‘Dark Matter Tomography'
Measuring the DM velocity distribution with directional detection

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University of Sheffield - 15th June 2016
Measure energy (and possibly direction) of recoiling nucleus

*Reconstruct the mass and cross section of DM?*

Need to know the velocity distribution of the DM particles.
The WIMP wind

In the halo:

\[ v_{\text{sun}} \sim 220 \text{ km s}^{-1} \]

WIMP: Weakly Interacting Massive Particle

In the lab:

\[ v_{\text{DM}} \sim 220 \text{ km s}^{-1} \]

‘WIMP wind from Cygnus’
The WIMP wind

In the halo:

$\nu_{\text{sun}} \sim 220 \text{ km s}^{-1}$

WIMP: Weakly Interacting Massive Particle

Cygnus constellation

But we don’t know the velocity distribution exactly!

In the lab:

‘WIMP wind from Cygnus’
What could go wrong?

Astrophysical uncertainties need to be accounted for!
While we’re at it, why not try to reconstruct the velocity distribution too?!
Need directionality!

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Outline

Directional event rate in DD
Mayet et al. [1602.03781]

Reconstructing $f(v)$ in non-directional experiments
BJK, Green [1207.2039, 1303.6868,1312.1852]; BJK, Fornasa, Green [1410.8051]

Discretising the DM velocity distribution
BJK [1502.04224]

Reconstructing $f(v)$ in directional experiments
BJK, O’Hare [in preparation]
Directional recoil rate
Directional recoil rate

Flux of particles with velocity \( \mathbf{v} \):

\[
v \left( \frac{\rho_\chi}{m_\chi} \right) f(\mathbf{v}) \, d^3\mathbf{v}
\]

Differential cross section for recoil energy \( E_R \):

\[
\frac{d\sigma}{dE_R} \sim \frac{1}{v^2}
\]

Kinematic constraint for recoil with momentum \( \mathbf{q} \):

\[
\hat{\mathbf{v}} \cdot \hat{\mathbf{q}} = \frac{v_{\text{min}}}{v}
\]

where

\[
v_{\text{min}} = \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}}
\]

Read (2014)
[arXiv:1404.1938]

\( \rho_\chi \sim 0.2-0.6 \text{ GeV cm}^{-3} \)

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Directional recoil spectrum

\[
\frac{dR}{dE_R d\Omega_q} = \frac{\rho_0}{4\pi \mu_{\chi p}^2 m_\chi} \sigma^p C_N F^2(E_R) \hat{f}(v_{\text{min}}, \hat{q})
\]

\[
v_{\text{min}} = \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}}
\]
### Directional recoil spectrum

\[
\frac{dR}{dE_R d\Omega_q} = \frac{\rho_0}{4\pi \mu^2_{\chi p} m_\chi} \sigma^p C_N F^2(E_R) \hat{f}(v_{\min}, \hat{q})
\]

**Enhancement for nucleus \( N \):**

\[
C_N = \begin{cases} 
|Z + (f^p / f^n) (A - Z)|^2 & \text{SI interactions} \\
\frac{4}{3} \frac{J+1}{J} |\langle S_p \rangle + (a^p / a^n) \langle S_n \rangle|^2 & \text{SD interactions}
\end{cases}
\]

**Form factor:** \( F^2(E_R) \)

**NB:** May get interesting directional signatures from other operators

BJK [1505.07406]

\[
v_{\min} = \sqrt{\frac{m_N E_R}{2\mu^2_{\chi N}}}
\]
Directional recoil spectrum

\[
\frac{dR}{dE_R d\Omega_q} = \frac{\rho_0}{4\pi \mu^2_{\chi p} m_\chi} \sigma^p C_N F^2(E_R) \hat{f}(v_{\text{min}}, \mathbf{q})
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\[v_{\text{min}} = \sqrt{\frac{m_N E_R}{2 \mu^2_{\chi N}}}\]

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\end{cases}\]

Form factor: \( F^2(E_R) \)

Radon Transform (RT):

\[\hat{f}(v_{\text{min}}, \mathbf{q}) = \int_{\mathbb{R}^3} f(\mathbf{v}) \delta(\mathbf{v} \cdot \mathbf{q} - v_{\text{min}}) \, d^3\mathbf{v}\]

NB: May get interesting directional signatures from other operators

BJK [1505.07406]
Radon Transform (RT):

\[ \hat{f}(v_{\text{min}}, \hat{q}) = \int_{\mathbb{R}^3} f(v) \delta (v \cdot \hat{q} - v_{\text{min}}) \, d^3v \]
What do we know about the velocity distribution?
Standard Halo Model (SHM) is typically assumed: isotropic, spherically symmetric distribution of particles with $\rho(r) \propto r^{-2}$.

