Modern cosmology 3: The Growth of Structure

- **Growth of structure in an expanding universe**
- **The Jeans length**
- **Dark matter**
- **Large scale structure simulations**
  - **effect of cosmological parameters**
- **Large scale structure data**
  - **galaxy surveys**
  - **cosmic microwave background**

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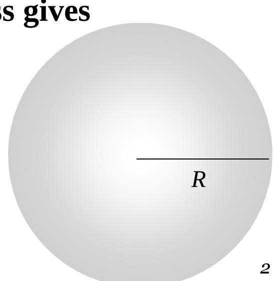
Growth of structure

- **Consider sphere of radius *R* and mass *M***
  - **add small amount of mass so that** $\rho = \bar{\rho}(1+\delta(t))$
  - **gravitational acceleration at edge of sphere is**

$$\ddot{R} = -\frac{GM}{R^2} = -\frac{4\pi}{3} G\bar{\rho}R(1+\delta(t))$$

  - **since** $\rho(t) = \rho_0 a^{-3}$**, conservation of mass gives** $R(t) \propto a(t)[1+\delta(t)]^{-1/3}$
  - **differentiating this twice gives**

$$\frac{\ddot{R}}{R} \approx \frac{\ddot{a}}{a} - \frac{\ddot{\delta}}{3} - \frac{2}{3}\frac{\dot{a}}{a}\dot{\delta}$$



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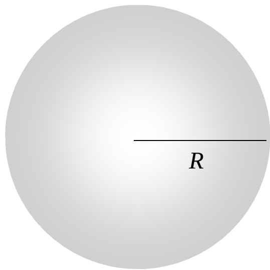
Growth of structure

- **Now have two equations for** $\ddot{R}/R$

$$\frac{\ddot{R}}{R} = -\frac{GM}{R^3} = -\frac{4\pi}{3}G\bar{\rho}\left(1+\delta(t)\right)$$

$$\frac{\ddot{R}}{R} \approx \frac{\ddot{a}}{a} - \frac{\ddot{\delta}}{3} - \frac{2}{3}\frac{\dot{a}}{a}\dot{\delta}$$



- **Combine to get**

$$-\tfrac{4}{3}\pi G\bar{\rho} - \tfrac{4}{3}\pi G\bar{\rho}\delta = \frac{\ddot{a}}{a} - \tfrac{1}{3}\ddot{\delta} - \tfrac{2}{3}\dot{\delta}\frac{\dot{a}}{a}$$

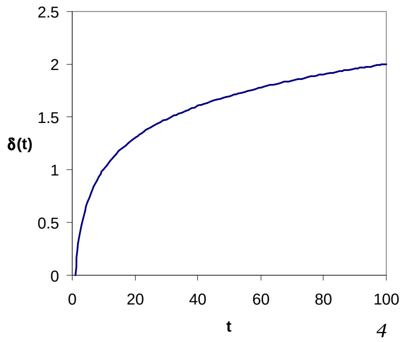
- **Subtract off** $\delta = 0$ **case to get**

$$\ddot{\delta} + 2H\dot{\delta} - \tfrac{3}{2}\Omega_{\mathrm{m}}H^2\delta = 0 \qquad \textbf{since} \qquad \Omega_{\mathrm{m}} = \frac{8\pi G\bar{\rho}}{3H^2}$$

Radiation dominated

$$\boxed{\ddot{\delta} + 2H\dot{\delta} - \tfrac{3}{2}\Omega_{\mathrm{m}}H^2\delta = 0}$$

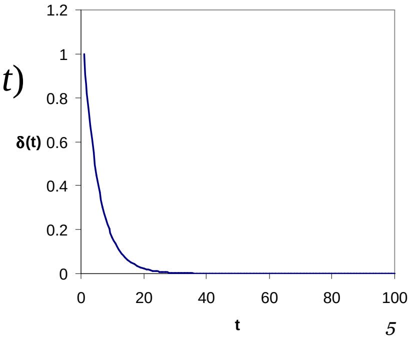
- $H = 1/2t$ **and** $\Omega_r >> \Omega_m$, **so** $\ddot{\delta} + \frac{1}{t}\dot{\delta} \approx 0$
  - **the solution of this is** $\delta(t) = C_1 + C_2 \ln t$
  - **any overdensity grows only logarithmically with time**



Inflation/Λ dominated

$$\ddot{\delta} + 2H\dot{\delta} - \tfrac{3}{2}\Omega_m H^2 \delta = 0$$

- ***H* = *H*$_\Lambda$ and Ω$_\Lambda$>>Ω$_m$, so** $\ddot{\delta} + 2H_\Lambda \dot{\delta} \approx 0$
  - **the solution of this is** $\delta(t) = C_1 + C_2 \exp(-2H_\Lambda t)$
  - **overdensity does not grow at all**



