PHY206 Problem class 1 - relativity problem.

Question PHY206, relativity problem 1.

The relativistic gamma and beta factors are defined as

$$\gamma \;=\; \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{and} \quad \beta \;=\; \frac{v}{c}.$$

(a) [2 points] For particles moving at a speed of $c-\varepsilon$, where $\varepsilon \ll c$ and c is the speed of light in a vacuum, show that

$$\gamma \;\simeq\; \sqrt{\frac{c}{2\epsilon}} \quad \text{and} \quad \beta\gamma \;\simeq\; \sqrt{\frac{c}{2\epsilon}}.$$

Hint: To get the expression for γ, either use a binomial expansion or multiply out $(c-\varepsilon)^2$ and use a suitable approximation to handle the smallest term.

Reminder: For radioactive particles moving at velocities comparable to that of light, the mean life is modified from the rest mean life τ to $\gamma\tau$. This is the mean life that is inserted into the radioactive decay law for relativistic particles.

(b) [2 points] Unstable radioactive particles with a mean life τ when at rest are produced in an accelerator and subsequently travel a distance x down a beam pipe at a velocity ε less than that of light, where again $\varepsilon \ll c$. Show starting from the radioactive decay law that the fraction N/N_0 of particles left undecayed after this distance is given approximately by

$$\frac{N}{N_0} \simeq \exp\left(\frac{-x\sqrt{\frac{2\varepsilon}{c}}}{c\tau}\right).$$

(c) [1 point] π mesons at rest have a mean life of 2.9×10^{-8} s. When produced in an accelerator they are given velocities only 2 m/s slower than the speed of light. Thirty metres from the production point, what fraction of the π mesons should have decayed?