Neutrino Mass, Flavour and CP Violation

- Introduction (neutrino masses, mixings, oscillation formulae)
- •Cosmological limits on neutrino mass
- •CP violation at a neutrino factory
- Theory of neutrino mass (Majorana, Dirac, See-saw, Majorana matrices)
- •Natural hierarchy (sequential right-handed neutrino dominance)
- •Relation between neutrino factory CP violation and leptogenesis?
- •SUSY and lepton flavour violation

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11/04/2003

Solar neutrino oscillations

 Energy of solar neutrinos is quite low (MeV).

 85% of the 40 billion solar neutrinos per cm² per sec reaching earth come from:

 $p + p \rightarrow d + e^+ + V_e$



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Total Rates: Standard Model vs. Experiment Bahcall-Pinsonneault 2000







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Global 3 neutrino analysis Fogli et al hep-ph/0212127



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 10^{-3} From Holanda and Smirnov hep-ph/0212270 Impact of Future SNO Measurements Dashed lines are ∆m² (eV²) SNO CC/NC Dotted lines are SNO N/D % Shaded region is 0.2 0.25 0,3 0.345 1sigma and 0.4 0.45 15 3 sigma fits 20 10^{-5} 0.2 0.3 0.4 0.5 0.6 0.7 0.1

11

0.01

0.1

0.5

1

2

3

4 5

0.5

0.9

1

10

0.8

tg²θ

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Totsuka, Neutrino Houches, 2001

Atmospheric neutrinos



Atmospheric Neutrino Oscillations

 ν_{μ} - ν_{τ} ∆m² (eV²) 5 -1 10 10 68% C.L. 90% C.L. 99% C.L. 10 ο.9 1 sin²2θ 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8

WMAP





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Cosmological limits on neutrino mass



Animations due to Tegmark

2dF Galaxy Redshift survey astro-ph/0204152

And WMAP implies

$$\sum_i m_{v_i} < 0.69 \, eV$$

Neutrino oscillations then imply

$$m_{\nu_i} < 0.23 \, eV$$

Per neutrino species

Possible three neutrino mass patterns with LMA $\Delta m_{21}^2 = (0.008 \ eV)^2$ and $|\Delta m_{32}^2| = (0.05 \ eV)^2$.





The Neutrino Mixing Matrix U

$$V^{E_{L}}m_{LR}^{E}V^{E_{R}^{\dagger}} = \begin{pmatrix} m_{e} & 0 & 0 \\ 0 & m_{\mu} & 0 \\ 0 & 0 & m_{\tau} \end{pmatrix} \xrightarrow{\text{Majorana matrix}} V^{\nu_{L}}V^{\nu_{L}}^{T} = \begin{pmatrix} m_{1} & 0 & 0 \\ 0 & m_{2} & 0 \\ 0 & 0 & m_{3} \end{pmatrix}$$

Constructing
$$U = V^{E_{L}}V^{\nu_{L}^{\dagger}}$$

Parametrising
$$\longrightarrow U = R_{23}U_{13}R_{12}P_{12}$$
 $P_{12} = \begin{pmatrix} e^{i\beta_1} & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \qquad U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \qquad R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Three Family Oscillation formulae in vacuum

$$P(v_{e} \rightarrow v_{\mu}) = P^{+}(v_{e} \rightarrow v_{\mu}) + P^{-}(v_{e} \rightarrow v_{\mu})$$

$$P(\overline{v}_{e} \rightarrow \overline{v}_{\mu}) = P^{+}(\overline{v}_{e} \rightarrow \overline{v}_{\mu}) + P^{-}(\overline{v}_{e} \rightarrow \overline{v}_{\mu})$$

$$CP \text{ even } P^{+}(\overline{v}_{e} \rightarrow \overline{v}_{\mu}) = P^{+}(v_{e} \rightarrow v_{\mu})$$

$$CP \text{ odd } P^{-}(\overline{v}_{e} \rightarrow \overline{v}_{\mu}) = -P^{-}(v_{e} \rightarrow v_{\mu})$$

