Modern cosmology 3: The Growth of Structure

- Growth of structure in an expanding universe
- The Jeans length
- Dark matter
- Large scale structure simulations
  - effect of cosmological parameters
- Large scale structure data
  - galaxy surveys
  - cosmic microwave background

Large scale structure simulations

- Simple theory only adequate for small changes in density
- Need big changes
  (e.g. $\rho_{\text{univ.}} \sim 10^{-27}$ kg m$^{-3}$, $\rho_{\text{galaxy}} \sim 10^{-20}$ kg m$^{-3}$)
- Therefore use numerical simulations
  - input cosmological parameters
  - evolve using general relativity
  - may include only dark matter or dark matter + gas
Simulations

- Information from simulations
  - strength of clustering on different scales
    - compare with galaxy surveys
  - evolution of clustering
    - compare with ages of structures such as galaxies and clusters

PHY306

The VIRGO Collaboration 1996

$z = 20.0$

dark matter density  

gas density  

gas temperature

kinetic SZ  

thermal SZ  

gas shocks

4 Mpc/h
Large scale structure data

- Galaxy surveys
  - pencil beam
    - e.g. Lyman α lines in quasar spectrum
  - slice of sky
    - e.g. 2dF galaxy redshift survey
  - whole sky (or large piece thereof)
    - e.g. Sloan Digital Sky Survey

Lyman α forest

- Study distribution of neutral hydrogen along particular lines of sight
  - potential information on clustering, metallicity, ionisation level, etc., at redshifts up to 6 or more
  - but systematic errors are difficult to control
**Sloan Digital Sky Survey**

- Dedicated 2.5-m telescope equipped with 120 megapixel camera and two multi-object spectrographs
- Imaged 8400 square degrees of sky
  - spectra of 930000 galaxies, 120000 quasars, 225000 stars

**SDSS Galaxy Map**

- Slice of SDSS survey around celestial equator
  - $-1.25^\circ < \delta < +1.25^\circ$
- Galaxies colour coded by stellar population
  - red = old
Results

- Sensitive to $\Omega_m H_0$, which is a different combination from nucleosynthesis
- Analysis is similar to CMB (see later), but expected shape differs
- Best results obtained by combining redshift surveys with WMAP

Analysis of survey data

- Survey data typically produce “maps”
- How do we analyse these?
  - they have finite resolution
  - they may not cover the whole sky
  - we probably don’t care about the actual locations of ‘hot’ and ‘cold’ spots
    - we want to look at strength of variation and characteristic size
The power spectrum

- Consider CMB data, i.e. map of temperature fluctuations $\delta T$ across sky
  - expand in spherical harmonics: $\frac{\delta T}{T}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi)$
  - consider correlation between pairs of points separated by angle $\theta$:
    $$C(\theta) = \left\langle \frac{\delta T}{T}(\mathbf{n}) \frac{\delta T}{T}(\mathbf{n}') \right\rangle \mathbf{n} \cdot \mathbf{n}' = \cos \theta$$
  - by applying the spherical harmonic expansion this can be expressed as a sum of Legendre polynomials:
    $$C(\theta) = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) C_l P_l(\cos \theta)$$

- Parameter describing characteristics of map is the coefficient $C_l$
  - customary to plot $\Delta_T \equiv \sqrt{\frac{l(l+1)}{2\pi} C_l \langle T \rangle}$ vs $l$
  - this is the contribution per logarithmic interval in $l$ to the total temperature fluctuation $\delta T$
  - the multipole number $l$ gives the angular scale: $\theta \sim 180^\circ/l$
The power spectrum

- 3D galaxy surveys are analysed in a similar way
  - expand as Fourier series
    \[ \delta(r) = \frac{V}{(2\pi)^3} \int \delta_k e^{-i k \cdot r} \, d^3 k \]
    where each Fourier component \( \delta_k \) is a complex number
  - construct power spectrum using mean square amplitude
    \[ P(k) = \left| \langle \delta_k \rangle \right|^2 \]

Will Percival et al., *MNRAS* 401 (2010) 2148

Conclusion

- Large scale structure is very sensitive to cosmological parameters
  - cold vs hot dark matter, \( \Lambda \), etc.
- 2D or 3D maps can be analysed by expanding as spherical harmonics or Fourier series
- Most significant contributors: galaxy surveys (especially SDSS) and CMB