Lecture 1 - Motivation, and some pre-relativity physics

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1 Motivational Introduction

Welcome to the relativity lectures of PHY206 here at Sheffield! This subject is one of the reasons people decide to study physics at University. It is interesting, profound, exciting, and powerful. It's principal architect, Albert Einstein, is one of the iconic figures of our time, and with good reason. You will have probably been taught before of some of the consequences of special relativity - perhaps you have been taught that things get more massive when they are moving at speeds approaching that of light. Perhaps you will have heard the saying 'moving clocks run slow'. Perhaps you know about Lorentz contraction - the observable fact that rigid bodies appear compressed parallel to their direction of motion when moving close to the speed of light. Certainly you will have seen the equation $E = mc^2$.

In this course, my main aim is to introduce these and other concepts very carefully and precisely, so that as many as possible of you understand how this subject fits in with the physics you have studied previously, and to equip those of you who wish to study the branches of physics that necessarily make use of special relativity with a solid foundation on which to build further later on. In particular, I wish to be careful to adopt modern conventions and modes of thought so that you will not later be confused by the conventions and choices made by research scientists. Finally, and most of all, I want to make you all think about this subject. The beauty of special relativity is that it is conceptually not all that difficult, but philosophically quite profound. Perhaps unlike quantum mechanics, you really can get your head around this subject, and come out, with some work, feeling that you have achieved a solid understanding of something truly marvellous and profound. I hope you enjoy these lectures, and I wish you the very best of luck with this course.

2 The language of special relativity - spacetime, events and world lines

Special relativity is an extension of the ordinary classical mechanics that existed before the quantum revolution in the 1920s. In this classical world, we describe the state of a system by the motion of its particles. As a particle moves around, we can use Cartesian coordinates (x, y, z) to describe where the particle is at any time t. How would we determine the motion of a particle? Suppose for example that we are only interested in the x–component of its position as it moved. Equipping ourselves with a notebook, we would write down the x–component of the particle's position, x, and the time on our watch t at which the particle was in that position. We would then make a table of these pairs of times and positions having two columns (or four if we were measuring y and z as well), and this table would give us information on the motion of the particle.

In this way of looking at things, the position x and the time t are on an equal footing - it takes a set of pairs of values (t, x) to describe the x–component of the particle's trajectory. The idea of thinking of position and time as coordinates on an equal footing occured to H. G. Wells who wrote in his book *The Time Machine*:

> if Time is really only a fourth dimension of Space, why is it, and why has it always been, regarded as something different?

Following Wells' suggestion, therefore, we make a plot of t versus x; it is conventional to put x on the horizontal axis. Figure 1 shows such a plot.

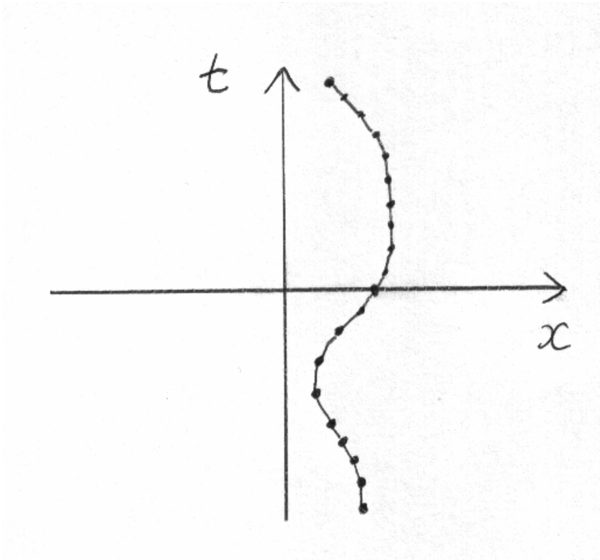

Figure 1: Our first spacetime diagram

This is our first space-time diagram, although the relativistic ones we will be using soon are slightly different in that the time axis has a factor of the speed of light inserted (ct instead of t), for reasons that I will make clear presently. Each point on the line represents an 'event' in the history of the particle's motion. Taken all together, the sequence of events trace out a continuous curve, called a 'world line', with a single value of x for each t, since at least in classical physics, particles do not travel backwards through time. The velocity of the particle is dx/dt, which is one over the gradient of the line in our diagram, so steeper lines represent slower motion, and stationary particles have vertical world lines.

3 Inertial observers

In Section 2 we made a tacit assumption about the observer making the measurements. Simplistically, we might claim that the observer is at rest. In fact even before relativity it was realized that concept of absolute rest is pretty meaningless. It is more useful to specify that our observer is not accelerating. You can make measurements to tell that you are accelerating, since acceleration of massive bodies produces forces that can be measured. Therefore any observer can test to see whether or not they are accelerating. A non–accelerating observer is called an inertial observer. Throughout this course all our observers will be inertial.

