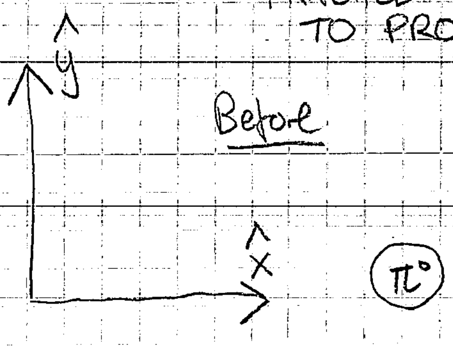
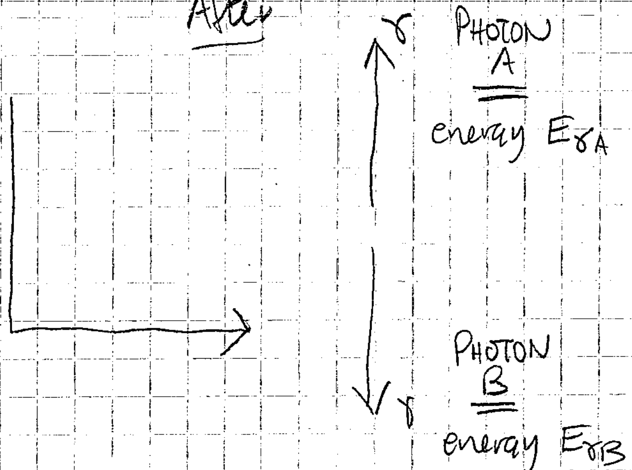
PRACTICE EXAM – SOLUTION TO PROBLEM 3



Before

Centre of mass frame of π meson



After

Centre of mass frame of π meson

(a) Photon A,

$$p_A = \begin{pmatrix} E_{\gamma A}/c \\ 0 \\ p_{\gamma A} \\ 0 \end{pmatrix} \qquad p_B = \begin{pmatrix} E_{\gamma B}/c \\ 0 \\ p_{\gamma B} \\ 0 \end{pmatrix}$$

But we need the components in terms of the mass m_π of the decaying π-meson

Conservation of energy, $E_{\gamma A} + E_{\gamma B} = m_\pi c^2$ (A)

Conservation of momentum, $\vec{p}_{\gamma A} + \vec{p}_{\gamma B} = 0$

$$\Rightarrow \vec{p}_{\gamma A} = -\vec{p}_{\gamma B}$$

$$\Rightarrow |\vec{p}_{\gamma A}| = |p_{\gamma B}| \quad \text{(B)}$$

But for a photon $E = |\vec{p}|c$

so (B) implies $E_{\gamma A} = E_{\gamma B} \left(= E_\gamma\right)$

substitute in (A), $2E_\gamma = m_\pi c^2$

$$E_\gamma = \frac{m_\pi c^2}{2}$$

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therefore the 4-momenta of the photons are

$$p_A = \begin{pmatrix} \frac{m_\pi c}{2} \\ 0 \\ +\frac{m_\pi c}{2} \\ 0 \end{pmatrix} \qquad p_B = \begin{pmatrix} \frac{m_\pi c}{2} \\ 0 \\ -\frac{m_\pi c}{2} \\ 0 \end{pmatrix}$$

3 (b) Boost parallel to the x axis

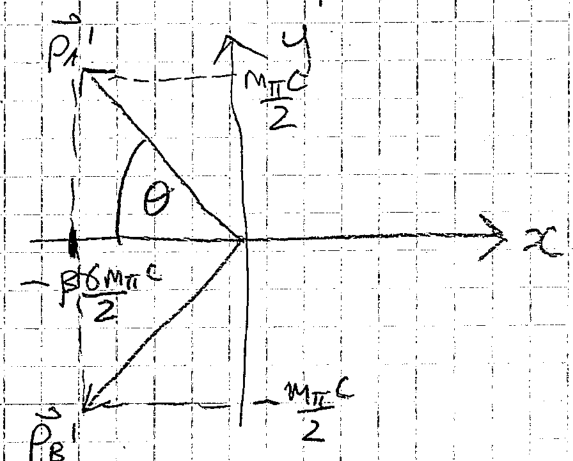
$$p_A' = \begin{pmatrix} E_A'/c \\ p_A^{x'} \\ p_A^{y'} \\ p_A^{z'} \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{m_\pi c}{2} \\ 0 \\ +\frac{m_\pi c}{2} \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\gamma m_\pi c}{2} \\ -\frac{\beta\gamma m_\pi c}{2} \\ \frac{m_\pi c}{2} \\ 0 \end{pmatrix}$$

