A Survey of

Monte Carlo Event Generators

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and

HERWIG $\geq$ 4.0

A Classification of Monte Carlos

	Fixed Order		**All Orders**
	'Tree' Level	**One Loop**	
	EUROJET	QCD two-jet	ARIADNE
	PAPAGENO	Drell-Yan	COJETS
	NJETS	prompt photon	DTUJET
	VECBOS	•	FIELDAJET
	VVJET	•	FRITIOF
	GGZZ		HERWIG
	•		HIJING
	•		ISAJET
			PYTHIA
Weighted	**yes**	yes	yes
Un-weighted	yes	**no**	**yes**

Cross-sections involve large logarithms: e.g. collinear, $L \sim \log(Q^2/Q_0^2)$. Some of these can be (re)summed to give the (**N**ext)^n-to-**L**eading **L**ogarithm **A**pproximation.

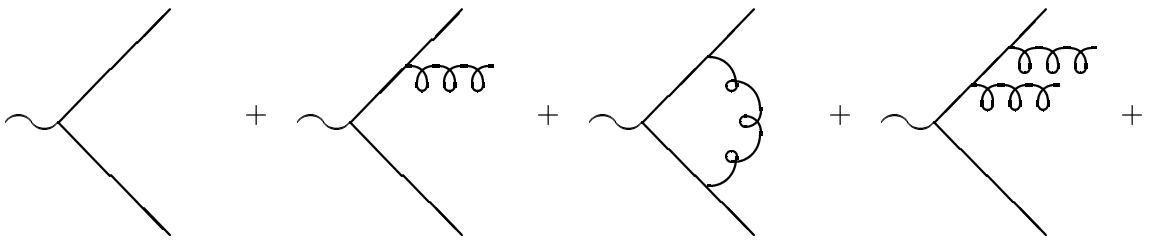
	LLA	NLLA	NNLLA			
Lowest order (Tree level)	$\alpha_s L$	α_s	$\alpha_s L^{-1}$	$\alpha_s L^{-2}$	•	•
Next-to-Lowest Order (One loop level)	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	$\alpha_s^2 L^{-1}$	•	•
	•	•	•			
	•	•		•		
	•	•			•	
	$(\alpha_s L)^n$	$\alpha_s(\alpha_s L)^{n-1}$	$\alpha_s^2(\alpha_s L)^{n-2}$	•	•	•

N.B. $\alpha_s \sim 1/L$

Cross-Sections at Next-to-Leading Order

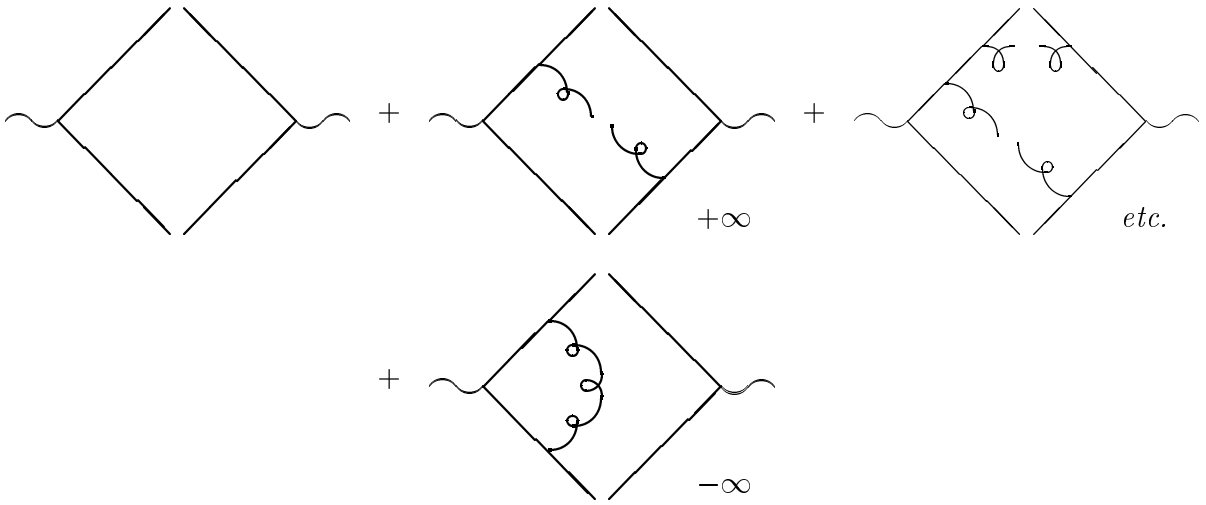
Consider the process $V^0 \longrightarrow q\bar{q}(ng)$:

$\mathcal{M}$:



$\mathcal{O}(1)$ $\mathcal{O}(g_s)$ $\mathcal{O}(g_s^2)$

$|\mathcal{M}|^{\mathbf{2}}$:



$\mathcal{O}(1)$ $\mathcal{O}(g_s^2)$ $\mathcal{O}(g_s^4)$

At $\mathcal{O}(g_s^2)$ negative divergences in the 1-loop $V^0 \to q\bar{q}$ process are cancelled by positive divergences in the tree level $V^0 \to q\bar{q}g$ process, which appear when the gluon is very soft or collinear: the result is finte. If a cut-off is introduced large logarithms remain.

$$\mathcal{M} \equiv \mathcal{M}_0 + g_s\mathcal{M}_1 + g_s^2\mathcal{M}_2 + \cdots$$

$$|\mathcal{M}|^2 = \underbrace{|\mathcal{M}_0|^2 + g_s^2\Big\{|\mathcal{M}_1|^2 + 2\mathcal{R}e\Big[\mathcal{M}_0\mathcal{M}_2^\star\Big]\Big\}}_{\text{Can be } \textbf{negative}} + g_s^4\Big\{|\mathcal{M}_2|^2 + \cdots\Big\} + \cdots$$

N.B. the one-loop cross-section can be **negative**.