4 Different Inertial Observers

It might occur to you that there are many different inertial observers. For a start, two different inertial observers may choose different origins for their coordinates. There is no interesting physics here. More interesting is the case where one observer is moving at a constant velocity with respect to another. Neither observer is accelerating, so both are inertial, but the coordinates of events seen by the two observers will be different. How do the coordinates of the same event written down by the two different observers differ?

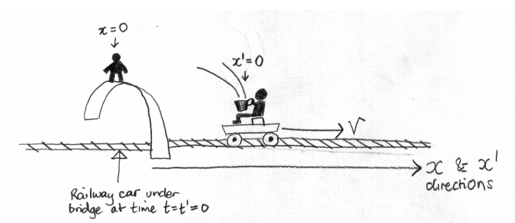

Figure 2: Two inertial observers viewing a coffee cup on a train.

It is useful to consider an example. Imagine that our two observers are a man on a bridge over a railway line, and a woman on a train passing under it at velocity v in a direction aligned with the x–axis of the man on the bridge. We will assume for now (in special relativity it will turn out that this assumption cannot be maintained) that the two observers have identical synchronized watches, and that at time zero, the woman is directly under the railway bridge, so that the origins of the coordinate systems of the two observers coincide at time zero. Let us denote the coordinates of the events seen by the man on the bridge by t and x, and the components of the same event seen by the woman on the train as t' and x'. It is common practice to refer to these two observers as the unprimed observer (the man on the bridge) and the primed observer (the woman on the train). In this case the primed observer is moving to the right, in the direction of increasing x, with respect to the the unprimed observer.

Suppose there is a coffee cup positioned on a table next to the woman on the train. What is the world line of this coffee cup to the two observers? To the primed, woman observer on the train, the coffee cup is not moving, so in her coordinate system we could decide that the coffee cup is at her origin at all times, so that $x' = 0$ for all t'. What about the world line of the same coffee cup in the coordinate system of the unprimed man on the bridge? To this man, the coffee cup is moving in the direction of increasing x at velocity v, so that $x = vt$. How then are t and x related to t' and x'? For a start, we decided that both observers have identical synchronised watches, so $t = t'$. And to make $x' = 0$ consistent with $x = vt$ we must have $x' = x - vt$. Finally, the y– and z– components of the coffee cup world line because the origins of the two coordinate systems coincide at $t = 0$, and there is no motion of the two observers parallel to the y– or z– axes subsequently. Therefore we have

$$\begin{aligned} t' &= t \\ x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned} \tag{1}$$

Equations 1 are called the Galilean transformations. We derived them using plain old common sense; they are named after Galileo because he was the first person to write about inertia. Before special relativity, they were considered obvious and not subjected to scrutiny.

In Figure 3 I have drawn a spacetime diagram for the motion of the coffee cup in the coordinate system of the unprimed observer, the man on the bridge. The world line has a constant slope equal to $1/v$, where v is the train's velocity, and it passes through the origin since we decided that the woman and the coffee cup passed under the bridge at time $t = 0$. For definiteness, let us suppose that the train is moving at $180\,\mathrm{km\,h^{-1}}$, which is $50\,\mathrm{m\,s^{-1}}$, and consider its motion for 20 seconds after it passes under the bridge.

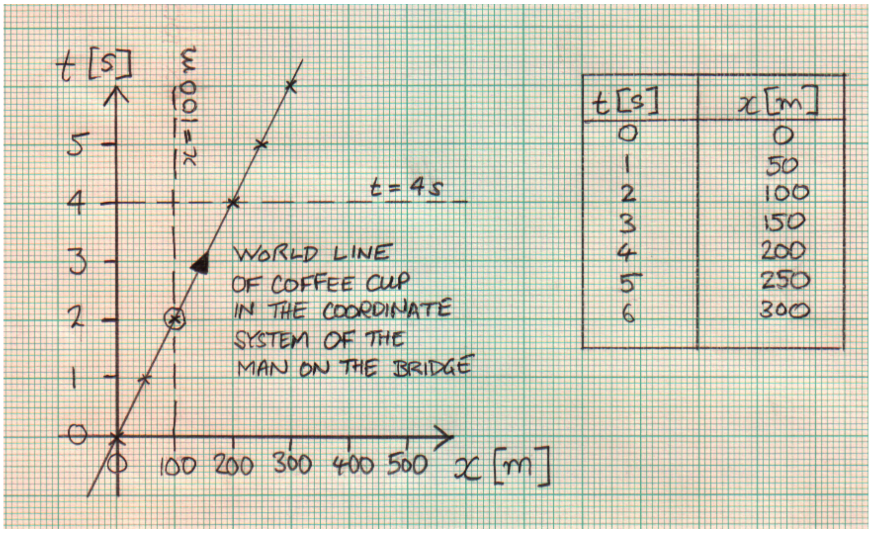

Figure 3: The world line of the coffee cup in the coordinate system of the unprimed observer on the bridge.

