A non-iterative mass constraint (NIMCO) for particle decays

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Content

1. Introduction

- To overcome the mass constraint problem of particle decays, a non-iterative method is developed.

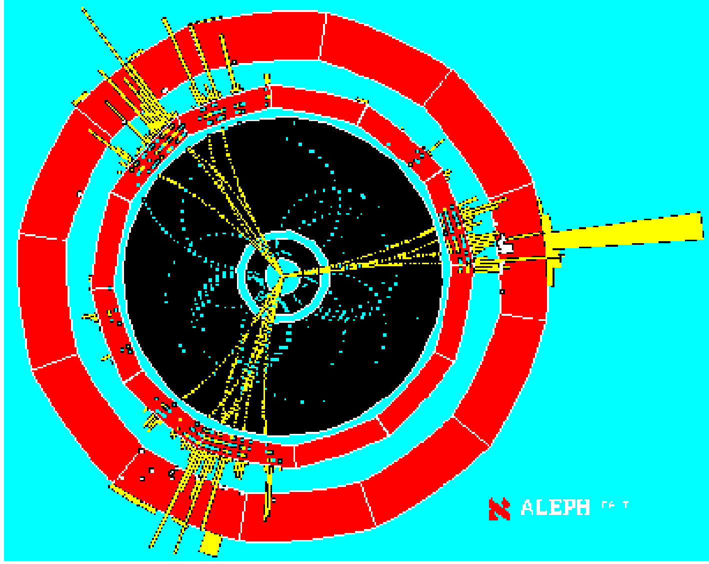
- The new method can be applied to any two-body decay or a many body decay.

- By using a toy detector simulation and the ALEPH full simulation data, the performance of the new method is compared with traditional iterative chi-square method for several decay types.

- No significant difference is obtained between the two methods.

- However, the non-iterative method is found to be much faster than the chi-square method.

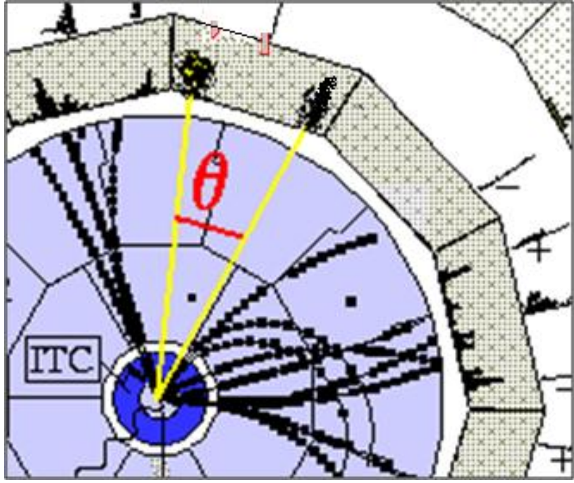
2. Particle Reconstruction



- In particle physics, mother particles are reconstructed from their decay products.
- If the *natural width* of the mother is very small then one can consider that the mother mass is a single scalar value called the *nominal mass*.
- But, the reconstructed momentum resolution of the mother is limited by the momentum and angular resolutions of the particle detector used.
- So, the invariant mass distribution is smeared (instead of giving a single peak).

Example: Pion Decay

$$\pi^0 \rightarrow \gamma + \gamma$$



Invariant mass:

$$M^2 = 2E_1E_2(1 - \cos\theta_{12})$$

Mass resolution:

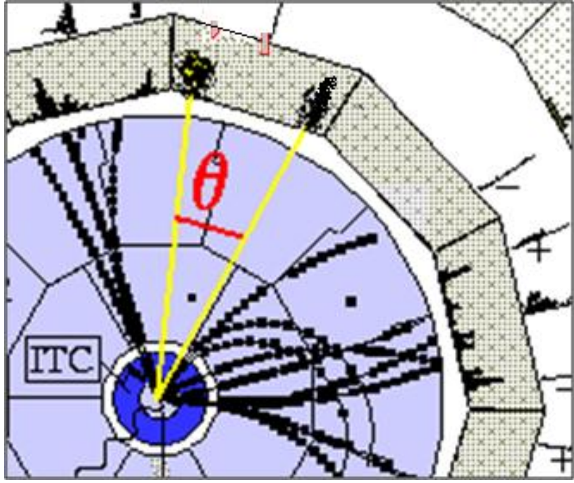
$$\frac{\sigma_M}{M} = \frac{1}{2}\left[\frac{\sigma_{E_1}}{E_1} \oplus \frac{\sigma_{E_2}}{E_2} \oplus \frac{\sigma_{\theta_{12}}}{\tan(\theta_{12}/2)}\right]$$