$$\begin{split} P^{+}(v_{e} \rightarrow v_{\mu}) &= -4 \operatorname{Re}(U_{e1}U_{\mu1}^{*}U_{e2}^{*}U_{\mu2}) \sin^{2}(1.27\Delta m_{21}^{2}L/E) \\ &- 4 \operatorname{Re}(U_{e1}U_{\mu1}^{*}U_{e3}^{*}U_{\mu3}) \sin^{2}(1.27\Delta m_{31}^{2}L/E) \\ &- 4 \operatorname{Re}(U_{e2}U_{\mu2}^{*}U_{e3}^{*}U_{\mu3}) \sin^{2}(1.27\Delta m_{32}^{2}L/E) \\ P^{-}(v_{e} \rightarrow v_{\mu}) &= -c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta \\ &\times \sin(1.27\Delta m_{21}^{2}L/E) \sin(1.27\Delta m_{31}^{2}L/E) \sin(1.27\Delta m_{32}^{2}L/E) \\ CP \text{ odd part is large for LMA and large } \theta_{13} \text{ and large } \delta \end{split}$$

CP Violation at a Neutrino Factory

e.g.Neutrino factory Golden Signature of "wrong sign" muons

Origin of neutrino mass

In the Standard Model neutrinos are massless, and a neutrino and anti-neutrino are distinguished by a (total) conserved lepton number L.



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See-saw mechanism

$$\begin{pmatrix} \overline{v_L} & \overline{v_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} v_L^c \\ v_R \end{pmatrix}$$

Heavy Majorana matrix

Diagonalise
$$\rightarrow m_{LL} \overline{\mathcal{V}}_L \mathcal{V}_L^c$$

Light Majorana matrix
$$\longrightarrow m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^T$$

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Natural Neutrino Hierarchy?

$$m_{LL}^{HI} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{m}{2}$$

Natural eigenvalues are $m_2^2 \approx m_3^2$ why $m_2^2 = m_3^2$?

Technically need a small 23 sub-determinant:

$$\det \begin{pmatrix} m_{22} & m_{23} \\ m_{23} & m_{33} \end{pmatrix} << m^2 \longrightarrow m_2 << m_3$$

But why should the sub-determinant be small ?
How can we get this from the see-saw mechanism?

See-saw with diagonal heavy Majorana matrix

Columns can be re-ordered without loss of generality

 $M_{RR} = \begin{pmatrix} X' & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & Y \end{pmatrix} \qquad m_{LR} = \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix} \qquad m_{LL} = m_{LR}M_{RR}^{-1}m_{LR}^{T} = \begin{pmatrix} \left(\frac{a'^{2}}{X'} + \frac{a^{2}}{Y} + \frac{d^{2}}{Y}\right) & \left(\frac{a'b'}{X'} + \frac{ab}{X} + \frac{de}{Y}\right) & \left(\frac{a'c'}{X'} + \frac{ac}{X} + \frac{df}{Y}\right) \\ \vdots & \left(\frac{b'^{2}}{X'} + \frac{b^{2}}{X} + \frac{e^{2}}{Y}\right) & \left(\frac{b'c'}{X'} + \frac{bc}{X} + \frac{ef}{Y}\right) \\ \vdots & \left(\frac{b'^{2}}{X'} + \frac{b^{2}}{X} + \frac{e^{2}}{Y}\right) & \left(\frac{b'c'}{X'} + \frac{bc}{X} + \frac{ef}{Y}\right) \\ \vdots & \vdots & \left(\frac{c'^{2}}{X'} + \frac{c^{2}}{X} + \frac{f^{2}}{Y}\right) \\ \vdots & \vdots & \left(\frac{c'^{2}}{X'} + \frac{c^{2}}{X} + \frac{f^{2}}{Y}\right) \end{pmatrix}$