Maxwell-Boltzmann distribution:

$$f_{\text{Lab}}(v) = (2\pi \sigma_v^2)^{-3/2} \exp \left[ -\frac{(v - v_e)^2}{2\sigma_v^2} \right] \Theta(\|v - v_e\| - v_{\text{esc}})$$

$v_e$ - Earth’s Velocity

$$v_e \sim 220 - 250 \text{ km s}^{-1}$$

$$\sigma_v \sim 155 - 175 \text{ km s}^{-1}$$

Feast et al. [astro-ph/9706293], Bovy et al. [1209.0759]

Piffl et al. (RAVE) [1309.4293]
Astrophysical uncertainties

High resolution N-body simulations can be used to extract the DM speed distribution

Non-Maxwellian structure

Debris flows

Dark disk

\[ f_1(v) = 10^{-3} \text{ km s}^{-1} \]

\[ v \text{ [km s]} \]

\[ f(v) = 10^{-3} \text{ km s}^{-1} \]

\[ v \text{ [km/s]} \]

\[ v \text{ [km/s]} \]

Vogelsberger et al. [0812.0362]

Kuhlen et al. [1202.0007]

Pillepich et al. [1308.1703], Schaller et al. [1605.02770]

However, N-body simulations cannot probe down to the sub-milliparsec scales probes by direct detection…

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Local substructure

May want to worry about ultra-local substructure - subhalos and streams which are not completely phase-mixed.

But from N-body simulations, expect lots of ‘sub-streams’ to form a smooth halo.

However, this does not exclude the possibility of a stream - e.g. due to the ongoing tidal disruption of the Sagittarius dwarf galaxy.

Freese et al. [astro-ph/0309279, astro-ph/0310334]

Helmi et al. [astro-ph/0201289], Vogelsberger et al. [0711.1105]

Measuring $f(v)$ may tell us something about galaxy formation and the history of our Milky Way!
Tomography

$I(\theta)$

www.fda.gov

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Tomography

\[ I(\theta) \]

RT

Invert

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DM Tomography

\[ f(v) \]

\[ \frac{dR}{dE_R d \cos \theta} \]
DM Tomography

\[ f(v) \]

\[ \frac{dR}{dE_R d \cos \theta} \]

\[ \theta \]
DM Tomography

\[ f(v) = -\frac{1}{8\pi^2} \int \frac{d^2}{d(v \cdot \hat{q})^2} \hat{f}(v \cdot \hat{q}, \hat{q}) d\Omega_q \]

Gondolo [hep-ph/0209110]
DM Tomography

But we don’t get to choose where to scan, we just get random samples!

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1-D reconstructions (Energy only)
Reconstructing $f(v)$

Many previous attempts to tackle this problem:

Numerical inversion (‘measure’ $f(v)$ from the data)
Fox, Liu, Weiner [1011.915], Frandsen et al. [1111.0292], Feldstein, Kahlhoefer [1403.4606]

Include uncertainties in SHM parameters in the fit
Strigari, Trotta [0906.5361]

Add extra components to the velocity distribution (and fit)
Lee, Peter [1202.5035], O’Hare, Green [1410.2749]

But can we be more general?
Write a *general parametrisation* for the speed distribution:

$$f(v) = \exp \left( - \sum_{k=0}^{N-1} a_k v^k \right)$$

This form guarantees a positive distribution function.

Now we attempt to fit the particle physics parameters \( (m_\chi, \sigma^p) \), as well as the astrophysics parameters \( \{a_k\} \).

BJK & Green [1303.6868,1312.1852]

Peter [1103.5145]
Testing the parametrisation

Assuming incorrect distribution

Tested for a number of different underlying speed distributions

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Testing the parametrisation

Using our parametrisation

Benchmark

Best fit

Assuming incorrect distribution

Tested for a number of different underlying speed distributions
Cross section degeneracy

Minimum DM speed probed by a typical Xe experiment

This is a problem for any astrophysics-independent method!

Can be solved by including data from Solar Capture of DM - sensitive to low speed DM particles

BJK, Fornasa, Green [1410.8051]
1-D reconstructions

This parametrisation allows us to fit the 1-D speed distribution in a general way. This means we can reconstruct the DM mass without bias!

Can also reconstruct the form of the speed distribution itself from the parameters (but we’ll leave that for later in the talk…)

But if we want to parametrise the full 3-D velocity distribution, we would need an infinite number of parameters!