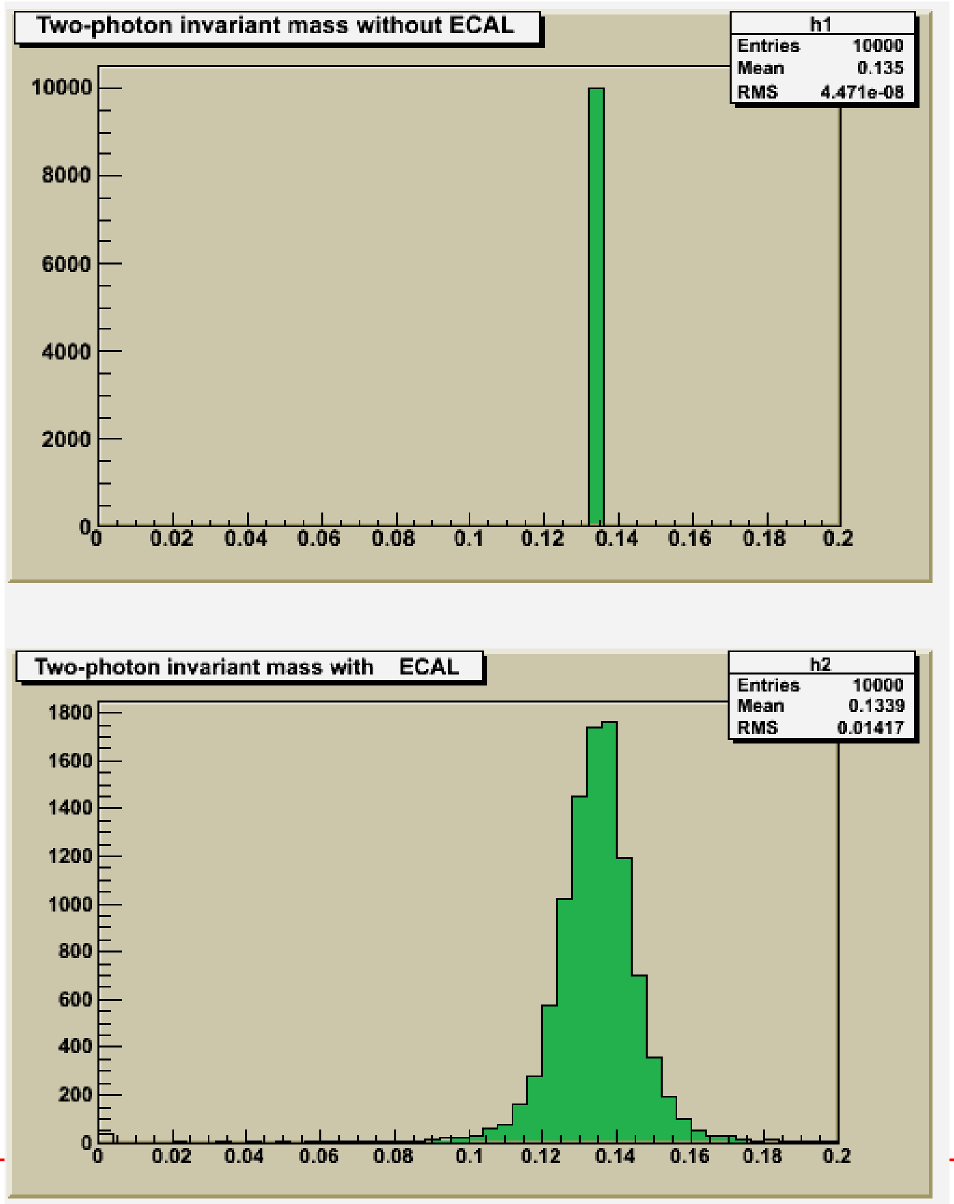
σ_E : *energy resolution*

$\sigma_{\theta 12}$: *opening angle resolution*

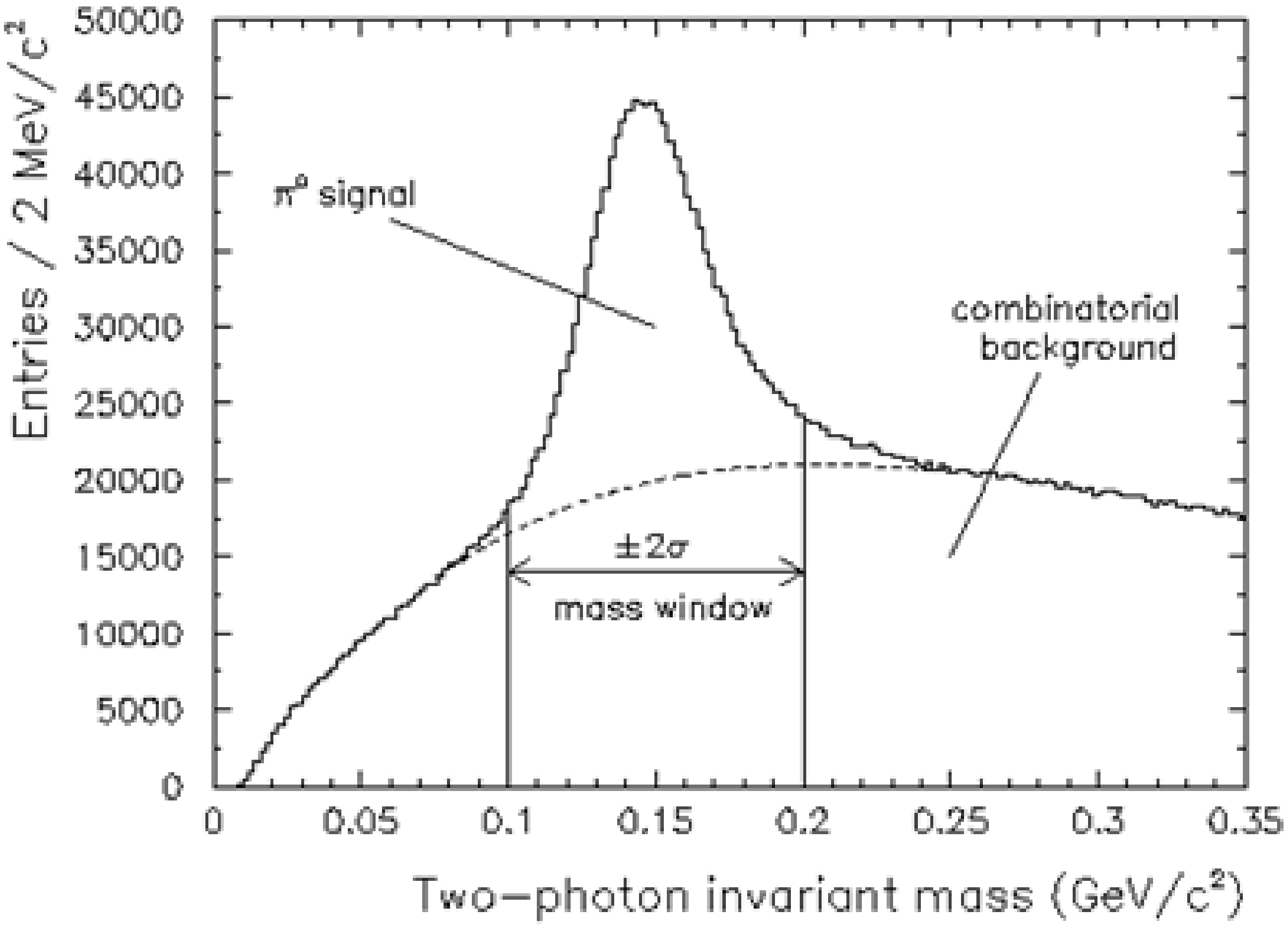
Toy Simulation:

$$\pi^0 \rightarrow \gamma + \gamma$$

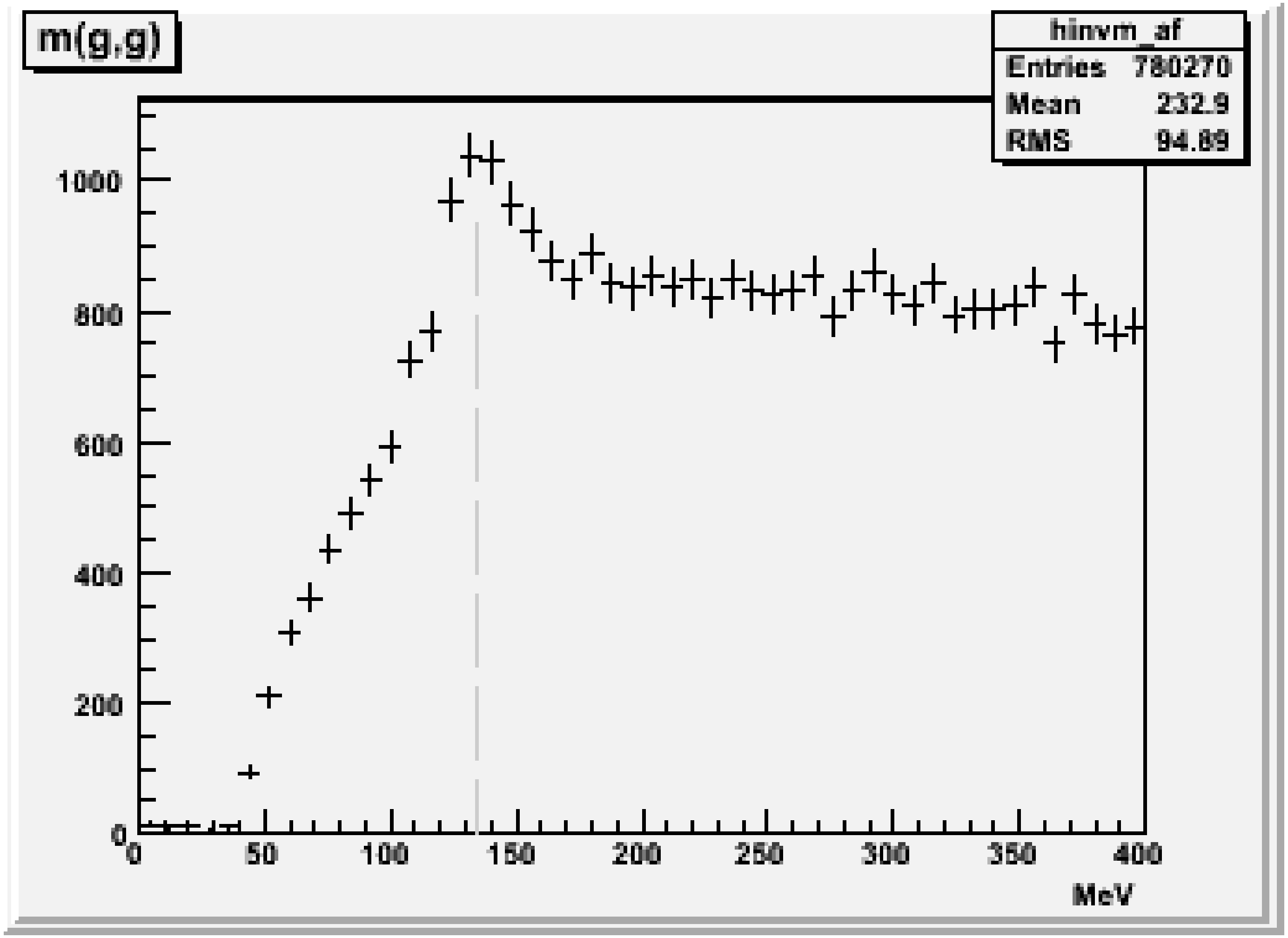




ALEPH Full Monte Carlo simulation:



ATLAS 7TeV data (using first data with rel17):



3. Mass Constraint

- In order to improve the momentum resolution of the mother particle candidate, one can apply a kinematic refit to the momentum vectors of the reconstructed daughters.

- To do that, the momenta of the daughters are adjusted such that their invariant mass ideally gives the *nominal mass* of the mother.

4. Iterative Methods for Mass Constraint

Consider a mother particle of mass *M* denoted by 0 is decaying to *n* daughters:

$$0 \longrightarrow 1 + 2 + \cdots + n$$

The invariant mass of the daughters (*W*):

$$W^2 = \Big(\sum_{i=1}^{n} [|\mathbf{p}_i|^2 + m_i^2]^{1/2}\Big)^2 - \Big(\sum_{i=1}^{n} \mathbf{p}_i\Big)^2$$

$\mathbf{p}_i$: reconstructed momentum vector

m_i : mass of the i^{th} daughter.

Traditionally refitting is performed by iterative methods.

==> minimise a chi-square function of the form:

$$\chi^2 = [\mathbf{p} - \mathbf{f}]^T V^{-1} [\mathbf{p} - \mathbf{f}]$$

p : vector of reconstructed momenta

f : vector of refitted momenta (target)

V : covariant matrix (contains detector resolutions)

- For *n*-body decay => we have 3*n* parameters in the refitting problem.
- These parameters are not entirely free since the model has to fulfill some special constraint(s).
- For one mass constraint, we can drop one of the parameters (3*n* – 1 partial derivatives!)

Another widely used method to solve such a minimisation problem is known as the Lagrange Multipliers.
==> For one constraint, minimise the function of the form:

$$\mathcal{L}(\mathbf{f}, \lambda) = \chi^2 + \lambda(M^2 - W^2)$$

λ : Lagrange mutiplier

- No parameter is eliminated!
- Partial derivative(s) with respect to *λ is* also computed.

5. Non-iterative Solutions

- Non-Iterative Mass COnstraint (NIMCO) can be applied to any two-body decay and certain many body decays!

- The solutions for the refitted values are obtained directly without minimising a chi-square.