The Major Monte Carlo Event Generator Programs

Monte Carlo Program	Hard M.E.		Perturbative Cascades	Underlying Soft Event	Hadronization Model	Unstable Hadron Decays	Colour Rearrangement
	QCD	E.W.					
ARIADNE 4.08	yes	yes	dipole	mini-jet	string	yes	○
COJETS 6.23	yes	yes	parton	param.	indep.	yes	•
FIELDAJET	yes	yes	parton	indep.	indep.	yes	•
HERWIG 5.9	yes	yes	parton	param.	cluster	yes	yes
ISAJET 7.27	yes	yes	parton	AGK	indep.	yes	•
PYTHIA 6.104	yes	yes	parton	mini-jet	string	yes	○
DTUJET 93	yes	•	•	DTU	string	yes	•
FRITIOF 7.2	yes	•	dipole	2 strings	string	yes	○
HIJING 1.0	yes	•	dipole	multi-jet	string	yes	○
EUROJET	yes	yes	•	indep.	indep.	yes	•
PAPAGENO	yes	yes	•	•	•	•	•

•'s indicate the absence of these components.

Standard Model Processes

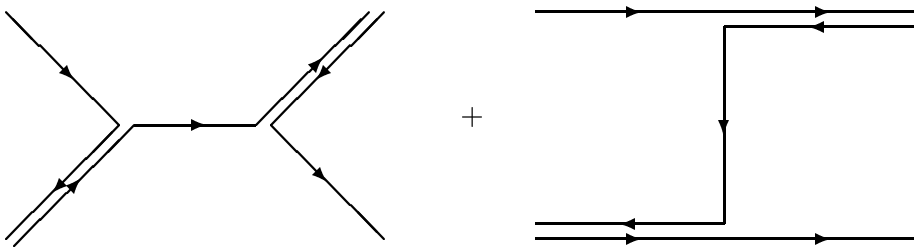
Process	Monte Carlo		
	HERWIG 5.9	ISAJET 7.27	PYTHIA 6.104
QCD			
QCD jets	yes	yes	yes
$q\bar{q}, gg \to Q\bar{Q}$	yes	yes	yes
$qQ \to qQ, gQ \to gQ$	yes	yes	yes
$q\bar{q}, gg \to t\bar{t}\gamma\gamma$	no	no	no
$qb \to q't$	yes	yes	yes
$gg \to g\chi_c, gJ/\psi, \gamma\chi_c, \gamma J/\psi$	no	no	yes
$qq' \to qq'$ (QED)	yes	no	no
double scattering	no	no	yes
minimum bias	yes	yes	yes
diffractive	no	yes	yes
elastic	no	no	yes
Drell-Yan			
$q\bar{q} \to V$	yes	yes	yes
$q\bar{q} \to Vg,\ gq \to Vq$	yes	yes	yes
$gb \to Wt, gt \to Zt$	no	yes	
$gg \to Vt\bar{t}$	no	no	yes
Prompt photon			
$qg \to q\gamma,\ q\bar{q} \to g\gamma$	yes	yes	yes
$gg \to \gamma g$	yes	no	yes
$q\bar{q} \to \gamma\gamma$	yes	yes	yes
$gg \to \gamma\gamma$	yes	no	yes
W/Z pair production			
$q\bar{q} \to VV, V\gamma$	no	yes	yes
$gg \to VV$	no	no	no
Standard Model Higgs h^0			
$q\bar{q}, gg, VV \to h^0$	yes	yes	yes
$q\bar{q} \to gh^0, \gamma h^0, qg \to qh^0, gg \to gh^0$	yes	no	yes
$qq' \to qq'h^0$	yes	no	yes
$q\bar{q} \to Vh^0$	no	no	yes
$q\bar{q}, gg \to t\bar{t}h^0$	no	no	yes
$VV \to VV$	yes	yes	yes
$h^0 \to VV, V^*V^*, f\bar{f}, \gamma\gamma, gg$	yes	yes	yes
$h^0 \to \gamma Z$	no	no	yes

Non-Standard Model Processes

Process	Monte Carlo		
	HERWIG 5.9 (HERBVI)	ISAJET 7.27 (ISASUSY)	PYTHIA 6.108 (SPYTHIA)
Non-standard Higgs			
$q\bar{q}, gg \to H^0, A^0$	no	no	yes
$q\bar{q}' \to H^+$	no	no	yes
$q\bar{q} \to VH^0, VA^0$	no	no	yes
$q\bar{q}, gg \to t\bar{t}H^0, t\bar{t}A^0$	no	no	yes
$qq' \to qq'H^0, qq'A^0$	no	no	yes
$gb \to H^-t$	no	no	yes
$V^0 \to h^0A^0, H^0A^0, H^+H^-$	no	no	yes
$t \to H^+b$	no	no	yes
Supersymmetry			
$q\bar{q} \to \widetilde{\chi}_i^0\widetilde{\chi}_j^0, \widetilde{\chi}_i^\pm\widetilde{\chi}_j^\mp$	no	yes	yes
$q\bar{q}' \to \widetilde{\chi}^0\widetilde{\chi}^\pm$	no	yes	yes
$q\bar{q} \to \tilde{g}\widetilde{\chi}^0$	no	yes	yes
$q\bar{q}' \to \tilde{g}\widetilde{\chi}^\pm$	no	yes	yes
$qg \to \tilde{q}\widetilde{\chi}^0, \tilde{q}'\widetilde{\chi}^\pm$	no	yes	yes
$q\bar{q} \to \tilde{\ell}\bar{\tilde{\ell}}, \tilde{\nu}\bar{\tilde{\nu}}, \tilde{q}\bar{\tilde{q}}, \tilde{g}\tilde{g}$	no	yes	yes
$q\bar{q}' \to \tilde{\ell}\bar{\tilde{\nu}} + \bar{\tilde{\ell}}\tilde{\nu}, \tilde{q}\bar{\tilde{q}}'$	no	yes	yes
$qq' \to \tilde{q}\tilde{q}'$	no	yes	yes
$qg \to \tilde{q}\tilde{g}$	no	yes	yes
$gg \to \tilde{q}\bar{\tilde{q}}, \tilde{g}\tilde{g}$	no	yes	yes
MSSM sparticle decays	no	yes	yes
Technicolour			
$q\bar{q} \to \rho^0_{\rm tech}$	no	yes	yes
$q\bar{q} \to \omega_{\rm tech}$	no	no	yes
$q\bar{q}' \to \rho^0_{\rm tech}$	no	yes	yes
$gg \to \eta_{\rm tech} \to$	no	no	yes
$\rho_{\rm tech} \to VV$	no	yes	yes
$\rho_{\rm tech} \to f\bar{f}, V\pi_{\rm tech}, \pi_{\rm tech}\pi_{\rm tech}$	no	no	yes
$VV \to \rho_T \to VV$	no	no	yes
Miscellaneous processes			
new gauge bosons	yes	no	yes
fourth generation	yes	yes	yes
contact interactions	no	no	yes
leptoquarks	no	no	yes
strongly interacting V	no	no	yes
excited quarks	no	no	yes
B (+L) number violation	yes	no	no

