ener­gy scale between figure 1 and figure 3, so there has been no change to the energies pre­sented in the data. Perhaps you mean the model predictions, but these are not “data”.

One of your data points looks wrong: in figure 1, the 40 MeV point is clearly lower than the 36 MeV point (they appear to be 33 and 32 events respectively), whereas in figure 3 these points are level. As the efficiency is the same for both, this should not have happened: they should come out at 41.8 and 40.5 events. In your code, the corrected numbers seem to be rounded to the nearest integer: this is not standard practice (though it’s defensible on the grounds that you were only given the efficiency to two significant figures), but in any case does not explain the discrepancy, since 41.8 should round to 42 and 40.5 to 41 (it’s actually 40.506…, so rounds up). Fortunately, this should not have affected the results significantly, since it’s in a region domi­nated by the Michel spectrum and the difference is well within the statistical errors.

I cannot reproduce your straight line fit: I get a gradient of 0.31±0.22 and an intercept of 0±16. However, the large errors suggest that small variations could modify this significantly, so I thought perhaps the issue was simply that you rounded your values to integers—but when I put your numbers into Excel, I still didn’t get your values: I got 0.23±0.25 and −5±18. I can’t see how you could possibly get 0.6±0.1, and I *definitely* do not believe −30±1: even if the central value is what came out of your fit, there is no possible way that the uncertainty could be as small as ±1. You have not explained how you did this fit, but your code just looks wrong: it can’t possibly be right to be calculating
as you do in your loop, since the factor of just cancels out. In fact, the gradient of the least-squares fit to a straight line is given by
note that . (Try it: ½ + ½ = 1, whereas (1+1)/(2+2) = ½.) So this is a bug, and I think there is a similar bug in your calculation of the uncertainty.

Also, you are calculating an unweighted fit, which is not strictly correct: the points have dif­ferent errors, so you should be doing a weighted fit. In fact, in this case it doesn’t make much difference: I tried an unweighted fit, and got a gradient of 0.28±0.27 and an intercept of 0±19 (as opposed to 0.31±0.22 and 0±16 for the weighted fit). But it is bad practice to ignore the error bars on your points when fitting: sometimes it *will* make a difference.

The Michel electron spectrum appears to be correctly done, although your equation (5) is incorrectly quoted: it should end (meaning “plus terms of order ”), not
“+ *O*”, which does not make sense (especially as you have not defined *O*). This spectrum is not quite right, because it is lacking the radiative corrections; these are complicated, and the effect is small (see figure 1), so you are probably right not to try to include them, but you should state that you have not done so.

*Figure 1: Michel electron spectrum showing effect of radiative corrections, from ICARUS Collaboration (S Amoruso et al., Eur. Phys. J.* ***C33*** *(2004) 233−241).*

Your conversion of the fluxes given in the model to numbers of events for comparison with the data also seems to be properly done, although it is missing some references (especially for the neutrino cross-section, which also needs units: is not a dimensionless number, so the units do matter—they are almost certainly cm2 (MeV)−2, but you need to *say* that). As noted above, saying “the data” when you mean “the mo­del predictions” is very confusing to the reader.

Having done all the component parts more-or-less right (with the exception of the code bug in the straight line fit), you unfortunately make a com­plete hash of the last part. The problems are:

1. you apparently used a fitting package (“JMP”), but you neither reference it nor explain what it does;
2. no errors are quoted for *A* and *B*, and *A* in particular does not make any sense at all (as you originally fitted your straight line *to the data*, the normalisation should be very close to 1—maybe a bit different, since you are now fitting the whole data sample rather than just the last five bins, but definitely *not* 67.68);
3. you do not quote the result for *C* at all—even if the central value was zero, the fit should have returned an uncertainty: 0±5 is different from 0.0±0.1, for example;
4. you have not even attempted task 2, in which you were asked to set limits on the number of events, and hence the flux, for each of the models.

Point 4 is the reason why the error on your fitted model normalisation *C* is important. If the errors on your fitted parameters are Gaussian—which is by no means guaranteed, but it is what you were told to assume—then 90% of the time you would observe fewer than events, where *μ* is the mean and *σ* is its uncertainty. Therefore, if your fit tells you that the model contributes, say, 0.0±1.5 events to your data, the 90% confidence limit is 1.5 × 1.282 = 1.9 events.