*But how do we extend this to directional detection?*
A directional parametrisation
From 1-D to 3-D

\[ f(\mathbf{v}) = f^1(\mathbf{v})A^1(\hat{\mathbf{v}}) + f^2(\mathbf{v})A^2(\hat{\mathbf{v}}) + f^3(\mathbf{v})A^3(\hat{\mathbf{v}}) + \ldots. \]

One possible basis is spherical harmonics:

\[ f(\mathbf{v}) = \sum_{lm} f_{lm}(\mathbf{v})Y_{lm}(\hat{\mathbf{v}}) \]

\[ \Rightarrow \hat{f}(\nu_{\text{min}}, \hat{\mathbf{q}}) = \sum_{lm} \hat{f}_{lm}(\nu_{\text{min}})Y_{lm}(\hat{\mathbf{q}}) \]

However, they are not strictly positive definite!

*If we try to fit with spherical harmonics, we cannot guarantee that we get a physical distribution function!*

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A discretised distribution

Divide the velocity distribution into N angular bins:

\[ f(v) = f(v, \cos \theta', \phi') = \begin{cases} 
  f^1(v) & \text{for } \theta' \in [0, \pi/N] \\
  f^2(v) & \text{for } \theta' \in [\pi/N, 2\pi/N] \\
  \vdots & \\
  f^k(v) & \text{for } \theta' \in [(k - 1)\pi/N, k\pi/N] \\
  \vdots & \\
  f^N(v) & \text{for } \theta' \in [(N - 1)\pi/N, \pi] 
\end{cases} \]

...and then we can parametrise \( f^k(v) \) within each angular bin.

In principle, we could also discretise in \( \phi' \), but assuming \( f(v) \) is independent of \( \phi' \) does not introduce any error.
Example: SHM

\[ v_{\text{lag}} = 220 \text{ km s}^{-1} \]

\[ \sigma_v = 156 \text{ km s}^{-1} \]
Examples: $N = 3$

WIMP wind

\[ k = 1 \quad k = 2 \quad k = 3 \]

**SHM**

\[ \sigma_{_{39}}^p f^k(v) \text{ [km}^{-3} \text{s}^{-1}] \]

- $k = 1$
- $k = 2$
- $k = 3$

**SHM + Stream**

\[ \sigma_{_{39}}^p f^k(v) \text{ [km}^{-3} \text{s}^{-1}] \]

- $k = 1$
- $k = 2$
- $k = 3$
Binned event rate

We want to try and calculate the event rate, binned in the same angular bins.

Need to calculate the integrated Radon Transform (IRT):

\[
\hat{f}^j(v_{\text{min}}) = \int_{\phi=0}^{2\pi} \int_{\cos(j\pi/N)}^{\cos((j-1)\pi/N)} \hat{f}(v_{\text{min}}, \hat{q}) \, d\cos\theta \, d\phi,
\]

The calculation of the Radon Transform is rather involved, but it can be carried out analytically in the angular variables for an arbitrary number of bins \( N \), and reduced to \( N \) integrations over the speed \( v \).

BJK [1502.04224]

So how well does this ‘approximation’ work?
Event numbers

\[ N_j \]

total number of events expected in each angular bin

\begin{align*}
\text{CF}_4 \text{ detector} \\
E_{th} &= 20 \text{ keV} \\
m_\chi &= 100 \text{ GeV} \\
N_S &= 50 \\
N_{BG} &= 1
\end{align*}
Event numbers

Could keep increasing N!

For now, try N = 3 angular bins
Bin the data in each experiment, depending on the direction of the recoil, into $N = 3$ bins

Simultaneously fit $(m_\chi, \sigma^p)$, and $N = 3$ sets of $\{a_k\}$ describing the speed distribution in each angular bin.

If an experiment is not directionally sensitive, just sum the three speed distributions to get the total.

We’ll use 4 terms to describe each of the 3 speed distributions. Some are fixed by normalisation, giving a total of 11 parameters for the fit.
Directional reconstructions

PRELIMINARY
Benchmarks

Mock data from 2 ideal experiments

**Xe detector**

\[ E_{\text{th}} = 5 \text{ keV} \]

1000 kg yr

\[ \sim 900 \text{ events} \]

**F detector**

\[ E_{\text{th}} = 20 \text{ keV} \]

10 kg yr

\[ \sim 50 \text{ events} \]

Mohlabeng et al. [1503.03937]

DRIFT [1010.3027]

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Reconstructions

Best Case
Assume underlying velocity distribution is known exactly.
Fit $m_\chi$, $\sigma_p$