$$p_B' = \begin{pmatrix} E_B'/c \\ p_B^{x'} \\ p_B^{y'} \\ p_B^{z'} \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{m_\pi c}{2} \\ 0 \\ -\frac{m_\pi c}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\gamma m_\pi c}{2} \\ -\frac{\beta\gamma m_\pi c}{2} \\ -\frac{m_\pi c}{2} \\ 0 \end{pmatrix}$$

$$\vec{p}_A{}' = -\frac{\beta\gamma m_\pi c}{2}\hat{x} + \frac{m_\pi c}{2}\hat{y}$$

$$\vec{p}_B = -\frac{\beta\gamma m_\pi c}{2}\hat{x} + \frac{m_\pi c}{2}\hat{y}$$



$$\tan\theta = \frac{\frac{m_\pi c}{2}}{\beta\gamma\frac{m_\pi c}{2}}$$

$$\tan\theta = \frac{1}{\beta\gamma}$$

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3c) $2\theta = 1^\circ$

$$\theta = 0.5^\circ = 0.5 \times \frac{\pi}{180} \text{ radians} = 8.7\times10^{-3} \text{ rad.}$$

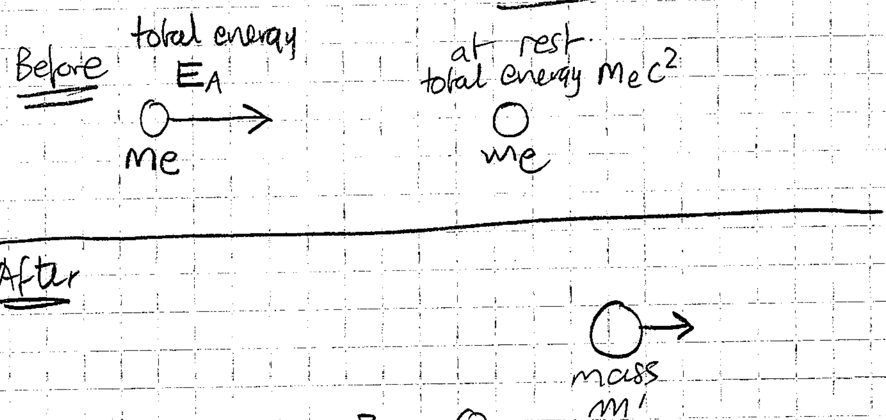
very small θ ($\theta \ll 1$ in radians)

$$\Rightarrow \tan\theta \approx \theta \text{ in radians}$$

$$\frac{1}{\beta\gamma} = 8.7\times10^{-3} \Rightarrow \beta\gamma = 115$$

But $\beta\gamma = 115 \Rightarrow \beta \approx 1$, $\underline{\gamma \approx 115}$. to very high order.

4)



Energy conservation $\quad E_A + m_e c^2 = \gamma m' c^2 \quad$ (A)

γ is the γ factor for positronium in the lab

Momentum of incoming electron is $\sqrt{E_A^2 - m_e^2 c^4}$

Momentum of outgoing positronium is given by $\beta\gamma m' c^2 = pc$

Equate momenta $\quad \sqrt{E_A^2 - m_e^2 c^4} = \beta(\gamma m' c^2)$

$$\text{from (A)} = \beta(E_A + m_e c^2)$$

$$\Rightarrow \beta = \frac{\sqrt{E_A^2 - m_e^2 c^4}}{E_A + m_e c^2}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1 - \left(\frac{\sqrt{E_A^2 - m_e^2c^4}}{E_A + m_e c^2}\right)^2}} = \frac{1}{\sqrt{\frac{(E_A + m_e c^2)^2 - E_A^2 + m_e^2 c^4}{(E_A + m_e c^2)^2}}}$$

$$\gamma = \frac{E_A + m_e c^2}{\sqrt{\cancel{E_A^2} + 2E_A m_e c^2 + m_e^2 c^4 - \cancel{E_A^2} + m_e^2 c^4}} = \frac{E_A + m_e c^2}{\sqrt{2 m_e c^2 (E_A + m_e c^2)}}$$

$$\gamma = \sqrt{\frac{E_A + m_e c^2}{2 m_e c^2}} \qquad \beta\gamma = \frac{\sqrt{E_A^2 - m_e^2 c^4}}{E_A + m_e c^2} \times \frac{\sqrt{E_A + m_e c^2}}{\sqrt{2 m_e c^2}}$$