5 The meaning of axes on a space-time diagram

Though you have drawn these kinds of plots lots of times, I want to encourage you to think more about them now. Consider first the set of points for which $x = 100\,\mathrm{m}$. All these points lie on a vertical line passing through the x–axis at the 100 label. I have drawn a dotted line on the figure corresponding to this set of points. Because the train is only 100 m after the bridge at a single instant, the world line intersects this vertical line once, so there is a single event where the train is $100\,m$ after the bridge, which I have circled on the plot. Let's now think about how to make this same argument algebraically. The vertical dotted line has the equation $x = 100$. Let us figure out the equation for the world line of the train. Start with the standard equation for a straight line in an x–y plane, $y = mx + c$, where m is the gradient of the line, and c is the intercept with the y axis. Now relate to the current set of axes, where y is replaced by t, and the gradient is $1/v$, where v is the velocity of the train, $50\,\mathrm{m\,s^{-1}}$. Finally, the y–axis intercept becomes the t–axis intercept, which is zero because we have decided to define zero seconds as the time when the train passes under the bridge. Therefore the equation of the world line of the train is

$$t = x/50 \tag{2}$$

To find out where this line intersects the vertical dotted line, substitute in the equation of this line, which is $x = 100$, and you get $t = 100/50$, or $t = 2$. Now notice that the event of the train being 100 m past the bridge has t–coordinate 2 seconds.

I have belaboured this example. as a way of stressing that you can use the geometry of spacetime diagrams in conjunction with simple algebra of straight lines in planes to solve problems. In this case, it was like cracking a walnut with a sledgehammer. Later on this sledgehammer will come in handy.

Using these methods, let's relate this spacetime diagram to what the woman on the train sees. There are two ways to do this, the first is to draw a spacetime diagram in the coordinate system of the woman on the train. We will do this in your problems class. The second is to consider how the coordinate system of the woman overlays onto the spacetime diagram of the man on the bridge. This is easy, but probably unfamiliar. What we would like is a grid to overlay on the spacetime diagram we have drawn from which we can read off the coordinates of events along the worldline as they would be observed by the woman on the train.

Let us figure out how to grid lines of constant time first. In the unprimed coordinates, these lines are horizontal. For example, I have drawn a dotted horizontal line that represents the set of events for which $t = 4\,\mathrm{s}$. But from the Galilean transformations of Equations 1, we know that $t' = t$, so that this horizontal line represents also the set of events for which $t' = 4s$. Therefore the lines of constant t' are coincident with the lines of constant t. Next, let us figure out where the x' axis is. The x axis is coincident with the line of constant $t = 0$ (if you can't see why this is, remember that in an x–y plane, the x axis is the set of points for which $y = 0$, if that makes it clearer to you). Similiarly, the x' axis will be coincident with the line of constant $t' = 0$, which is coincident with the line of constant $t = 0$. Therefore the x axis and the x' axis lie on top of each other. We can therefore draw half of the grid we require, the lines of constant t'. This half of the grid is shown in Figure 4.

Next let us repeat this exercise to work out the lines of constant x'. Suppose we want the line coincident with all events where $x' = 100\,\mathrm{m}$. What does this line look like in the coordinate system of the man on the bridge? Just substitute the equation $x' = 100$ into the expression for x' from Equations 1 and we get $100 = x - vt$, or $t = x/v - 100/v$. With $v = 50\,\mathrm{km\,s^{-1}}$, this

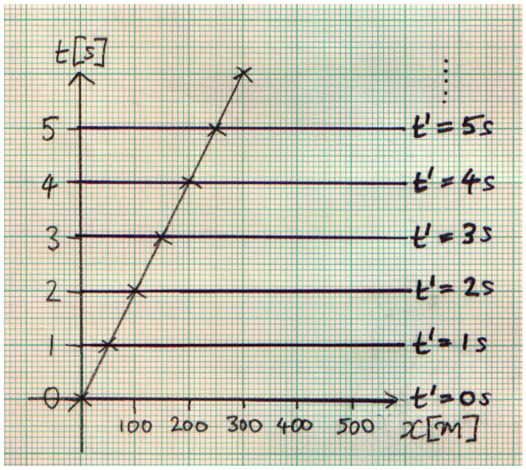

Figure 4: A spacetime diagram for the moving train in the coordinate system of the observer on the bridge, with lines of constant t' (the time coordinate for the woman on the train) overlaid.

becomes $t = x/50 - 2$. This is a straight line with gradient 1/50 and t–axis intercept -2. We can repeat this exercise for all other lines of constant x', and you get the grid lines representing constant x' that we require. Note that they are not perpendicular to the lines of constant t' that we already found. I have drawn the entire grid of the coordinate system of the woman on the train, overlaid on our coordinate system of the man on the bridge, in Figure 5.