Consider the two-body decay
(with mother mass M and daughter masses are m_1 and m_2)

$$M \rightarrow m_1 + m_2$$

Invariant mass of products:

$$W^2 = \left(\sum_{i=1}^{2} [|\mathbf{p}_i|^2 + m_i^2]^{1/2}\right)^2 - \left(\sum_{i=1}^{2} \mathbf{p}_i\right)^2$$

Rearranging yields:

$$\begin{aligned} W^2 - m_1^2 - m_2^2 &= 2p_1p_2\left(t - \cos\theta\right) \\ &= 2p_1p_2K \end{aligned}$$

$$\theta = \text{opening angle between the daughters}$$

$$p_i = |\mathbf{p}_i|$$

$$t = \sqrt{\left(1 + m_1^2/p_1^2\right)\left(1 + m_2^2/p_2^2\right)}$$

$$K = t - cos\theta$$

We have unconstrained mass:

$$W^2 - m_1^2 - m_2^2 = 2p_1p_2K \quad \textbf{(1)}$$

After constraining *W* to *M* by modifying the momentum of decay products and opening angle:

$$M^2 - m_1^2 - m_2^2 = 2f_1f_2L \quad \textbf{(2)}$$

where
f_1 and f_2 are the refitted momenta and
L is the refitted value of *K*.

* Refitted values are obtained by rescaling measured values *

$$f_1 = s_1p_1, \;\; f_2 = s_2p_2, \;\; L = s_3K \quad \textbf{(3)}$$

Subsituting (3) -> (2):

$$M^2 - m_1^2 - m_2^2 = 2(s_1 p_1)(s_2 p_2)(s_3 K) \qquad \textbf{(4)}$$

Dividing (4) by (1):

$$\frac{M^2 - m_1^2 - m_2^2 = 2(s_1 p_1)(s_2 p_2)(s_3 K)}{W^2 - m_1^2 - m_2^2 = 2 p_1 p_2 K}$$

$$\boxed{\frac{M^2 - m_1^2 - m_2^2}{W^2 - m_1^2 - m_2^2} = s_1 s_2 s_3 = s^2} \qquad \textbf{(5)}$$

Aim: find scales subjected to the constraint of (5).

SOLUTION of EQUAL SCALES!

We have:

$$s_1 s_2 s_3 = s^2$$

Assume that angular resolution is insignificant (*K* = *L*).

The most simple mass constraint can be formed by setting equal scales:

$$\boxed{s_1 = s_2 = s}$$

and

$$\boxed{s_3 = 1}$$

N I M C O

An important consideration is to include the influence of detector resolutions to the constraint.
=> each scale s_i has to be a function of detector resolutions!

Let:

σ_1 : momentum resolution of particle 1

σ_2 : momentum resolution of particle 2

σ_3 : resolution of $K = t - \cos(\theta)$

Define relative resolutions: ($s_1 s_2 s_3 = s^2$)

$$r_1 = \sigma_1/p_1\,, \quad r_2 = \sigma_2/p_2\,, \quad r_3 = \sigma_3/K$$

The limiting forms of the scales can be obtained as follows:

- as $r_1 \rightarrow 0$ then $s_1 = 1$ and $s_2 s_3 = s^2$
- as $r_2 \rightarrow 0$ then $s_2 = 1$ and $s_1 s_3 = s^2$
- as $r_3 \rightarrow 0$ then $s_3 = 1$ and $s_1 s_2 = s^2$

- as $r_1 \rightarrow 0$ and $r_2 \rightarrow 0$ then $s_1 = s_2 = 1$ and $s_3 = s^2$
- as $r_1 \rightarrow 0$ and $r_3 \rightarrow 0$ then $s_1 = s_3 = 1$ and $s_2 = s^2$
- as $r_2 \rightarrow 0$ and $r_3 \rightarrow 0$ then $s_2 = s_3 = 1$ and $s_1 = s^2$

The following ansàtzse satisfy all of the requirements:

$$\begin{aligned} s_1 &= s(r_1 s + r_2 + r_3)/(r_1 + r_2 s + r_3 s) \\ s_2 &= s(r_1 + r_2 s + r_3)/(r_1 s + r_2 + r_3 s) \\ s_3 &= s(r_1 + r_2 + r_3 s)/(r_1 s + r_2 s + r_3) \\ &= s^2/s_1 s_2 \end{aligned} \quad (6)$$

where $s^2 = (M^2 - m_1^2 - m_2^2)/(W^2 - m_1^2 - m_2^2)$

Note: final mother momenta is given by:

$$\mathbf{p}_{mot} = s_1 \mathbf{p}_1 + s_2 \mathbf{p}_2 \quad (7)$$

6. Event and Detector Simulation

Two types of events are generated

1) Pythia 8.1: pp collisions at 14 TeV, with the MinBias pro.
=> select ~40k D*0 particles from the successive decays:

$$D^{*0} \to D^0 \gamma$$
$$D^0 \to K_S^0 \pi^0$$
$$\pi^0 \to \gamma\gamma$$
$$K_S^0 \to \pi^+\pi^-$$

$E_\gamma > 1$ GeV , $|\eta|<3.2$

$p_T > 0.5$ GeV , $|\eta|<2.5$

The selected events are passed though
a toy detector simulation whose parameterasitions are:

ECAL
$\sigma_E/E = 10\%/\sqrt{E} \oplus 1\%/E$
$\sigma_{\theta,\phi} = 5\ mrad/\sqrt{E}$

Tracker
$\sigma_{p_T}/p_T = 0.05\% p_T \oplus 1\%$

2) Using Jetset 7.4 following decays are selected

$$\pi^0 \to \gamma\gamma, \quad \eta \to \gamma\gamma, \quad \eta \to \pi^+\pi^-\gamma\,, \quad \eta \to \pi^+\pi^-\pi^0$$

in the momentum range between 1 and 30 GeV/c
from ~4M events hadronic Z decays at the LEP collider.

The events have been passed through the ALEPH full simulation (realistic momentum/energy resolution)

ALEPH parameterizations:

ECAL

$$\sigma_E/E = 0.18/\sqrt{E} + 0.009/E$$
$$\sigma_{\theta,\phi} = 2.5/\sqrt{E} + 0.25\ (mrad)$$