Reasonable Case
Assume functional form of underlying velocity distribution is known.
Fit $m_\chi$, $\sigma_p$ and theoretical parameters of $f(v)$

Worst Case
Assume nothing about the underlying velocity distribution.
Fit $m_\chi$, $\sigma_p$ and empirical parameters of $f(v)$
Reconstructing the DM mass

Experiments w/ directional sensitivity

None
Xe
Xe,F

30 40 50 60 70 80 90

$m_\chi$ / GeV

SHM
SHM+Stream

No uncertainties

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Reconstructing the DM mass

![Graph showing DM mass reconstruction with different experimental sensitivities and known functional forms.](image)

No uncertainties

Known functional form

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Reconstructing the DM mass

![Graph showing the reconstruction of DM mass with different experiments and sensitivities.](image)

- **None**: No uncertainties
- **F**: Known functional form
- **Xe**: Empirical parametrisation

Experiments with directional sensitivity:
- **Xe, F**: SHM
- **Xe**: SHM + Stream

Parameters:
- $m_\chi$ / GeV

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Reconstructing the DM cross section

Experiments w/ directional sensitivity

No uncertainties
Known functional form
Empirical parametrisation

$\sigma_{SD}^p / 10^{-39} \text{ cm}^2$
Reconstructing the velocity distribution

True velocity distribution
Best fit distribution
(+68% and 95% intervals)

Directional F and non-directional Xe
Reconstructing the velocity distribution

**Directional F and non-directional Xe**

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Reconstructing the velocity distribution

**Directional F and Directional Xe**

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Caveats

Only sensitive to speeds inside the energy window of the detector.

We don’t know the true cross section (or local DM density) in advance, difficult to compare with a given velocity distribution.

Fraction of DM particles in each angular bin is less sensitive to changes in overall normalisation.

Use directionality of $f(v)$ as a discriminator between different distributions.
Distinguishing distributions

Underlying SHM distribution
No directionality

SHM

SHM + Stream

Forward

Transverse

Backward

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Distinguishing distributions

Underlying SHM distribution
*Directional Xe + F*

SHM
SHM + Stream

Forward
懋verse
Backward

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The strategy

In case of signal break glass

Perform parameter estimation using two methods: ‘known’ functional form vs. empirical parametrisation

- Compare reconstructed parameters

Estimate fraction of DM particles in each angular bin

- Check for consistency with SHM

In case of inconsistency, look at reconstructed shape of f(v)

- Hint towards unexpected structure?
Conclusions

Astrophysical uncertainties are a problem for parameter estimation in direct detection

- Use halo-independent, general parametrisation

This can be extended to directional detection (with angular binning)

- Naturally account for angular resolution

Doesn’t spoil the reconstruction of the DM mass

- But lose information about cross section

May allow us to distinguish different velocity distributions (and tell us something about the Milky Way)

- Much harder to do without directionality
Conclusions

Astrophysical uncertainties are a problem for parameter estimation in direct detection

Use halo-independent, general parametrisation

This can be extended to directional detection (with angular binning)

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May allow us to distinguish different velocity distributions (and tell us something about the Milky Way)

Much harder to do without directionality

Thank you
Backup Slides
Reconstructing the mass (1-D)

Ideal experiments

‘Real’ experiments
Different speed distributions (1-D)

• Generate 250 mock data sets

• Reconstruct mass and obtain confidence intervals for each data set

• True mass reconstructed well (independent of speed distribution)

• Can also check that 68% intervals are really 68% intervals
Incorporating IceCube

IceCube can detect neutrinos from DM annihilation in the Sun

Rate driven by solar capture of DM, which depends on the DM-nucleus scattering cross section

Crucially, only low energy DM particles are captured:

$$\frac{dC}{dV} \sim \sigma \int_0^{v_{\text{max}}} \frac{f_1(v)}{v} \, dv$$

But Sun is mainly spin-1/2 Hydrogen - so we need to include SD interactions…
How many terms do we need?

\[ f_1(v) \text{ [km s}^{-1}\text{]} \]

\[ v \text{ [km s}^{-1}\text{]} \]

- **Bump**
- **Double-peak**

Graphs showing the variation of BIC and m_{\text{rec}} (GeV) with the number of basis functions, N.
Compare:

- **Exact IRT** - calculated from the true, full distribution
- **Approx. IRT** - calculated from discretised distribution

![Graphs comparing exact and approximate IRT for N = 2.](attachment:image.png)
$N = 3$
Number of angular bins

- For $N = 5$
  - SHM
    - Exact
    - Approximate
  - Stream
    - Exact
    - Approximate

$N_{\text{events}}$ vs Bin number, $j$