QCD Two-to-Two

Not all programs are the same: HERWIG $\neq$ PYTHIA $\neq$ FRITIOF



$T^aT^b(\mathcal{M}_s + \mathcal{M}_t)$ $\qquad$ $T^bT^a(\mathcal{M}_u - \mathcal{M}_t)$

	PYTHIA		HERWIG
T^aT^b	$\frac{C_F}{N_c}\frac{-u}{s}\frac{(u^2+s^2)}{t^2}$	$+\frac{1}{N_c^2}\frac{u^2}{t^2}$	$\frac{u^2}{2}\frac{(u^2+s^2)}{-ust^2} + \frac{1}{2N_c^2}\frac{u}{s}$
T^bT^a	$\frac{C_F}{N_c}\frac{-s}{u}\frac{(u^2+s^2)}{t^2}$	$+\frac{1}{N_c^2}\frac{s^2}{t^2}$	$\frac{s^2}{2}\frac{(u^2+s^2)}{-ust^2} + \frac{1}{2N_c^2}\frac{s}{u}$
Sum	$\frac{C_F}{N_c}\frac{(u^2+s^2)^2}{-ust^2}$	$+\frac{1}{N_c^2}\frac{(u^2+s^2)}{t^2}$	$\frac{(u^2+s^2)}{-2ust^2}\left[u^2+s^2-\frac{t^2}{N_c^2}\right]$

- the pole structure and crossing symmetry are preserved;
- each weight is seperately positive definate;
- the sum over colour configurations yields the exact differential cross-section.

$W^{\pm}$ + Jets

Showers (HERWIG) V **Matrix Elements** (VECBOS)

(at $\sqrt{s} = 40$ TeV/c^2, $E_T^{jet} > 50$ GeV/c^2)

Consider two cases: 1) Start showers from $W^{\pm} + 0$ jet;

2) Start showers from $W^{\pm} + 1$ jet.

Then compare resulting $W^{\pm} + n$ jet distributions to M.E. calc.

Find:

1) $W^{\pm} + 0$ jet gives **poor** fits;
2) $W^{\pm} + 1$ jet much **better** – however E_T^{jet} and P_T^W not great, especially for many jets.

Explanation:

Scale branching < scale hard sub-process

$\Longrightarrow$ 1) No emission above M_W
$\Longrightarrow$ 2) Problems if $W^{\pm}$ **not** either of two hardest particles.

Solution:

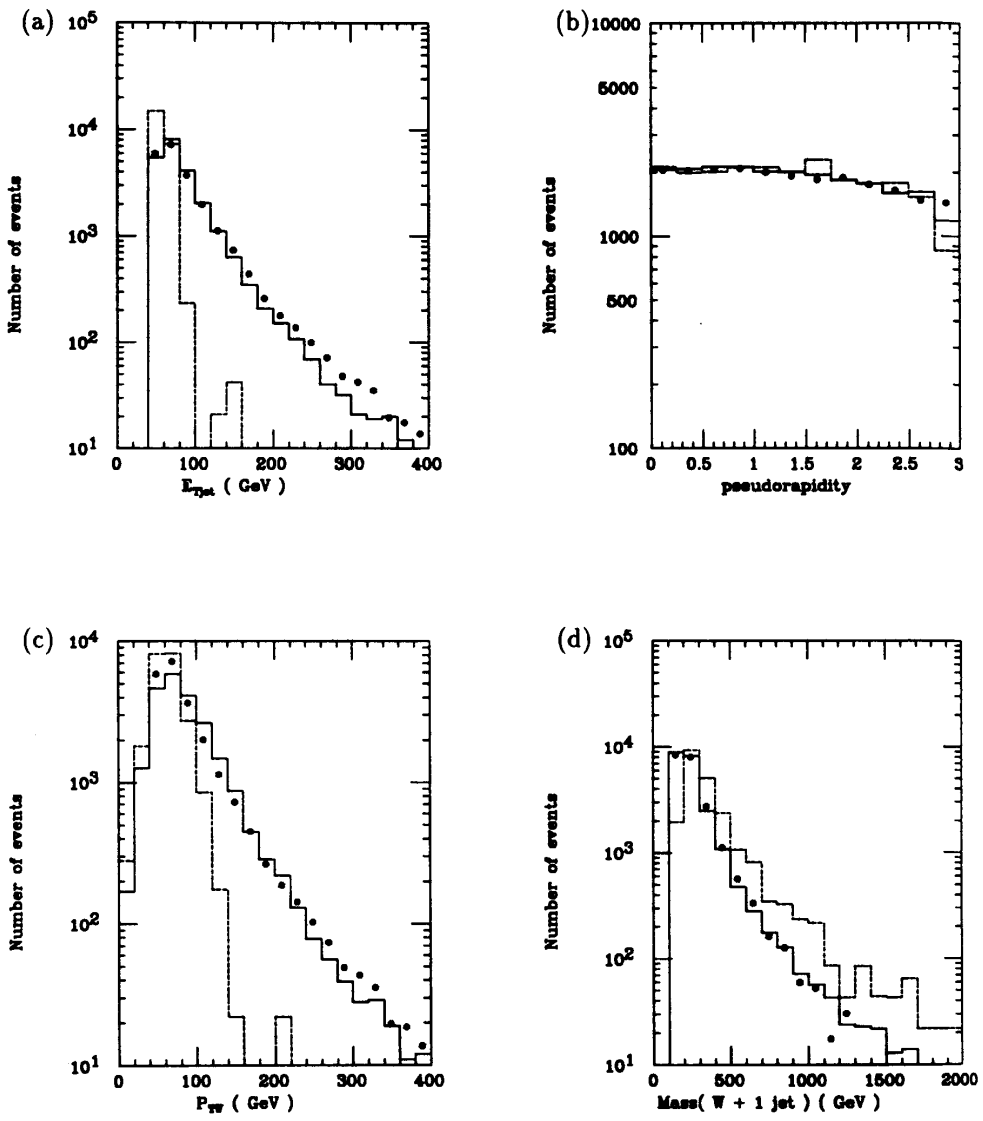
Include $W^{\pm}$, Z^0 and γ in showers.