If one right-handed neutrino of mass Y dominates then subdeterminant is naturally small: single right-handed neutrino dominance (SFK PLB 98, NPB 99)

$$\frac{|e|^{2}, |f|^{2}, |ef|}{Y} \quad \frac{|xy|}{X} \quad \frac{|x'y'|}{X'}, \quad \text{Sequential dominance}$$

$$x, y \in a, b, c, \quad x', y' \in a', b', c'$$

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Analytic estimates for neutrino masses and mixing angles (valid for sequential dominance) SFK hep-ph/0204360 Sequential dominance Mass hierarchy $\frac{|e|^2, |f|^2, |ef|}{Y} \quad \frac{|xy|}{X} \quad \frac{|x'y'|}{X'}, \qquad |d| \quad |e| \approx |f|$ $x, y \in a, b, c, \quad x', y' \in a', b', c'$ (set d=0 below) (set d=0 below) and large mixing angles naturally! $m_1 \quad O(\frac{x'y'}{X'}) \quad \tan \theta_{23} \approx \frac{|e|}{|f|}$ $m_{1} \quad m_{2} \quad m_{3} \begin{cases} m_{2} \approx \frac{|a|^{2}}{Xs_{12}^{2}} & \tan \theta_{12} \approx \frac{|a|}{c_{23}|b|\cos(\tilde{\phi_{b}}) - s_{23}|c|\cos(\tilde{\phi_{c}})} \\ m_{3} \approx \frac{|e|^{2} + |f|^{2}}{Y} & \theta_{13} \approx e^{i(\tilde{\phi} + \phi_{a} - \phi_{e})} \frac{|a|(e^{*}b + f^{*}c)Y}{[|e|^{2} + |f|^{2}]^{3/2}X} \end{cases}$ Phases fixed by real 12 angle condition $c_{23} | b | \sin(\tilde{\phi}_b) \approx s_{23} | c | \sin(\tilde{\phi}_c)$ where $\tilde{\phi}_b \equiv \phi_b - \phi_a - \tilde{\phi} + \delta$ $\tilde{\phi}_c \equiv \phi_c - \phi_a + \phi_e - \phi_f - \tilde{\phi} + \delta$ and $\tilde{\phi}$ is fixed by the real 13 angle condition $\tilde{\phi} \equiv \phi_e - \phi_a - \arg(e^*b + f^*c)$ 11/04/2003 S.F.King, Sheffield 27

Light or Heavy Sequential Dominance?

Heavy Sequential Dominance (HSD) X' X Y

$$M_{RR} = \begin{pmatrix} X' & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & Y \end{pmatrix} \qquad Y_{LR}^{\nu} = \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{``Lop-sided''}$$
(large 23 element)

Light Sequential Dominance (LSD) Y X X'

$$M_{RR} = \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X' \end{pmatrix} \quad Y_{LR}^{\nu} = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
(no large off-diagonal entries)

Note that X' is irrelevent for neutrino masses, mixings (and leptogenesis-see later), and the model reduces to an effective 2 right-handed neutrino model SFK 2000; Frampton, Glashow, Yanagida 2002; Raidal, Strumia 2002.

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Fukugita, Yanagida; Buchmuller, Plumacher; Di Bari,...

•Right-handed neutrinos are produced in early universe and decay out of equilibrium giving net lepton number L

- •CP violation from complex Yukawa couplings
- ·L is processed into B via B-L conserving sphalerons



Relation between CP violation at a neutrino factory and in leptogenesis for LSD SFK hep-ph/0211228

LSD

$$X' X Y \approx 10^{12} GeV$$
 $M_{RR} = \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X' \end{pmatrix}$ Assume 11
texture zero $Y_{LR}^{\nu} = \begin{pmatrix} 0 & a & a' \\ e & b & b' \\ f & c & 1 \end{pmatrix}$
Di Bari,Buchmuller,Plumacher

