

Circular Motion of Particle Orbits in a Circular Storage Ring

Aims and Objectives

To simulate the conditions that affect the characteristics of a storage ring using computational methods. During the project the student should develop an understanding of the issues surrounding charged particle transport in a storage ring along with an understanding of terms like *acceptance*, *phase space*, *resonance* and *field index*.

Method

First it will be necessary to develop an understanding of the trajectory of a charged particle in a magnetic field comprising dipolar and quadrupolar elements, for a full discussion of this refer to the References below. Writing these equations in matrix form allows the particle's trajectory at an instantaneous point in the storage ring to be determined. It will then be possible, by performing small step-wise increments with the equations of motion, to follow the particle as it travels around the ring.

Starting off with the equations of motion in an ideal storage ring, these are simulated with a suitable computer program (written in C, C++, FORTRAN). Once working, the program is refined (and so becomes more realistic!) by considering phenomena such as:

- Non-uniform quadrupolar field
- Breaks in the magnet configuration due to
 - Particle injection and extraction
 - Non-continuous magnets with regular spacing
- Introduction of sextupolar field
- Effect of off-momentum particles
- Resonant conditions

References

Banford A.P., The transport of Charged Particle Beams

Rossbach J., Schmueser P., Basic Course on Accelerator Optics, CERN Yellow Report

Kollath, R., Particle Accelerators

In order to aid the student, the project is sub-divided into tasks with an approximate timescale allocated to each task. It is assumed that the project will start in week 3 (project allocations having taken place in week 2) and that the project will terminate in week 10, allowing 2 weeks for the project to be written up and handed in before the end of week 12.

Task 1: Introduction and Basic Equations of Motion (weeks 3 and 4)

Using the references listed above, along with other resources (such as the CERN Web site: www.cern.ch), develop an understanding of :

- Why storage rings are used by particle physicists;
- The basic equations of motion of a charged particle in a magnetic field;
- The use of dipolar and quadrupolar magnets in particle acceleration and storage;

Some hints :

- When developing the basic equations of motion of a charged particle in a magnetic field start from first principles i.e. a particle of charge e moving with velocity \underline{v} in a magnetic field \underline{B} will be subjected to a force \underline{F} given by:

$$\underline{F} = e\mathbf{v} \times \mathbf{B}$$

- The starting point comes from the realisation that the field is not constant across the aperture that defines the storage ring. For *small* displacements away from the central ring radius (r_0) the B field varies as

$$\circ B_y(r_0 + x) = B_y(r_0) + x \frac{dB_y}{dx} \quad \text{in the radial direction and}$$

$$\circ B_x(y) = y \frac{dB_x}{dy} \quad \text{in the vertical direction;}$$

- Consider the radial motion and the vertical motion separately;
- Work in local Cartesian co-ordinates when considering small displacements;
- Introduce n , the *field index* into the equations where

$$n = -\frac{r_0}{B_0} \frac{dB_y}{dx} \quad \text{in the radial case and } n = -\frac{r_0}{B_0} \frac{dB_x}{dy} \quad \text{in the vertical case,}$$

consider the boundary conditions on n ;

- A small displacement from the central ring radius will generate a force which is the difference of the centripetal force and the Lorentz force at that point, write the resulting expression out;

- Use the Binomial expansion $\frac{1}{(r_0 + x)} = \frac{(r_0 - x)}{r_0^2}$ to simplify this expression;

- Next change the differential with respect to time to one with respect to distance, z , along the storage ring using

$$\frac{d^2x}{dz^2} = \frac{d^2x}{dt^2} \frac{dt^2}{dz^2} \quad \text{noting that } \frac{dx}{dz} = x' \text{ is called the } \textit{divergence} \text{ of } x;$$

- This leads, in both the radial and vertical motion cases to equations for simple harmonic motion about the central orbit where the greater the displacement is away from the central orbit the greater the resulting force is *towards* the central orbit;
 - By expressing the solutions to these equations in the classic forms
 - $x = A \cos\left(\frac{z}{r_0} \sqrt{1-n}\right) + B \sin\left(\frac{z}{r_0} \sqrt{1-n}\right)$ and
 - $y = A \cos\left(\frac{z}{r_0} \sqrt{n}\right) + B \sin\left(\frac{z}{r_0} \sqrt{n}\right)$
- and by differentiating these two equations to get those for the divergences in x and y (x' and y' respectively) it is then possible to deduce the position and divergence of a particle at $z=z$ given that the position and divergence at position $z=0$ are known (separately in x and y);
- Finally express this relationship in matrix form.

Task 2: Converting the basic equations of motion into a computer program, simulating a perfect storage ring, acceptance (weeks 5 and 6)

The next step is to take these equations of motion and to convert them into computer code using a suitable procedural language such as FORTRAN, C or C++.

When performing this task it will be necessary to consider the boundary conditions for the simulation which may be written x_{\max} , y_{\max} , x'_{\max} and y'_{\max} respectively. x_{\max} and y_{\max} correspond to the physical dimensions of the storage ring, assume this is 0.06m x 0.06m by 2m in circumference and so $-0.03 \leq x_{\max}$, $y_{\max} \leq 0.03$. Limits on x'_{\max} and y'_{\max} can be derived in a similar way by considering the equations from Task 1.

In simulating a particle's trajectory around the storage ring it will be necessary to randomly choose a point in x, y, x' and y' that satisfies the above criteria and then track the particle around the ring in small steps of z (say 40).

The *acceptance* of the storage ring is a measure of how many particles maintain a stable orbit (that remains *inside* the storage ring) whilst circulating around it.

Calculate the acceptance as a function of n, the field index, for several different values of n. For each n simulate 10,000 particles and make each particle perform 50 complete revolutions inside the storage ring. Make a graph of acceptance versus field index.

Some questions to ask yourself :

- Is the choice of 40 steps per rotation a good one? Does your answer change if this value is changed?
- Will your answer change with a) the number of particles simulated or b) the number of rotations simulated? If so why?

- Your graph of acceptance versus field index n has a distinctive shape, do you understand this shape in terms of the physical properties of the dipole and quadrupole magnets?
- How do the horizontal and vertical acceptances vary?

Task 3: Check of results against theoretical value of acceptance (week 7)

A purely theoretical consideration of the acceptance can be developed as follows:

- Rewrite the solution to the simple harmonic motion as $x = A \sin\left(\frac{z}{r_0} \sqrt{1-n} + \Phi\right)$;
- Differentiate to get x' ;
- Square and add to get something in terms of A^2 ;
- Consider limits on A (A_{\max}) and develop an inequality for A ;
- Identify the resulting equation as that of an ellipse;
- Repeat for y and y' ;
- Evaluate overall acceptance in x, x', y and y' ;
- Express this as a fraction (proportional acceptance) of available four-dimensional phase space.

Compare the theoretical acceptance calculated this way with the results from Task 2.

Task 4: Introduction of a Ten Percent gap in the Storage Ring (week 8)

Up until now all the simulations have been performed using an idealised set of particle transport matrices. In real life there are gaps in a storage ring for inserting and extracting particles.

Simulate the effect of a ten percent gap in the storage ring. In the gap the quadrupolar field will fall to zero whilst the dipolar field will remain.

Plot the acceptance versus field index in this case. How does it differ from the findings in Task 2?

Project Supervision

- Dr Lee Thompson, Room E41, l.thompson@sheffield.ac.uk