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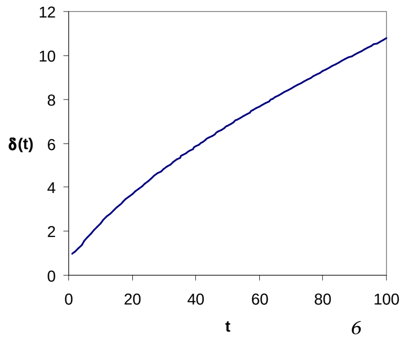
Matter dominated

$$\ddot{\delta} + 2H\dot{\delta} - \tfrac{3}{2}\Omega_m H^2 \delta = 0$$

- ***H* = 2/3*t*, so if Ω$_m$= 1** $\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta \approx 0$
  - **try a power law solution** $\delta(t) = C\,t^n$
  - **get** $3n^2 + n - 2 = 0$
  - $n = -1$ **or** $n = \tfrac{2}{3}$
  - $\delta(t) = C_1\, t^{-1} + C_2\, t^{2/3}$
  - **overdensity does grow**
  - **growth** $\propto t^{2/3} \propto a$



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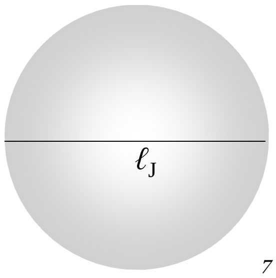
The Jeans length

- **Astrophysical objects are stabilised against collapse by pressure forces**
  - **pressure forces travel at the speed of sound, $c_s = c\,(dP/d\varepsilon)^{1/2} = w^{1/2}\,c$**
  - **collapse occurs if the size of the object is smaller than the *Jeans length***

$$\ell_J = c_s\sqrt{\frac{\pi c^2}{G\bar{\varepsilon}}} = c^2\sqrt{\frac{w\pi}{G\bar{\varepsilon}}}$$

where $\bar{\varepsilon}$ is the mean energy density
    - (exact numerical factor depends on details of derivation)
  - **can also be expressed as *Jeans mass***



The Jeans length

- **We know that for a flat matter-dominated universe $H = (8\pi G\varepsilon/3c^2)^{1/2}$**
  - **so** $\ell_J = c^2\sqrt{\frac{w\pi}{G\bar{\varepsilon}}} = \sqrt{w}\times 2\pi\sqrt{\frac{2}{3}}\frac{c}{H}$
- **At last scattering $H^2 = \Omega_{m0}H_0^2/a^3$, giving $c/H \sim 0.2$ Mpc for $\Omega_{m0} = 0.3$**
  - **so for radiation $\ell_J \sim 0.59$ Mpc**
    - ~ horizon size, so nothing below horizon size collapses
  - **for matter $w = kT/\mu c^2 = 2.3 \times 10^{-10}$, so $\ell_J \sim 15$ pc**
    - equivalent to current size ~ 15 kpc, a small galaxy

Conclusions (radiation-dominated)

- **Below Jeans length (~horizon size), nothing collapses**
  - **supported by pressure**
- **Above Jeans length**
  - **fluctuations in *matter* grow only logarithmically**
  - **fluctuations in *radiation* grow ∝ a^2**
    - derivation similar to p3 gives $\ddot{\delta} + 2H\dot{\delta} - 4\Omega_r H^2 \delta = 0$
    - solve as in p6 to get $\delta \propto t$
    - but $\ell_J \propto a^2$, so as universe expands fluctuations will "enter the horizon" and stop growing

Conclusions (matter-dominated)

- **Before matter-radiation decoupling, photons and baryons form a single gas**
  - **this is stabilised by radiation pressure**
  - **density perturbations cannot grow**
- **After decoupling, baryon gas has galaxy-scale Jeans length**
  - **galaxy-sized objects start to grow ∝ *a***
  - **but it turns out that this is too slow to produce structures we see**
    - (note that our derivation is only valid for small δ)

Dark Matter

- **In fact we know Ω_{baryon} ~ 0.04 from light element abundances, whereas Ω_m ~ 0.25 from cluster X-ray data**
  - **most matter is not made of baryons (non-baryonic dark matter)**
  - **this does not couple to photons**
  - **dark matter structures can grow from time of matter-radiation equality (z ~ 3200)**
  - **this is much more promising**

Structure formation with dark matter

- **Assume dark matter consists of a weakly interacting particle with mass m_{dm}**
  - **such a particle will be relativistic until the temperature drops to $T_{dm} \sim m_{dm}c^2/3k$**
  - **if this is less than $T_{rm} \sim 10^4$ K, the dark matter is hot; if more, it is cold**
    - for hot dark matter the minimum scale for collapse is given by $[ct_{dm} = ct_{rm}(T_{dm}/T_{rm})^2] \times (T_{dm}/2.74)$
    - for cold dark matter it is dwarf galaxy sized

Conclusions (dark matter)

- **Hot dark matter candidate: neutrino with mass ~2 eV/c^2**
  - **T_{dm} = 7750 K**
  - **minimum length scale (for t_{rm} ~ 50000 yr) ~ 70 Mpc**
  - **supercluster sized**
- **Large structures form first**

- **Cold dark matter candidate: WIMP with mass ~100 GeV/c^2**
  - **$T_{dm} = 4 \times 10^{14}$ K**
  - **minimum length scale set by Jeans length for matter**
  - **small galaxy sized**
- **Small structures form first**