Revision lecture 2.

1. (a) Two neutrons each of rest mass M_0 have velocities equal in magnitude $v = \beta c$ but opposite in direction as measured in their centre–of–mass frame. Show that in the rest frame of one of the neutrons, the other neutron has energy $E' = M_0c^2(1+\beta^2)/(1-\beta^2)$. [3]

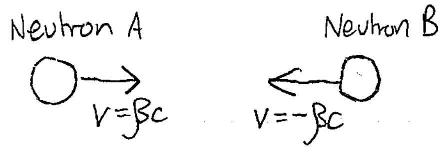
   (b) A photon rocket works by converting a portion of its mass into photons. If the photon rocket starts at rest with respect to some observer with a rest mass m_i, and subsequently is observed moving with some velocity $v = \beta c$ with respect to the same observer, and having a rest mass m_f having converted some of its rest mass into photons, show that

$$\frac{m_f}{m_i} = \sqrt{\frac{1-\beta}{1+\beta}}.$$

   You may wish to assume something particularly simple about the emission of the photons, in particular about the number of photons that is actually emitted. [4]

   (c) A photon of energy E collides with a stationary particle of rest mass m_0 and is absorbed. Express the velocity of the resulting composite particle in terms of m_0, E and c. [3]

1(a)



<u>In the lab frame</u>

Boost to the rest frame of neutron A using a lorentz transformation

$$ct' = \gamma(ct - \beta x) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$x' = \gamma(x - \beta ct)$$

or, $\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$

Apply to energy & momentum of neutron B $\quad \begin{pmatrix} E' \\ p'c \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E \\ pc \end{pmatrix}$

$E = \gamma M_0 c^2$

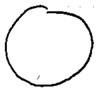
$pc = -\gamma M_0 v c = -\beta\gamma M_0 c^2$

Negative because neutron B travelling to left.

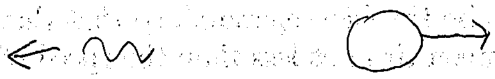
$$\begin{pmatrix} E' \\ p'c \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \gamma \\ -\beta\gamma \end{pmatrix} M_0c^2$$
$$= (\gamma^2 + \beta^2\gamma^2) M_0c^2$$
$$= \left(\frac{1+\beta^2}{1-\beta^2}\right) M_0c^2$$

1(b) before

$p = 0$
$E = m_i c^2$



after



Energy E_γ Energy $\gamma m_f c^2 = E_f$

Momentum $-\frac{E_\gamma}{c}$ Momentum $p_f c = \beta \gamma m_f c^2$

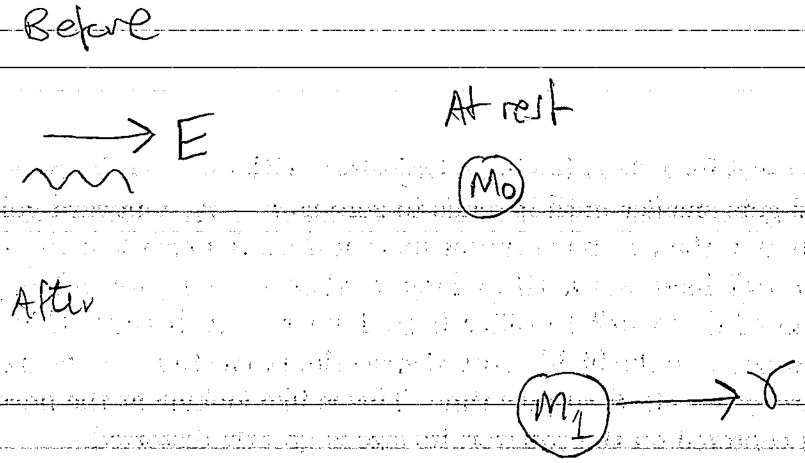
conserve energy & momentum

energy $$E_\gamma + \gamma m_f c^2 = m_i c^2 \quad (A)$$

momentum $$p_\gamma c = \left(\frac{E_\gamma}{c}\right) c = E_\gamma = \beta \gamma m_f c^2 \quad (B)$$

substitute (B) in (A) to eliminate E_γ
$$\beta \gamma m_f c^2 + \gamma m_f c^2 = m_i c^2$$
$$\gamma m_f c^2 (1+\beta) = m_i c^2$$
$$\frac{m_f}{m_i} = \frac{1}{\gamma(1+\beta)} = \frac{\sqrt{1-\beta^2}}{(1+\beta)}$$
$$= \frac{\sqrt{(1+\beta)(1-\beta)}}{(1+\beta)}$$
$$\frac{m_f}{m_i} = \sqrt{\frac{1-\beta}{1+\beta}}$$

1c)



Conserve energy

$$E + M_0 c^2 = \gamma M_1 c^2$$

Conserve momentum

$$\gamma M_1 V = \frac{E}{c}$$

$$\beta \gamma M_1 c^2 = E$$

$$\gamma M_1 c^2 = \frac{E}{\beta}$$

$$E + M_0 c^2 = \frac{E}{\beta}$$

$$\beta = \frac{E}{E + M_0 c^2}$$

$$V = \frac{Ec}{E + M_0 c^2}$$