Tracker

$$\sigma_{p_T}/p_T = 8 \times 10^{-4}\, p_T$$

7. Performance Analysis

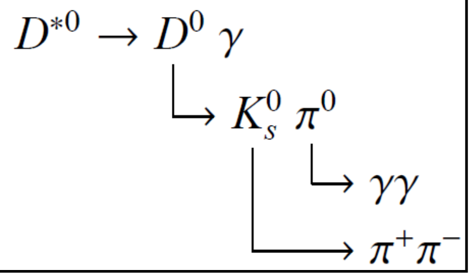
Figure of merit: *Momentum resolution*

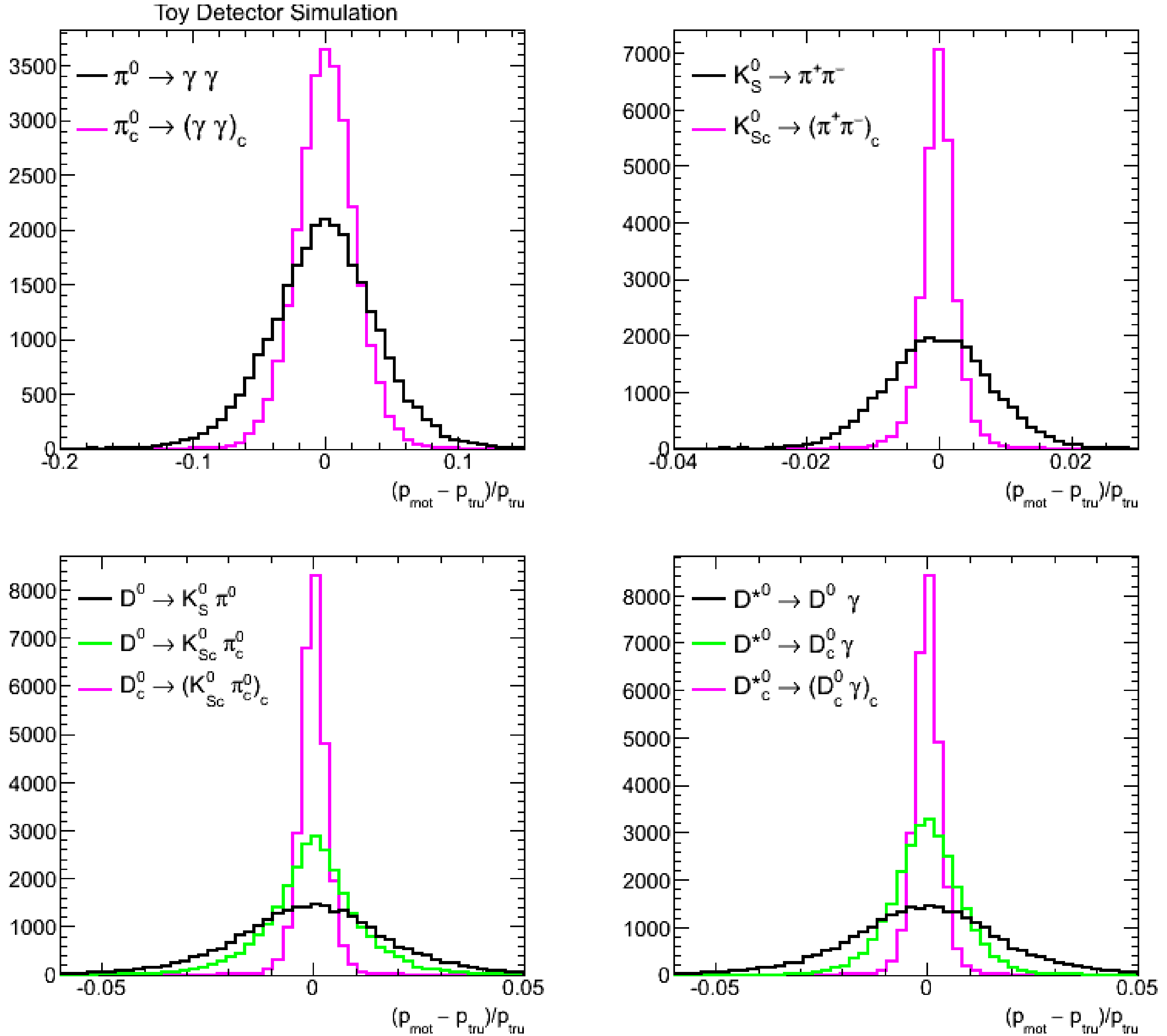
defined as the standard deviation of the distribution:

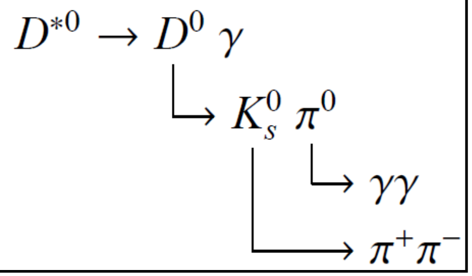
Constrained or unconstrained momentum

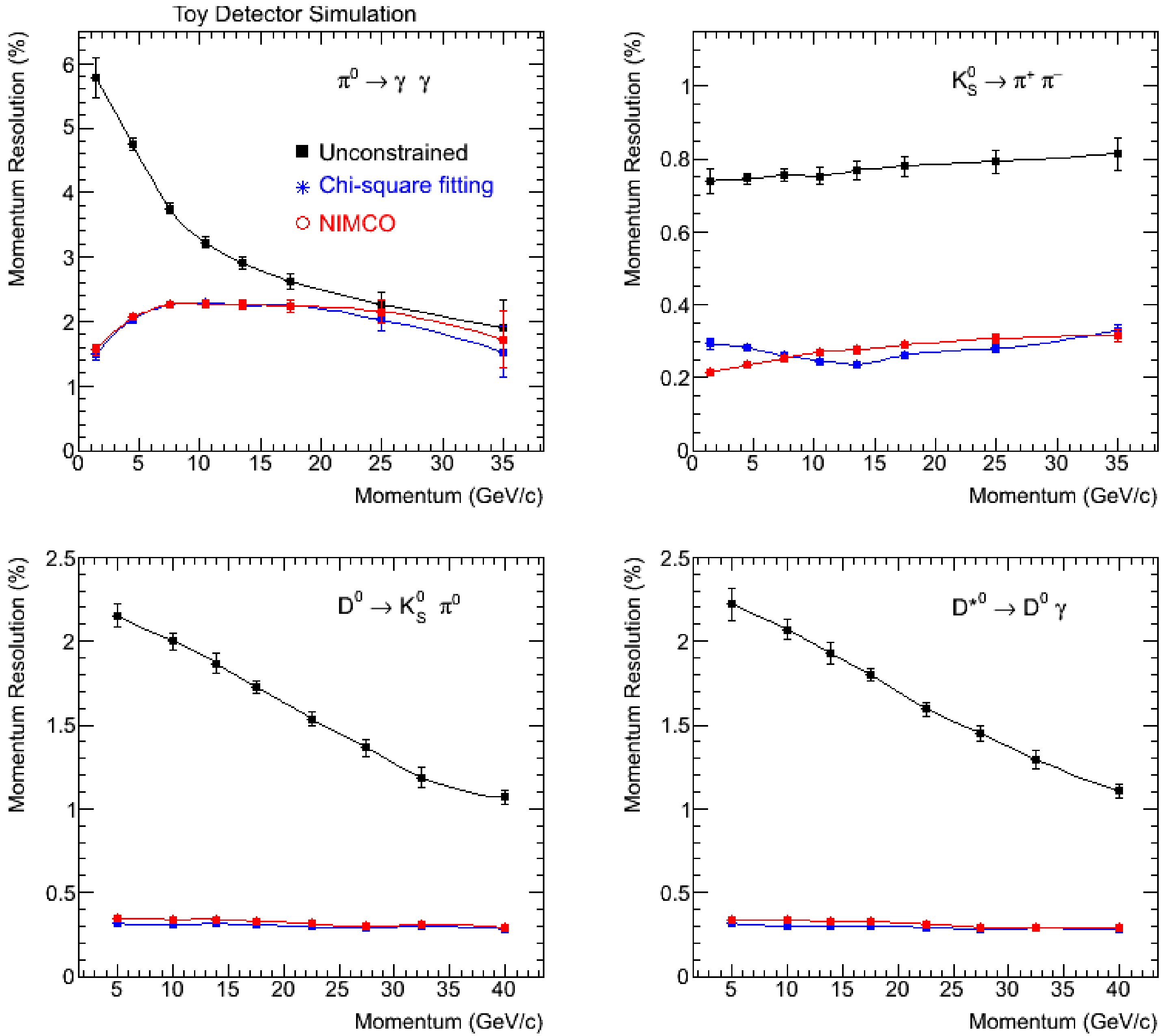
Mother true momentum

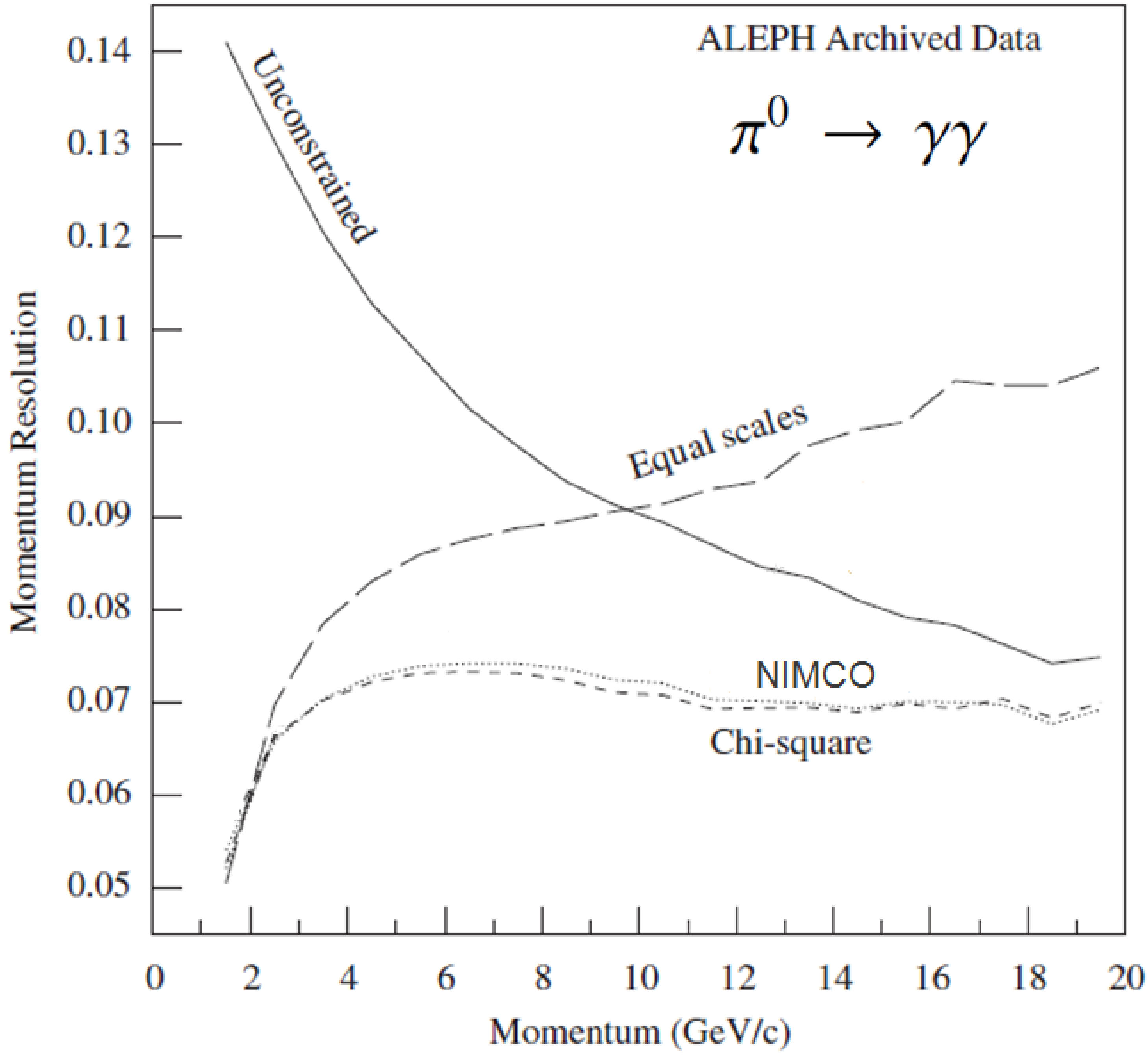
$$\frac{p_{mot} - p_{tru}}{p_{tru}}$$

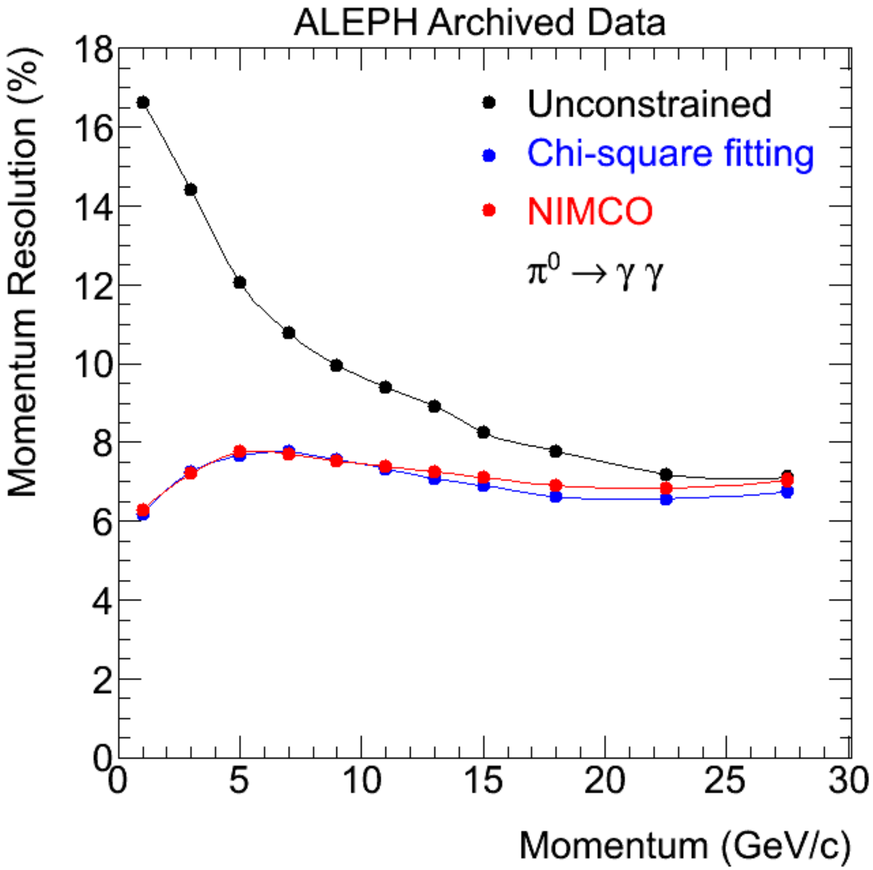


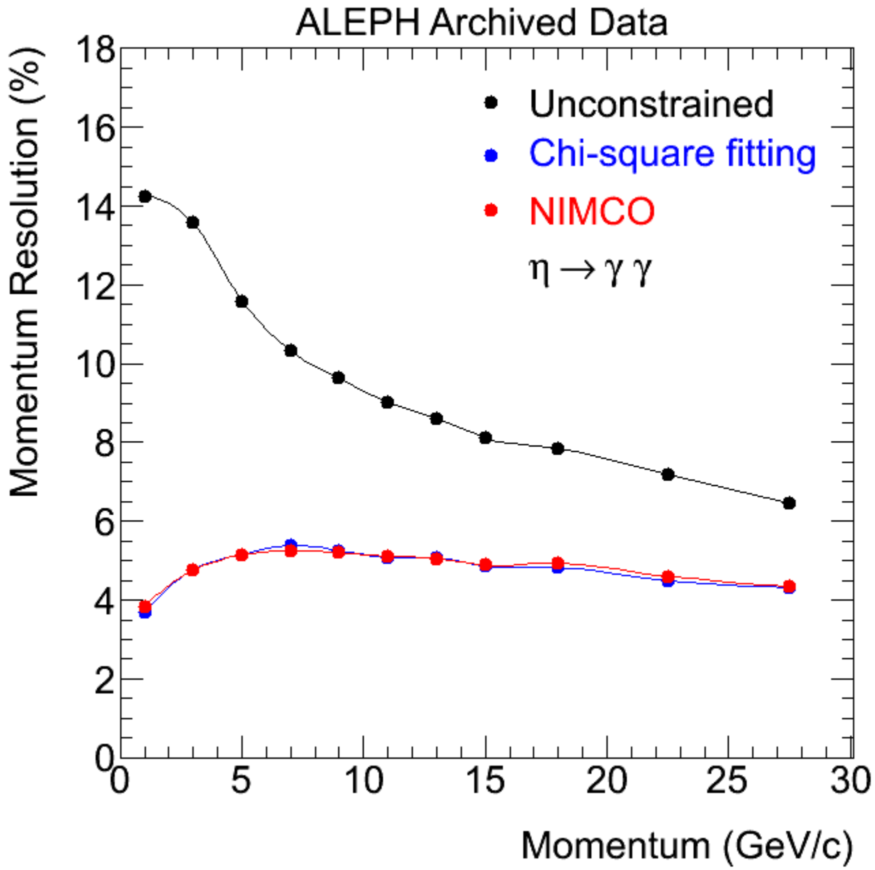

Toy Detector Simulation
π0 → γ γ
π0c → (γ γ)c
K0S → π+π−
K0Sc → (π+π−)c
D0 → K0S π0
D0 → K0Sc π0c
D0c → (K0Sc π0c)c
D*0 → D0 γ
D*0 → D0c γ
D*0c → (D0c γ)c
(pmot − ptru)/ptru


$D^{*0} \to D^0 \gamma$
$K_s^0 \pi^0$
$\gamma\gamma$
$\pi^+\pi^-$


Toy Detector Simulation
$\pi^0 \to \gamma\ \gamma$
Unconstrained
Chi-square fitting
NIMCO
$K_S^0 \to \pi^+ \pi^-$
$D^0 \to K_S^0\ \pi^0$
$D^{*0} \to D^0\ \gamma$
Momentum Resolution (%)
Momentum (GeV/c)


ALEPH Archived Data
$\pi^0 \rightarrow \gamma\gamma$
Unconstrained
Equal scales
NIMCO
Chi-square
Momentum Resolution
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0
2
4
6
8
10
12
14
16
18
20
Momentum (GeV/c)


ALEPH Archived Data
Momentum Resolution (%)
Unconstrained
Chi-square fitting
NIMCO
π0 → γ γ
Momentum (GeV/c)


ALEPH Archived Data
Momentum Resolution (%)
Unconstrained
Chi-square fitting
NIMCO
η → γ γ
Momentum (GeV/c)

The idea of NIMCO for the two-body decay can be extended to a tree-body decay.