N.B. Can argue showers give correction to a given process not generate a new one.
P.S. Expect photon showers to be poor approx., due to isolation cuts.

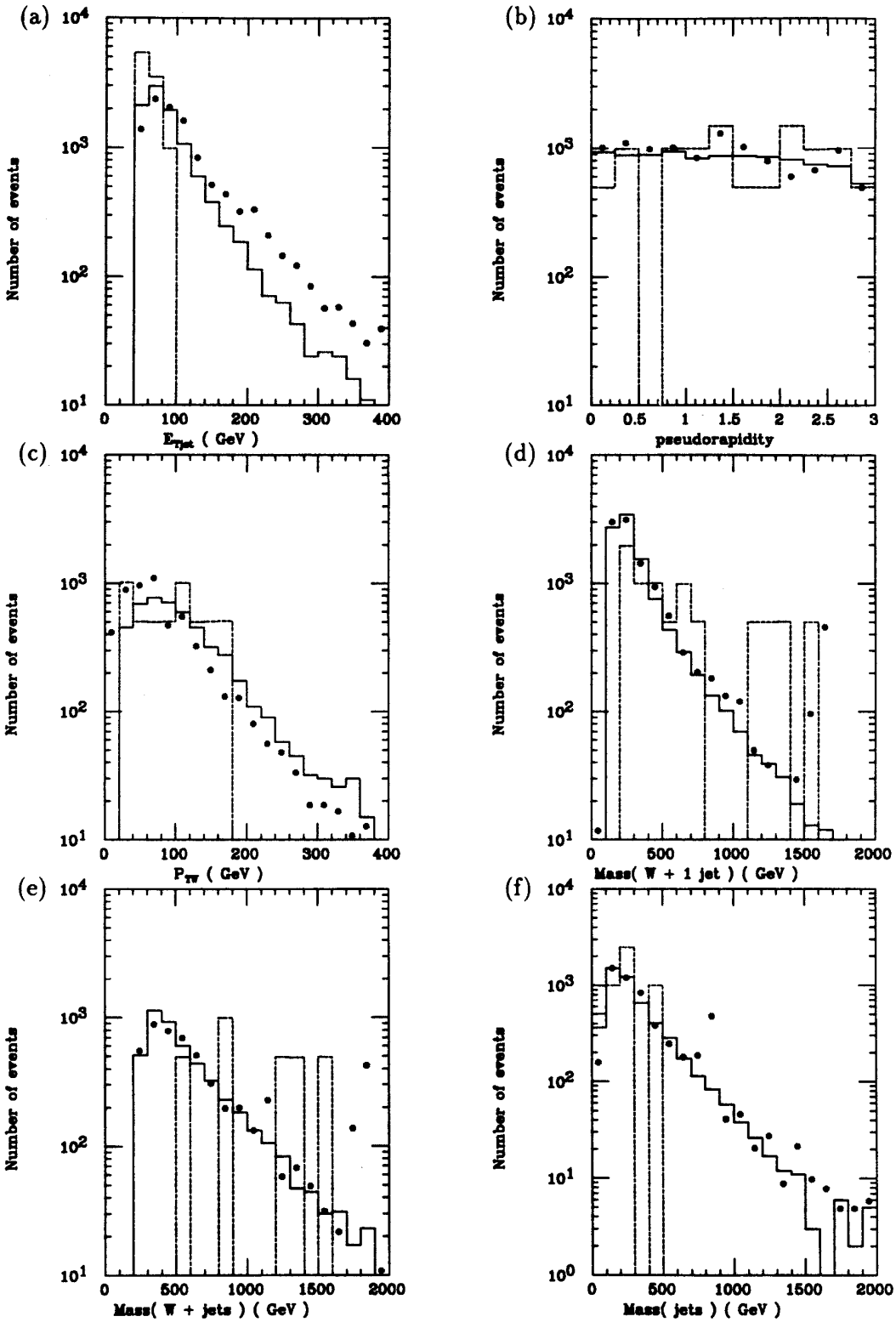
W + n-jet event distributions

In the following three sets of graphs the results of shower Monte Carlo HERWIG are compared to the tree level predictions of the VECBOS program, shown as circles. The solid line corresponds to starting from the W + 1-jet hard subprocess and the dotted line to starting from the the W + 0-jet hard sub-process.

