PHY306 Introduction to Cosmology
Numerical Answers to Problems

1. No numerical answer.

2. No numerical answer.

3. No numerical answer.

4. (a), $z \simeq 0.025$; (b)(i), 100–200 Mpc; (ii), just about, but the effect will be small ($z = 0.033 > 0.025$: difference between d_L and d_A is significant compared to 5% systematic error, but difference of either from d_P is not).

5. No numerical answer.

6. No numerical answer.

7. (i), 8.3×10^{-10} J m^{-3}; (ii), 9.2×10^{-27} kg m^{-3}; (iii), $1.4 \times 10^{11}\ M_\odot$ Mpc^{-3}; (iv), 5.2 GeV m^{-3}.

8. $R_0 = 20$ Gpc; $r = 10.7$ Gpc, for which $x_k = 10.2$ Gpc.

9. No numerical answer.

10. No numerical answer.

11. (c), condition is $a^3 > \Omega_{\mathrm{m0}}/2(1-\Omega_{\mathrm{m0}})$, where the flatness condition requires that $\Omega_{\Lambda 0} = 1 - \Omega_{\mathrm{m0}}$. This is either a minimum value of a (for fixed Ω_{m0}) or a maximum value of Ω_{m0} (if you take $a = 1$).

12. (b), $a = \Omega_{\mathrm{r0}}/\Omega_{\mathrm{m0}}$; (c), $a = (\Omega_{\mathrm{m0}}/\Omega_{\Lambda 0})^{1/3}$.

13. (a), 6400; (b), 0.73.

14. No numerical answer.

15. No numerical answer.

16. No numerical answer.

17. No numerical answer.

18. 49 ± 8 km s^{-1} Mpc^{-1}.

19. No numerical answer.

20. No numerical answer.

21. (i), 11 Gyr (SCDM 8.8 Gyr); (ii), 9.0 Gyr (SCDM 7.7 Gyr).

22. (a), 12.0 Gpc; (b), 4.9 Gpc.

23. (a), 59 ± 10 km s^{-1} Mpc^{-1}, which is larger than the SCDM value (note that the errors aren't relevant for this comparison, because they are not independent), but less than Riess et al. (admittedly, the difference (15 ± 10) is not statistically significant).

    (b), $H_0 t_0 = 1.02 \pm 0.17$; $\Omega_{\mathrm{m}0} = 0.19$.

24. (e), $r = \dfrac{c}{H_0\sqrt{\Omega_{\mathrm{m}0} - 1}}\left(\theta(t_0) - \theta(t_z)\right)$ where t_z is the time corresponding to redshift z, and $\theta(t_z)$ is the corresponding value of θ.

25. (b), $a^3 = \Omega_{\mathrm{m}0}/2\Omega_{\Lambda 0}$; (d), $\frac{27}{4}\Omega_{\mathrm{m}0}^2\Omega_{\Lambda 0} = (\Omega_{\mathrm{m}0} + \Omega_{\Lambda 0} - 1)^3$;

    (e), useful form of expression is

$$\Omega_{\Lambda 0} = \frac{4}{27}\frac{(\Omega_{\mathrm{m}0} - 1)^3}{\Omega_{\mathrm{m}0}^2}\left(1 + \frac{\Omega_{\Lambda 0}}{\Omega_{\mathrm{m}0} - 1}\right)^3,$$

which can be solved iteratively by using the factor outside the brackets to calculate a first value of $\Omega_{\Lambda 0}$ and then substituting this into the factor inside the brackets. Repeat, each time substituting in the new value of $\Omega_{\Lambda 0}$, until the result doesn't change to whatever number of significant figures you're interested in. This is easy to do on a spreadsheet. It will converge if $\Omega_{\mathrm{m}0} > 1$ and $\Omega_{\Lambda 0} \ll \Omega_{\mathrm{m}0} - 1$ (so that the second term in the brackets is small).

26. (a), $R_0 = c^2/\sqrt{4\pi G \mathcal{E}_{\mathrm{m}0}}$.

27. No numerical answer.

28. (i), 13 Gyr; (ii) 11 Gyr.

29. Horizon distance, 13.7 Gpc (matter-only 8.1 Gpc, open universe 11.9 Gpc).

    Distance to object with $z = 3$, 6.1 Gpc (matter-only 4.1 Gpc, open universe 4.9 Gpc).