In LSD the heavy RH neutrino of mass X' is irrelevant for both leptogenesis and neutrino masses, mixings $Y_R \propto + \operatorname{Im}\{[(Y_{IR}^{\dagger}Y_{IR})_{12}]^2\} \propto + \sin 2\phi_{COSMO}$ $\phi_{COSMO} = \arg(e^*b + f^*c)$

$$\tan(\phi_{COSMO}) \approx \frac{|b|s_{23}\sin\eta_2 + |c|c_{23}\sin\eta_3}{|b|s_{23}\cos\eta_2 + |c|c_{23}\cos\eta_3} \qquad \tan(\phi_{COSMO} + \delta) \approx \frac{|b|c_{23}\sin\eta_2 - |c|s_{23}\sin\eta_3}{-|b|c_{23}\cos\eta_2 + |c|s_{23}\cos\eta_3}$$

Neutrino factory phase and leptogenesis phase are $\eta_2 \equiv \phi_b - \phi_e$, $\eta_3 \equiv \phi_c - \phi_f$ linked in terms of two invariant see-saw phases

Remarkably the leptogenesis phase is equal to the phase which enters neutrinoless double beta decay ! $|\phi_{COSMO}| = |\phi_{\beta\beta0\nu}|$

In HSD $Y = X = 10^9 GeV$ (Hirsch,SFK) and gravitino bound may be respected. In this case there is no leptogenesis-MNS link since X' couplings are relevant for leptogenesis but not neutrino masses, mixings.

SUSY and Lepton Flavour Violation

If SUSY is present then neutrino masses inevitably lead to lepton flavour violation due to radiatively generated off-diagonal slepton masses

$$BR(\tau \to \mu\gamma) \approx \frac{\alpha^3}{G_F^2} f_{32}(M_2, \mu, m_{\tilde{\nu}}) \mid m_{\tilde{L}_{32}}^2 \mid^2 \tan^2 \beta$$

$$\frac{dm_{\tilde{L}}^2}{dt} \approx \left(\frac{dm_{\tilde{L}}^2}{dt}\right)_{Y_{LR}=0} - \frac{(3m_0^2 + A_0^2)}{16\pi^2} \left[Y_{LR}Y_{LR}^{\dagger}\right] \implies m_{\tilde{L}_{32}}^{2(LLA)} \approx -\frac{(3m_0^2 + A_0^2)}{8\pi^2} C_{32}$$
$$C_{32} \approx b'c' \ln\left(\frac{M_{GUT}}{X'}\right) + bc \ln\left(\frac{M_{GUT}}{X}\right) + ef \ln\left(\frac{M_{GUT}}{Y}\right)$$

$$M_{RR} = \begin{pmatrix} X' & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & Y \end{pmatrix} \quad Y_{LR}^{\nu} = \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{array}{c} \mathsf{HSD/lopsided will lead to} \\ \mathsf{large} \ BR(\tau \to \mu\gamma) & \mathsf{due to} \ \mathsf{Blazek}, \mathsf{SFK} \\ \mathsf{large 23 Yukawa}. \end{array}$$

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Lepton Flavour Violation in Constrained Minimal Supersymmetric Standard Model

Blazek,SFK hep-ph/0211368



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Conclusion

- LMA MSW is established, along with atmospheric neutrino mixing $\theta_{12} \approx 30^{\circ}, \quad \theta_{23} \approx 45^{\circ}, \quad \theta_{13} \le 10^{\circ} \quad |\Delta m_{32}^2| \approx (0.05 \ eV)^2 \quad \Delta m_{21}^2 \approx (0.008 \ eV)^2$
- Heaviest neutrino mass from cosmology and osc. $0.05 eV < m_v^{heaviest} < 0.23 eV$
- LMA MSW allows CP phase to be measured (maybe) at neutrino factory
- Small neutrino masses can be elegantly explained by the see-saw
- Sequential dominance then provides a natural explanation of a neutrino mass hierarchy and large mixing angles
- The dominant right-handed neutrino may be either the heaviest (HSD) or the lightest one (LSD)
- LSD links leptogenesis phase and the CP phase which is measurable at a neutrino factory, but is inconsistent with SUSY models
- HSD has no such link, but may be consistent with SUSY models
- SUSY leads to lepton flavour violating processes such as mu→e+gamma and especially tau →mu+gamma which can discriminate HSD from LSD.