Mathematicians would say that the woman's train coordinate grid is skewed with respect to the man's bridge coordinate system. Notice that the world line of the cup of coffee coincides with the line of constant $x' = 0$, so we have found out the hard way that in the coordinate system of the woman with the cup in front of her, it stays fixed in space. Walnut with a sledgehammer again, but once again the point is to have the sledgehammer for when we will need it.

6 Trouble with the Galilean transformations

Einstein was probably unaware of many of the experimental results that appeared to contradict the tenets of non relativistic

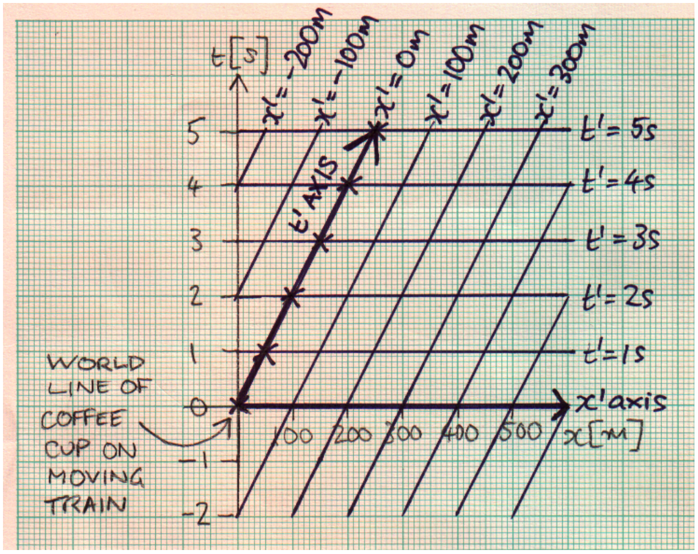

Figure 5: A spacetime diagram for the moving train in the coordinate system of the observer on the bridge, with lines of constant t' and lines of constant x' overlaid.

mechanics at the time he wrote his revolutionary 1905 paper. Instead, he made use of thought experiments to see that a more sophisticated mechanics might be needed under some circumstances. Let us try and do the same thing here. You have heard of the principle of conservation of charge. If we have a physical system the total charge in it is conserved. So suppose our system consists of a single electron. To an inertial observer, this electron starts out at the origin. Now let us imagine that the electron suddenly vanishes, reappearing at a different position at exactly the same instant. Charge is conserved. Is this allowed? Actually, this would be absurd. Suppose the place where the charge reappears is in the Andromeda galaxy. To a local observer, it would appear that charge conservation was violated, even though it apparently is not. Something is wrong.

It is not sufficient to flail our arms and make statements that it would be impossible to realize the physics of the charge moving instantaneously from one place to another - the fact is that the circumstances outlined here do not violate any known principle of classical theory. Furthermore, all inertial observers will agree that the electron disappears in one place and appears in a different place simultaneously - in Figure 6 I have drawn the world line of the electron along with the coordinate axes of various different intertial observers. To each of them, the world

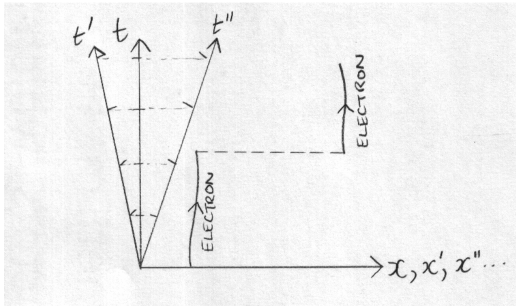

Figure 6: The world line of a jumping electron

line jumps between different positions at a single time. This suggests, maybe something is wrong with the Galilean transformations. The cause of the trouble here is that the electron has been allowed to move infinitely fast from one place to another. In classical physics, there is no problem with this. Einstein wondered if the world actually works this way. We will eventually return to this problem once we have some of Einsteins tools in our arsenal and see that in special relativity the instantaneous teleportation of charge (or mass, or any other conserved quantity) through space is explicitly forbidden.

7 About the course

I am handing out my own notes for the course, but I also recommend the the book 'Special Relativity' by Prof. A. P. French of the Massachusetts Institute of Technology, published by Chapman and Hall. This nice little book costs £ 40 new or about half that used from Amazon. It is particularly good on experimental evidence supporting special relativity. Also, it turns out that the University of Sheffield has the entire book available from on-campus computers through this link:

`http://lib.myilibrary.com/browse/open.asp?id=1972&loc=`

There are also several copies in the Western bank library and information commons.

I do hope you enjoy the course.