In your conclusion, you say that “the value returned from the χ2 test of 1.945, showing a greater than 90% confidence, indicates…”. Apart from the dodgy sentence structure (it should read “the value of 1.945 returned from the χ2 test”—you aren’t testing the 1.945), this does not make any sense. The table you used states that, for 13 degrees of freedom, the probability of getting a χ2 value greater than or equal to 7.042 is 90%: this is *not* “a 90% confidence limit level.” (In fact, the 90% confidence limit is given by : it’s the χ2 value that *includes* (not *excludes*) 90% of the χ2 distribution. A total χ2 of 1.945 is really a ridiculously small number for 13 degrees of freedom, and looks distinctly unlikely to be right—in fact, looking at your code, it’s *not* right. You have calculated
However, the definition of χ2 is actually
where *σi* is the uncertainty on the *i*th point. *If and only if* the *yi* are raw counts, with Poisson errors of , you can replace by *y*fit (***NOT*** ), rather than *yi*, because you assume that *y*fit is a better esti­mate of *y*true than the observed value, and so its square root is a better estimate of the Poisson error than . This is Pearson’s χ2, as quoted in Wikipedia; it does *not* apply if the data are not raw counts. In fact your data are *not* raw counts: they are counts corrected for ef­fi­ciency, so their errors are larger than (because the actual number of entries is less than the corrected number of entries). Your numbers have a few errors in them, as noted above (and there’s another in the last five points, where the 76 MeV point rounds to 14 not 13), but using your numbers with the appropriate Poisson errors gives a total χ2 of 23.55.

I don’t believe your count of 13 degrees of freedom, because you used the data to define your straight line for the atmospheric neutrinos, so that’s another two degrees of freedom lost. How­ever, neglecting that, the χ2 probability for your fit is 3% (i.e. 97% of chance results would have a lower χ2). The reason it’s so bad is almost entirely just one point: at 56 MeV you have 19 points (with a statistical error of ±4.9) but expect only 2.4. This point contributes half your total χ2 (11.5 out of the total of 23.6). The reason for this is not that you have a supernova relic neu­trino signal: it’s that in calculating your Michel spectrum you have not allowed for the resolution of the Super-Kamiokande detector (in fairness, you weren’t asked to consider this). As you can see in figure 2, the ef­fect of this is that the upper cut-off of the Michel spec­­trum is not sharp, as shown in figure 1, but smeared out by the resolution (which is about 8 MeV at this en­ergy).

*Figure 2: Michel spectrum measured by Super-Kamiokande, from S Fukuda et al., Nucl. Inst. Meth.* ***A501*** *(2003) 418− 462. Points are data, line is MC.*

You had the right idea in this section, but your execution was not good. You really ought to know better by now than to quote a measured number without an associated error, and your understanding of χ2 and confidence limits needs work.

You also did not attempt task 3, the model-independent limit, but that was not compulsory.

Average mark for this section: 26/50

# Data presentation

Apart from the fact that you did not quote the uncertainties on your fit parameters *A* and *B*, and therefore probably quoted them to excessive precision (I am willing to bet that the uncertainty is not in the second decimal place!), your data presentation is generally good: your plots are clearly presented, with numbers and captions. The models in figure 2, and the Michel spectrum, should really be referred back to your primary sources, not simply to the literature search: it is not guaranteed that the reader has access to your literature search, since it is not a published do­cu­ment, and citing only yourselves gives the impression that you are the originators of the models. It is usually a good thing to distinguish lines by style as well as colour, in case your reader happens to print them out on a monochrome printer (but at least you did give us a colour copy: one group distinguished lines only by colour, *and then printed out the copies they handed in in black and white*)*.* Making diagrams readable in monochrome is a good habit to get into, because paper copies of journals are still usually black and white, even though the electronic versions are in colour, and journal editors therefore want figures that are readable in mono­chrome; also, a significant fraction of the popu­lation (about 8% of men) has defective colour vision and may not see the lines as you intend them to look.

Average mark for this section: 19.2/30

# Style

The general appearance and style of the report have both improved compared to the literature search: well done! That said, it would still have benefited from careful and thorough proof-reading. At the beginning of section 2, you state that “the given experimental data had units of flux, while the models found in the literature search were given in event rates.” This is exactly the wrong way round: the data are event rates, the models are expressed in terms of flux. We are sure you know this, since your plots are all in terms of event rates, so the problem lies in the proof-reading (or lack of it) and not in the understanding. As noted above, your cross-section needs units, and the order of presentation needs to be reconsidered: you can’t quote the cross-section until you have explained what interaction you are quoting the cross-section *for*, and you can’t quote it as until you have explained that at these energies (since in fact the cross-section is ). A water molecule contains hydrogen *atoms*, not hydrogen *molecules*, and it is not at all clear what you mean by “outer” hydrogen (water is H2O, and both the H atoms are available). You may mean that the protons inside the oxygen nucleus aren’t involved; if that *is* what you mean, then *say* that. In equation (4), you mean , not or (I’m not sure exactly what that subscript is, but it definitely isn’t *ν*). You also scatter capital letters round at random: why are “Kinetic Energy”, “Neutrino” and “Electron” all capitalised on page 2?

There are one or two punctuation errors, and a few odd word choices: “a fitting” is something you get at a dressmaker’s; what you are doing is “a fit”. As noted in the previous sections, sometimes you don’t quite say what you intend to say, usually because of poor sentence struc­ture. But the improvement over the literature search report is very significant, so you are on the right track.

Average mark for this section: 11.8/20

Overall average mark: 57%