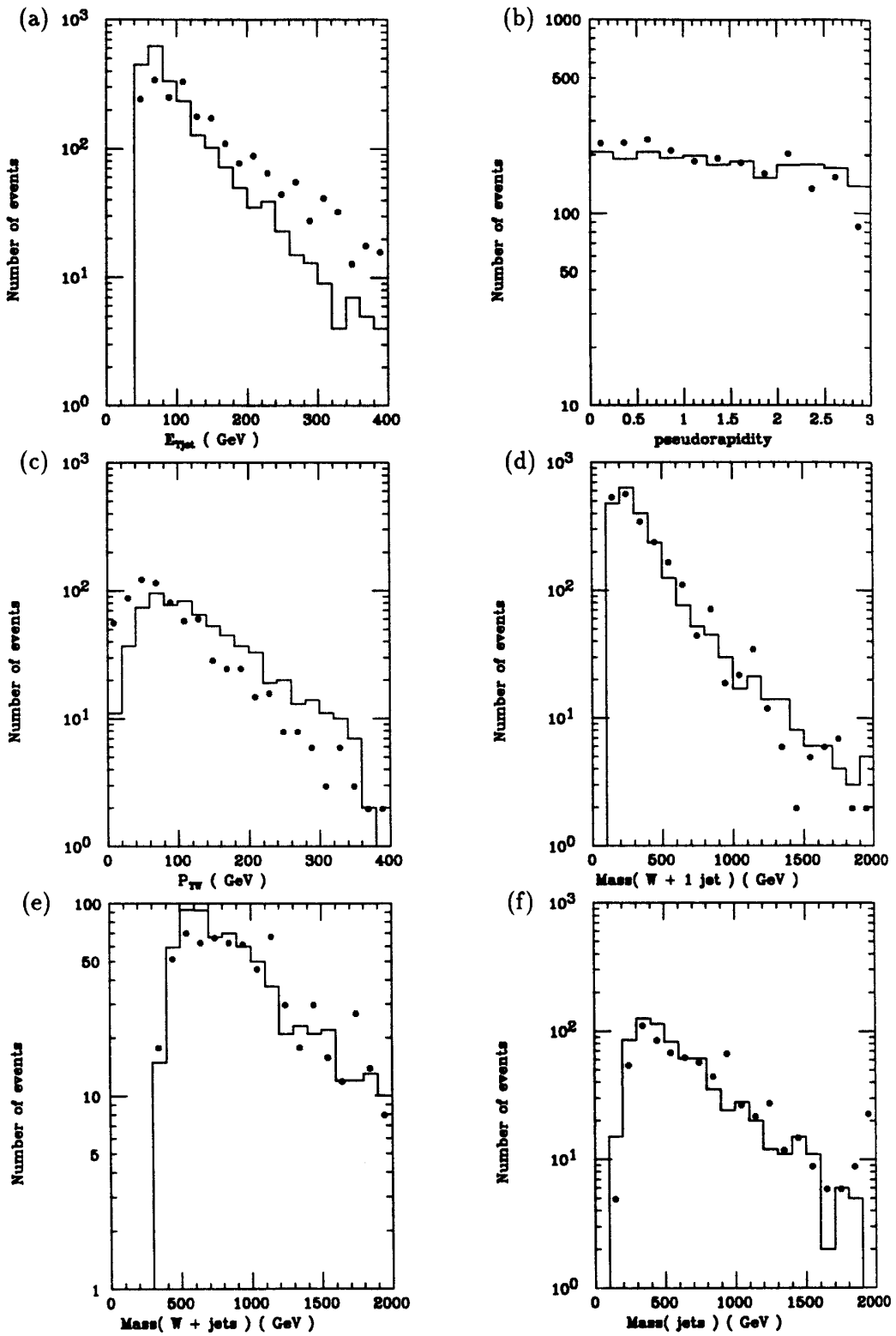
W + 1-jet event distributions



W + 2-jet event distributions



W + 3-jet event distributions

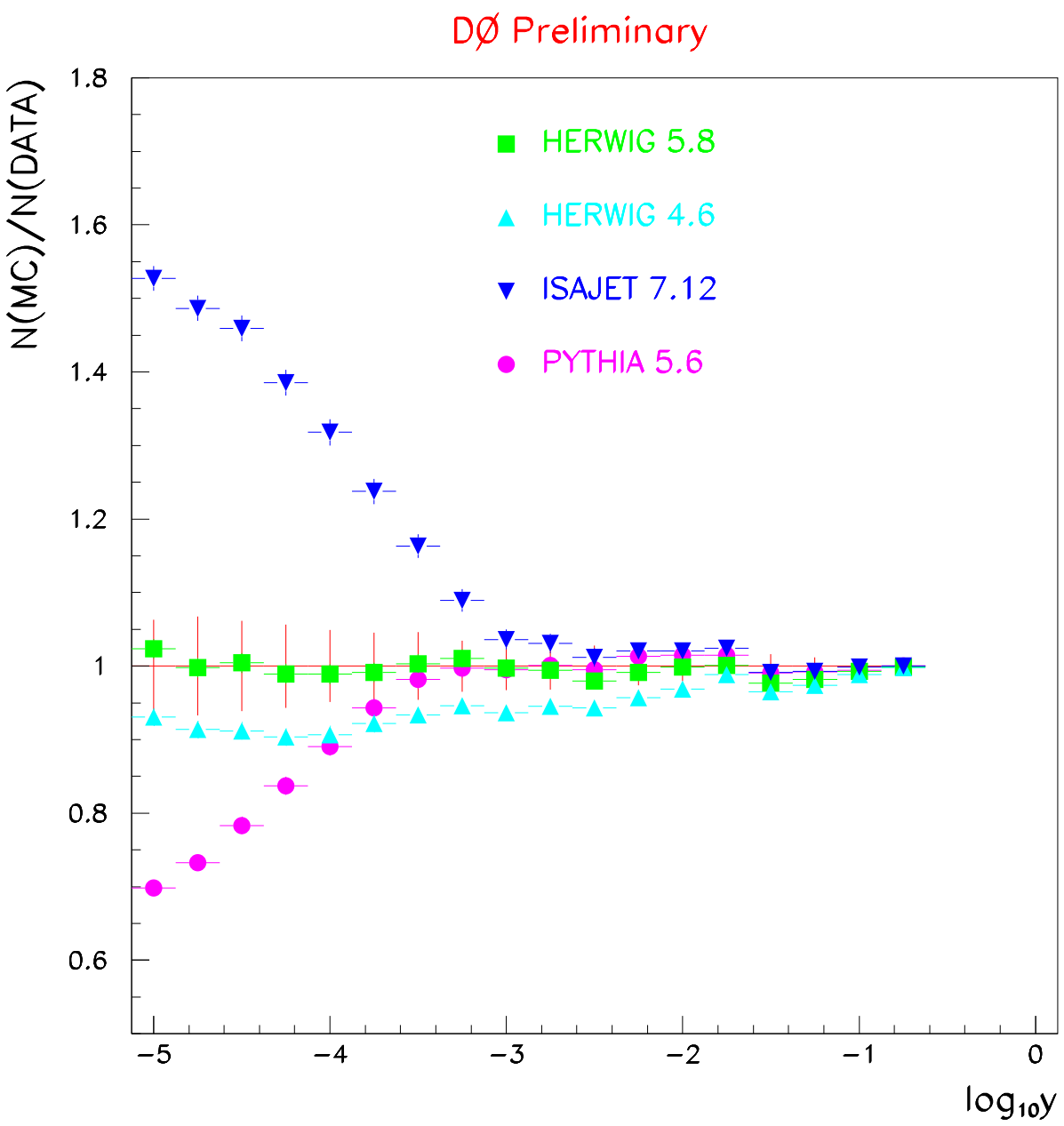


Parton Shower Algorithms

Feature	Monte Carlo				
	ARIADNE 4.08	COJETS 6.23	HERWIG 5.9	ISAJET 7.27	PYTHIA 6.108
Time-like Cascades: $i \longrightarrow jk$					
Evolution var. t	p_T^2	Q^2	$E_i^2 \frac{p_j \cdot p_k}{E_i E_k}$	Q^2	Q^2
Splitting frac. z	y	P_j^+/P_i^+	E_j/E_i	P_j^+/P_i^+	E_j/E_i
α_s scale	p_T^2	Q^2	$z(1-z)t$	Q^2	$z(1-z)Q^2$
Ang. ordering	automatic	no	automatic	no	veto on: $\frac{z_j(1-z_j)}{Q_j^2} > \frac{1-z_i}{z_i Q_i^2}$
Dead Cone	yes	no	yes	no	no
Soft Correlations	dipole approx.	no	dipole approx.	no	∘ dipole approx.
Spin Correlations	nearest neighbour	no	fully included	no	∘ nearest neighbour
photon rad.	∘ yes	1γ from M.E.	yes	yes, plus $W^\pm$, Z^0	yes
Space-like Cascades: $j(+k) \longleftarrow i$					
Direction	as above with an extended source	forward	backward	backward	backward
Evolution var. t		Q^2	$E_i^2 \frac{p_k \cdot n_k}{E_k}$	Q^2	Q^2
