1. (a) Two neutrons each of rest mass M_0 have velocities equal in magnitude $v = \beta c$ but opposite in direction as measured in their centre–of–mass frame. Show that in the rest frame of one of the neutrons, the other neutron has energy $E' = M_0 c^2(1+\beta^2)/(1-\beta^2)$. [3]

(b) A photon rocket works by converting a portion of its mass into photons. If the photon rocket starts at rest with respect to some observer with a rest mass m_i, and subsequently is observed moving with some velocity $v = \beta c$ with respect to the same observer, and having a rest mass m_f having converted some of its rest mass into photons, show that

$$\frac{m_f}{m_i} = \sqrt{\frac{1-\beta}{1+\beta}}.$$

You may wish to assume something particularly simple about the emission of the photons, in particular about the number of photons that is actually emitted. [4]

(c) A photon of energy E collides with a stationary particle of rest mass m_0 and is absorbed. Express the velocity of the resulting composite particle in terms of m_0, E and c. [3]

2. π^0–mesons decay to two gamma rays via $\pi^0 \rightarrow 2\gamma$. The rest mass of a π^0 meson is $(135.1\,\mathrm{MeV})/\mathrm{c}^2$. In a particular lab experiment, the π^0 mesons have energies in the range 6 GeV to 18 GeV. Take the mean life τ_0 of a π^0–meson to be $2.9 \times 10^{-16}\,\mathrm{s}$. For this question, use $c = 2.9974 \times 10^8\,\mathrm{m\,s^{-1}}$.

   (a) What are the lowest and highest possible velocities in the π^0–meson beam? [2]

   (b) What are the lowest and highest possible π^0–meson lifetimes as measured in the lab, in seconds? [2]

   (c) For π^0–mesons at the lower and upper edges of the energy range, what distances between production and decay do the mean lives correspond to, in micrometres? Is it possible for a π^0 to survive have a longer range between production and decay than the greater of these two numbers? [2]

   (d) In the lab, what is the maximum possible energy achievable by a photon from a π^0–meson decay in this particular beam? [4]

3. In the centre of mass frame, a π^0–meson decays to two photons each of which travels parallel to the y–axis.

   (a) Write down the four components of the 4–momenta of the emitted photons in the centre of mass frame of the decaying π^0–meson expressing your answer in terms of the π^0–meson rest mass m_π and the velocity of light c. [3]

   The centre of mass frame is related to the lab frame by a Lorentz boost parallel to the x–axis in the direction of decreasing x. In this frame, the directions of emission of the two photons are at an acute angle θ to the x–axis.

   (b) If the velocity of the lab frame with respect to the centre of mass frame is $v = \beta c$, show that $\tan\theta = 1/(\beta\gamma)$, where $\gamma = 1/\sqrt{1-\beta^2}$. [3]

   (c) Making suitable approximations to speed up the calculation, work out what value of γ would yield an opening angle 2θ between the two photons of $1°$. [4]

4. A positron of kinetic energy 0.51 MeV collides inelastically with an electron at rest, forming a state called a positronium, consisting of the electron and the positron bound together. The positronium recoils freely, and then the electron and positron annihilate, producing two photons in the final state. Take the electron and positron masses as $(0.511\,\mathrm{MeV})/c^2$.

   (a) What is the kinetic energy of the recoiling positronium, in MeV? [5]

   (b) What is the maximum possible energy of a single photon produced in the final state, in MeV? [5]

5. If a proton of kinetic energy 437 MeV in the lab frame collides elastically with a proton at rest, and the two protons rebound with equal energies as measured in the same lab frame, what is the opening angle between the emission directions of the two protons, in degrees? Assume a proton rest mass of $(938\,\mathrm{MeV})/c^2$. [10]

END OF QUESTION PAPER