    Result for horizon distance is rather sensitive to how finely binned your numerical integration is; result for $z = 3$ is much less so.

30. $z = 0.73$ (not dependent on H_0); 6.2 Gyr.

31. 50 km s^{-1} Mpc^{-1}.

z	5.0	4.0	3.0	2.0	1.0
Look-back time (Gyr)					
Matter-only	12.1	11.8	11.4	10.5	8.4
Benchmark	11.9	11.5	10.9	9.8	7.4
Proper distance (Gpc)					
Matter-only	7.1	6.6	6.0	5.1	3.5
Benchmark	7.5	6.9	6.1	5.0	3.2

32. (b), if $M' = M + \Delta M$, then $H_0' = H_0 \times 10^{\Delta M/5}$.

    (c), if $\ell' = \ell + \Delta\ell$ (linear error) then H_0 is not changed, but the intercept on the Hubble plot isn't zero. If $\ell' = \ell(1 + \Delta\ell)$ (fixed percentage error, more plausible), then $H_0' = H_0/(1 + \Delta\ell)$.

33. No numerical answer.

34. (c)(i), 71.5 ± 1.3 km s^{-1} Mpc^{-1} (statistical errors only) with outlier, 70.9 ± 1.3 km s^{-1} Mpc^{-1} without.

35. No numerical answer.

36. 1.4×10^{-14} u m^{-3} s^{-1}.

37. No numerical answer.

38. (a), $(-8 \pm 12) \times 10^{-7}$; (b), $(-5.2 \pm 7.5) \times 10^{-54}$; (c), 61 e-foldings.

39. (a), $\Omega_k(10^{-35}\,\text{s}) = (-2.2 \pm 3.1) \times 10^{-38}$, 43 e-foldings;
    (b), $\Omega_k(10^{-35}\,\text{s}) = (-6.2 \pm 8.9) \times 10^{-56}$, 63 e-foldings.

40. (a)(i), 12.0 Gpc; (ii), 330 kpc; (iii), $330 \times 10^{-6} \times 1100/12 = 0.03$ rad $= 1.7°$.
    (b)(i), $9.0 \times 10^{-27}\,\text{m} = 2.9 \times 10^{-43}$ pc; (ii), $z = 1.2 \times 10^{35}$; 1.0×10^{-25} pc; 40 e-foldings.

41. No numerical answer.

42. No numerical answer.

43. No numerical answer.

44. No numerical answer.

45. No numerical answer.

46. No numerical answer.

47. (b), $w \leq -\frac{1}{3}$; (c)(i) $z = 0.73$ (same as benchmark); (ii) $n = 20/3$ and $t_0 = 20/(3H_0)$; (iii) $r = \frac{20c}{17H_0}\left((1+z)^{17/20} - 1\right)$.

48. No numerical answer.

49. No numerical answer.

50. No numerical answer.

51. No numerical answer.

52. (a), 1.25; (b)(i), 25.3; (ii), 17 kpc; (c)(i), 13 Gyr; (ii), 12 Gpc; (iii), 0.4 Gpc.

53. (i), 12 Gyr; (ii), 7.8 Gyr; (iii), 26; (iv), $2.6''$.

54. (i), 0.0152; (ii), 0.0066; (iii), 0.0034; (iv), 0.0023; (v), 0.0018.

55. (a), $\pm 20\%$ (more precisely, $^{+20}_{-17}\%$); (b), $\sim$100.

56. (a), 13.8 Gyr; (b), 8.60×10^{-5}; $\mathcal{E}_{\rm BB} = 4.17 \times 10^{-14}$ J m^{-3} $\Longrightarrow$ $\Omega_{\rm BB} = 5.09 \times 10^{-5}$; (c), $z = 0.720$, $t_{\rm lb} = 6.5$ Gyr; (d), $H = 1.54 \times 10^6$ km s^{-1} Mpc^{-1}, $\Omega_{\rm r} = 0.249$, $\Omega_{\rm cdm} = 0.628$, $\Omega_{\rm b} = 0.122$, $\Omega_\Lambda = 1.48 \times 10^{-9}$. [Note: they don't quite add to 1 because of rounding errors.]