Splitting frac. z		P_j^+/P_i^+	E_j/E_i	P_j^+/P_i^+	$\hat{s}_j/\hat{s}_i$
α_s scale		Q^2	$(1-z)t$	Q^2	$(1-z)Q^2$
Ang. ordering		no	automatic	no	first branching
Soft correlations		no	dipole approx.	no	dipole approx.
Spin correlations		no	fully included	no	∘ nearest neighbour
photon rad.		no	no	no	$q \to q + \gamma$ only

∘ indicates a non-default option.

Parton Showers are Not Equivalent



The subjet multiplicity in a centrally produced 300 GeV jet ($p\bar{p}$ collisions at $\sqrt{s} = 1800$ GeV) as a function of y_{cut}, the data are normalized to unity. The different treatments of (softer) gluon radiation leads to sizeable discrepancies with the data.

Soft Gluon Coherence

Cross-section for $\{p_i\}$ hard partons and one soft gluon, $k_\mu = \omega(1, \underline{\hat{e}})$, $\omega \ll E_i$, is:

$$d\sigma^{(N+g)} = -\frac{\alpha_s}{2\pi} \sum_{i \neq j} \hat{T}^i . \hat{T}^j \omega^2 \left[\frac{p_i . p_j}{p_i . k p_j . k} - \frac{1}{2} \frac{p_i^2}{(p_i . k)^2} - \frac{1}{2} \frac{p_j^2}{(p_j . k)^2} \right] \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} d\sigma^{(N)}$$