Consider the decay

$$\eta \rightarrow \pi^{+}\pi^{-}\gamma$$

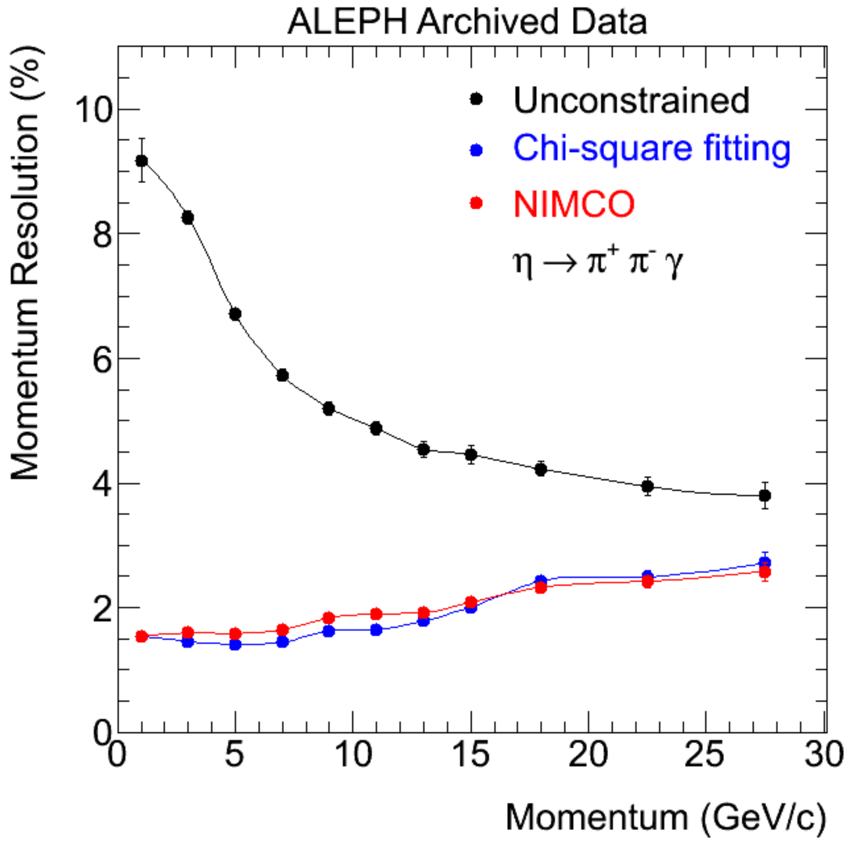
the pions are combined to build a *pseudo particle x* reducing the decay to a two-body decay of the form

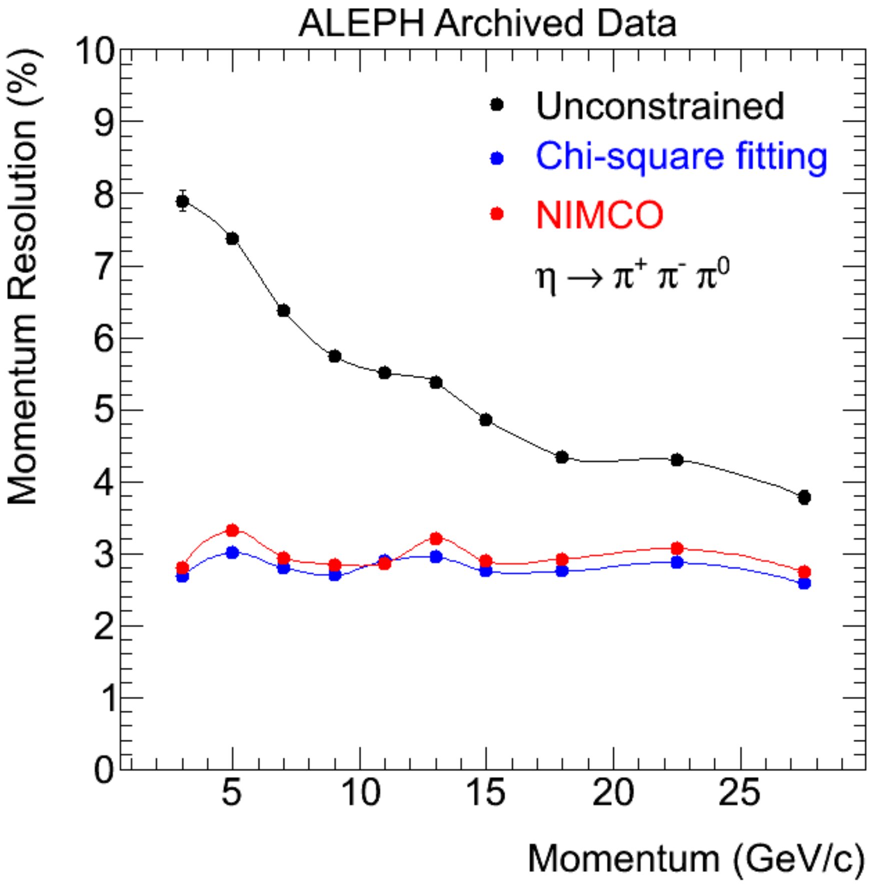
$$\eta \rightarrow x\gamma$$

- The mass of the pseudo particle x is taken from the reconstructed invariant mass of the pair.

- The same strategy can be applied to the decays:

$$\eta \rightarrow \pi^{+}\pi^{-}\pi^{0} \quad \text{or} \quad D_s^{+} \rightarrow K_s^{0}K^{-}2\pi^{+}$$


ALEPH Archived Data
Momentum Resolution (%)
Unconstrained
Chi-square fitting
NIMCO
$\eta \to \pi^+ \pi^- \gamma$
Momentum (GeV/c)


ALEPH Archived Data
Momentum Resolution (%)
Unconstrained
Chi-square fitting
NIMCO
$\eta \to \pi^+ \pi^- \pi^0$
Momentum (GeV/c)

8. Conclusion

- A simple and robust non-iterative method for the mass constraint of two-body decays is developed.

- The new method called NIMCO is shown to perform equally well to that of standard iterative chi-square method.

- The chi-square method provides a significant CPU overhead! NIMCO is more than 100 times faster than the iterative chi-square minimisation method provided by the author

- NIMCO can be adapted to the many-body particle decays which can be reduced to a two-body decay of well known daughter masses.

- NIMCO may also be employed to initialise very sophisticated algorithms used to improve the top quark mass resolution in the decay channel $t \longrightarrow Wb$

Teşekkürler ...

9. References

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