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$$\beta\gamma = \sqrt{\frac{(E_A^2 - m_e^2c^4)}{2m_ec^2(E_A + m_ec^2)}}$$

$$K.E = (\gamma - 1)m'c^2$$

from (A) $$m'c^2 = \frac{E_A + m_ec^2}{\gamma} = E_A + m_ec^2 \times \sqrt{\frac{2m_ec^2}{E_A + m_ec^2}}$$

$$m'c^2 = \sqrt{2m_ec^2(E_A + m_ec^2)}$$

$$KE = (\gamma - 1)m'c^2$$

$$= \left(\sqrt{\frac{E_A + m_ec^2}{2m_ec^2}} - 1\right)\sqrt{2m_ec^2(E_A + m_ec^2)}$$

$$KE = E_A + m_ec^2 - \sqrt{2m_ec^2(E_A + m_ec^2)}$$

where $E_A = 1.02$ MeV

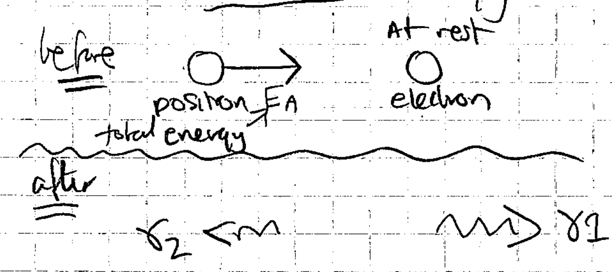
$m_ec^2 = 0.51$ MeV

$$KE = 1.02 + 0.51 - \sqrt{1.02(1.02 + 0.51)} \text{ MeV}$$

$$= 0.28 \text{ MeV}$$

Maximum energy of decay photon is in the case where one of the photons is emitted parallel to the direction of incidence of the positron

Conserve energy $\longrightarrow$ $E_A + m_ec^2 = E_{\gamma 1} + E_{\gamma 2}$



conserve momentum

$$\sqrt{E_A^2 - m_e^2c^4} = E_{\gamma 1} - E_{\gamma 2}$$

$$2E_{\gamma 1} = E_A + m_ec^2 + \sqrt{E_A^2 - m_e^2c^4}$$

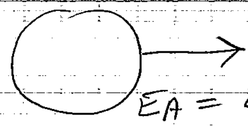
$$E_{\gamma 1} = \frac{E_A}{2} + \frac{m_ec^2}{2} + \frac{1}{2}\sqrt{E_A^2 - m_e^2c^4}$$

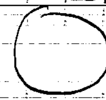
$$E_{\gamma 1} = \frac{1.021}{2} + \frac{0.511}{2} + \frac{1}{2}\sqrt{1.021^2 - 0.511^2} = 1.208 \text{ MeV}$$

PRACTICE EXAM SOLUTION TO PROBLEM 5

Before collision

Target at rest





$E_A = 437\,\text{MeV} + 938\,\text{MeV}$
$= 1.38\ \text{GeV}$

$E_B = m_p c^2$
$p_B = 0$

$m_p c^2 = 0.938\ \text{GeV}$

so E is <u>not</u> much greater than $m_p c^2$

$$\gamma = \frac{E}{m_p c^2} = 1.47$$

$$1-\beta^2 = \frac{1}{(1.47)^2} = 0.462 \qquad \beta = 0.887$$

$$p_A c = \beta\gamma m_p c^2$$

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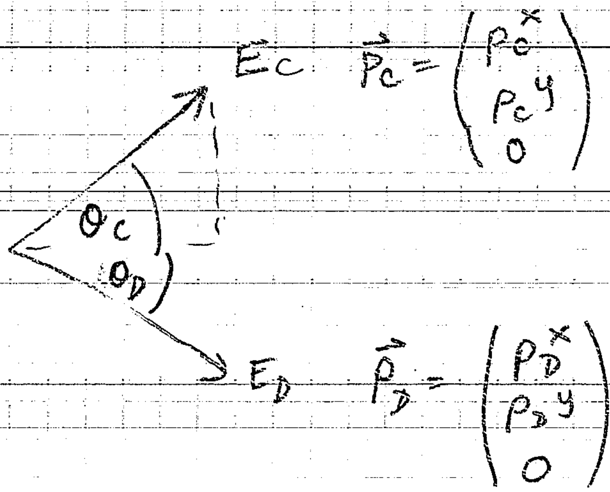
Q5.