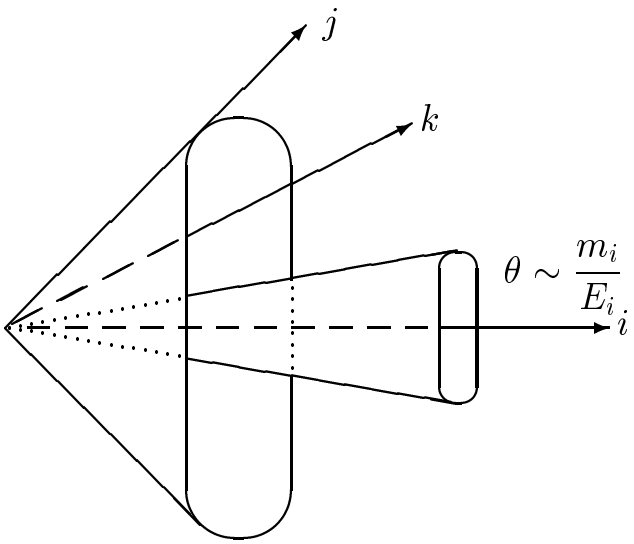
Using angles ($\omega^2 \times$) term in square brackets is:

$$\frac{1}{2} \left[\frac{1}{p_i . k} \left\{ 1 - \frac{p_i^2}{p_i . k} + \frac{(p_i . p_j - p_i . k)}{p_j . k} \right\} + (i \leftrightarrow j) \right]$$

$$= \frac{1}{2} \left[\frac{E_i \omega}{p_i . k} \left\{ \frac{v_i - \cos\theta_{ik}}{1 - v_j \cos\theta_{ik}} + \frac{(\cos\theta_{ik} - v_j \cos\theta_{ij})}{1 - v_j [\cos\theta_{ij} \cos\theta_{ik} - \sin\theta_{ij} \sin\theta_{ik} \cos\phi_i]} \right\} + (i \leftrightarrow j) \right]$$

Look at ϕ_i dependence.

$\Longrightarrow$ **no** net radiation for: $\theta_{ik} > \theta_{ij}$ $(\cos\theta_{ik} < \cos\theta_{ij})$, massless case.
The massive case is slightly more complicated.



Note kinematically suppressed **dead cone** — seen experimentally.

Eikonal Function Azimuthal Angular Dependence

$\theta_{ij} > \theta_{ik}$

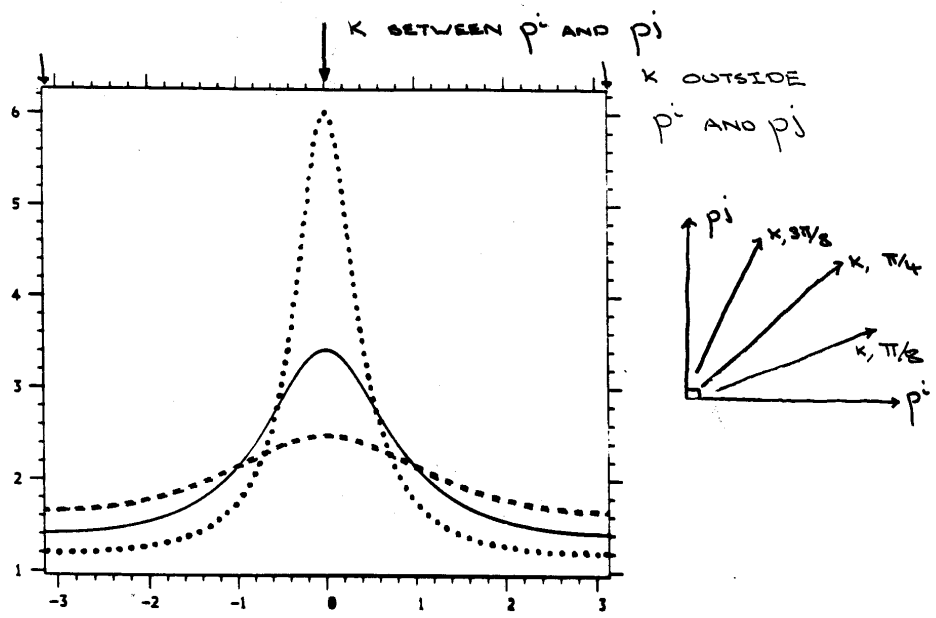


Figure 6.3: The variation of $W_j^{(i)}(\phi_i)$ for $\theta_{ij} > \theta_{ik}$. Here $\theta_{ij} = \pi/2$ in each case, whilst $\theta_{ik} = \pi/8$ dashed line, $\pi/4$ solid line and $3\pi/8$ dotted line. For convenience $1/(2\omega^2\zeta_{ik})$ has been set equal to one.

$\theta_{ij} < \theta_{ik}$

NOTE DIFFERING VERTICAL SCALES

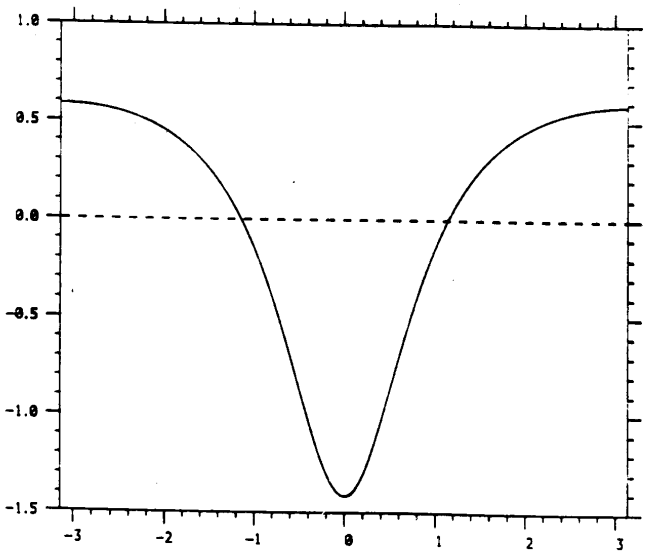


Figure 6.4: The variation of $W_j^{(i)}(\phi_i)$ for $\theta_{ij} < \theta_{ik}$. Here $\theta_{ij} = \pi/4$ and $\theta_{ik} = \pi/2$ and again $1/(2\omega^2\zeta_{ik}) = 1$.

Final State State Radiation

Order successive opening angles: $\theta_{i+1} < \theta_i$
Suggests evolution variable $\sim (1 - \cos\theta_{ij}) = \dfrac{p_i . p_j}{E_i E_j}$ **c.f.** HERWIG

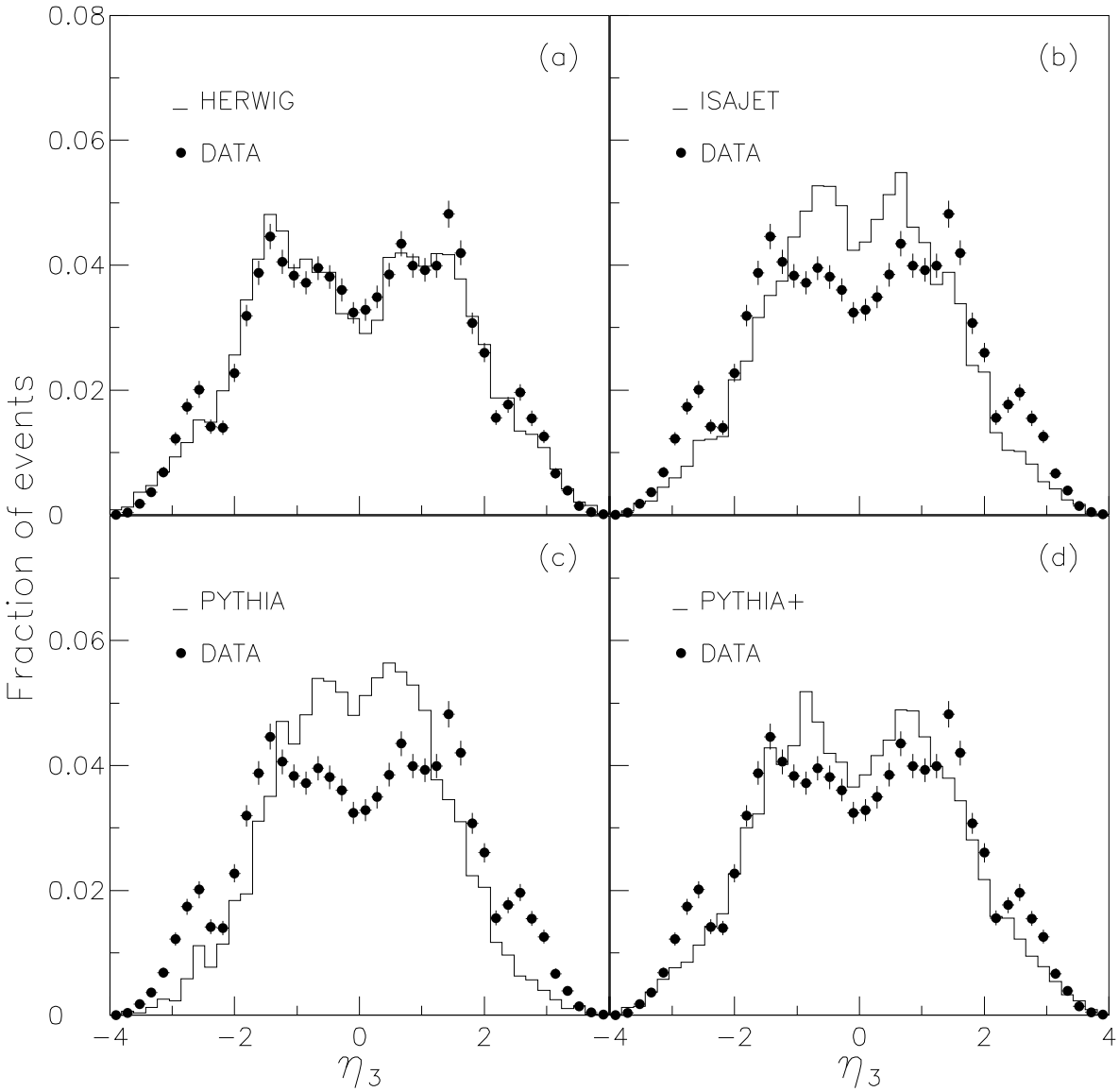
Initial State Radiation

Much harder to analyse – asymetric kinematics.
Again order angles φ_i =angle between incoming hadron and time-like emitted parton, decreases away from hard scatter.

Adequate for $x \to 1$, for $x \to 0$ also need **new** non-local, non-Sudakov form factor.

Note: only a specialist program exists for $x \to 0$ case.

Initial State Coherence is Important



The pseudo-rapidity distribution of the softest of three jets seen in the central region of $p\bar{p}$ collisions at $\sqrt{s} = 1800$ GeV. Comparison is made with the fully coherent HERWIG, incoherent ISAJET and partially coherent PYTHIA(+).

Comparison of the Main Hadronization Models

Feature	Hadronization Model			
	Cluster	Independant	String	
			Lund	UCLA
Principle	very simple	simple	complex	less complex
Lorentz invariant	yes	no	yes	
Flavour, charge, *etc.*, cons.	automatic	*ad hoc*	automatic	
Mass dep. via	hadrons	quarks	quarks	hadrons
Strangeness supp.	predicted	free param.	rest. params.	predicted
Baryon supp.	predicted	free param.	rest. params.	predicted
J^P ratios	predicted	free params.	rest. params.	predicted
Limited P_T	natural	built in	built in	natural
Frag. fn.	not applic.	free	restricted by L-R symmetry	
Cut-off (Q_0) dep.	significant	very strong	modest	
Stability	infrared prob.	collinear prob.	stable	
Limitations	massive clusters treated like strings	requires large Q_0	light strings treated like clusters	

Also exist:

- Montevay scheme – covariant indep. frag.
- CALTECH-II – hybrid string/cluster model