After



Conserve energy $\quad E_A + m_p c^2 = E_C + E_D \quad$ (A)

Conserve momentum y component

$$p_C^y + p_D^y = 0 \quad \text{(B)}$$

Conserve momentum x component

$$c p_C^x + c p_D^x = c p_A = \beta \gamma m_p c^2 \quad \text{(C)}$$

from (A)

Equal energies $\Rightarrow$ $E_C = E_D = E \qquad 2E = E_A + m_p c^2$ (D)

$$\left.\begin{aligned} p_C^2 c^2 &= E_C^2 - m_p^2 c^4 \\ p_D^2 c^2 &= E_D^2 - m_p^2 c^4 \end{aligned}\right\} E_C = E_D \text{ implies } |\vec{p}_C| = |\vec{p}_D| = p$$

In terms of p, $\quad p_C^y = p \sin\theta_C$

$$p_D^y = p \sin\theta_D$$

from (B) $\quad p_C^y = -p_D^y \Rightarrow \sin\theta_C = -\sin\theta_D$

$$\Rightarrow |\theta_C| = |\theta_D| = \theta.$$

hence

$$p_C^x = p\cos\theta = p_D^x$$

and (C) $\Rightarrow 2pc\cos\theta = \beta\gamma m_p c^2$ momentum of particle A

$$= \sqrt{E_A^2 - m_p^2 c^4}$$

PRACTICE EXAM SOLUTION TO PROBLEM 5

Q5

$$2pc\cos\theta = \beta\gamma m_p c^2 = \sqrt{E_A^2 - m_p^2 c^4} = \frac{p_A c}{pc}$$

$$cp = \sqrt{E^2 - m^2c^4}$$

$$\cos\theta = \frac{\sqrt{E_A^2 - m_p^2 c^4}}{2\sqrt{E^2 - m^2c^4}}$$

$$\cos^2\theta = \frac{E_A^2 - m_p^2 c^4}{4E^2 - 4m^2c^4} \quad (E)$$

$$\text{but } 4E^2 = (E_A + mc^2)^2 \text{ from } (D)$$

$$= E_A^2 + 2E_A mc^2 + m^2c^4$$

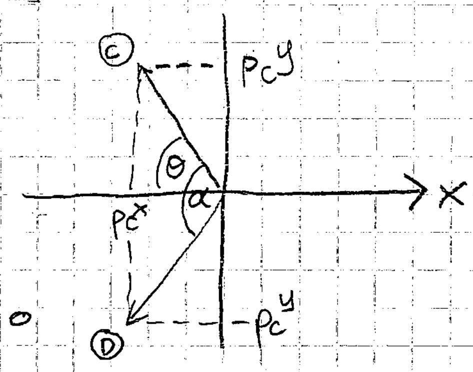
$$\text{so } 4E^2 - 4m^2c^4 = E_A^2 + 2E_A mc^2 - 3m^2c^4$$

substitute in to (E)

$$\cos^2\theta = \frac{E_A^2 - m_p^2c^4}{E_A^2 + 2E_A mc^2 - 3m^2c^4}$$

$$\cos^2\theta = \frac{(1.38\ \text{GeV})^2 - (0.938\ \text{GeV})^2}{(1.38\ \text{GeV})^2 + 2(1.38\ \text{GeV})(0.938\ \text{GeV}) - 3(0.938\ \text{GeV})^2}$$

$$= \frac{1.02\ \text{GeV}^2}{1.85\ \text{GeV}^2}$$



$$\cos^2\theta = 0.55$$

$$\cos\theta = 0.74$$

$$\theta = 0.735\ \text{rad} = 42.1^\circ$$

opening angle is $\alpha = 2\theta = 84.2^\circ$