ON THE INTERPRETATION OF RADIO SOURCE COUNTS

D. W. Sciama

(Received 1963 March 22)

Summary

It is shown that, contrary to the claims of Ryle and his co-workers, their counts of radio sources can be interpreted in terms of the steady state model of the universe. The interpretation is based on the assumption that at a given flux level there are approximately equal numbers of extragalactic and galactic sources, before corrections are made for the red shift effect and for a postulated local irregularity in the distribution of the galactic sources. The main parameters of the proposed distribution of sources are determined from various radio data, especially the number-flux density relation of Scott and Ryle. There is no discordance with any of these data.

The validity of the model is tested by applying it to the recent survey of Ryle and Neville, which extends the counts down to lower flux levels. In this new flux range cosmological effects are substantial—for instance, the red shift effect of the steady state theory reduces the number of extragalactic sources by a factor of about four. Nevertheless our model is in agreement with these recent observations without needing to invoke any evolutionary effects.

1. *Introduction*

It has been claimed by Ryle and his co-workers (**1, 2, 3**) that their counts of radio sources provide conclusive evidence against the steady state model of the universe. This claim has been challenged by Hoyle and Narlikar (**4, 5**) and by Hanbury Brown (**6**). These authors accept Ryle's conclusion that most of the sources are extragalactic, and criticise his detailed interpretation of the observations. The aim of this paper is to point out that many of the sources may in fact be inside our Galaxy, and that if this is the case the present data are consistent with the steady state theory.

In the next section we construct a mixed model of the distribution of radio sources, that is, one containing comparable numbers of galactic and extragalactic sources in a given flux range. The basic parameters of this model are estimated from various radio data, especially the number-flux density relation derived by Scott and Ryle (**7**) in 1961, and from the red shift relations of the steady state theory. The resulting parameters are consistent with all the present radio data.

In Section 3 our model is applied to the recently published survey of Ryle and Neville (**8**), which extends the counts down to a lower range of flux levels, namely, from 1·2 to 0·25 flux units (1 flux unit $= 10^{-26}\,\mathrm{w(c/s)^{-1}\,m^{-2}}$). According to our model the red shift effect at 0·25 flux units reduces the number of extragalactic sources by a factor of about four, so that one is in a region where the cosmological effects are large. Nevertheless our model agrees with the results of the Ryle–Neville survey without the introduction of any evolutionary effects.

2. *A mixed model of the distribution of radio sources*

2.1. *The log N–log S relation for S > 2 flux units.*—A mixed model of the distribution of radio sources is suggested by the results of Scott and Ryle (**7**), which imply that there is an excess of weak sources or a deficit of intense ones, as compared with a random distribution of stationary sources in Euclidean space. This latter distribution would lead to the relation

$$N = \tfrac{1}{3}\rho P^{3/2} S^{-3/2}, \tag{1}$$

where N is the number of sources per steradian whose flux density exceeds S, and ρ and P are their spatial density and luminosity. If we assume that the sources have a luminosity function of the form* (**3**)

$$\rho(P) = \left(\frac{P_0}{P}\right)^{3/2} \rho(P_0)\, e^{-l^2/2} \tag{2}$$

where

$$l = \log_{10}\left(\frac{P}{P_0}\right),$$

then (1) becomes modified to

$$N = \frac{\sqrt{2\pi}}{3} \rho(P_0)\, P_0^{3/2} S^{-3/2}.$$

Scott and Ryle found that the slope of the observed log N–log S relation is significantly steeper than $-1\cdot5$. This result was interpreted by Ryle and Clarke (**3**) as arising from an excess of weak sources, which they attributed to cosmological effects. Now the first-order effect of the red shift in any of the usual cosmologies is to make the slope *less* steep. Accordingly Ryle and Clarke had to appeal to evolutionary effects to counteract this, and to lead to a net increase of slope (cf. also **9, 10**). This was their reason for rejecting the steady state model.

By contrast Hoyle and Narlikar (**4, 5**) and Hanbury-Brown (**6**) explored the alternative hypothesis, namely that the steepening of the slope arises from a deficit of intense sources, occurring as a result of a local irregularity in their distribution. If the sources were mainly extragalactic, with an average luminosity P_0 of 10^{25} w(c/s)$^{-1}$ ster^{-1} (Section 2.2), the scale of this irregularity would have to be at least 300 Mpc (**11**). Although not much is known about the large-scale distribution of extragalactic objects, it is perhaps somewhat premature to postulate the existence of an irregularity as large as this. On the other hand, if the deficit occurs within a galactic population of sources the irregularity in their distribution occurs within about 17 pc of the Sun (Section 2.3). This distance is small compared with the scale of the Galaxy or even of a spiral arm, and would appear to be quite acceptable. Accordingly we shall assume that a substantial fraction of the sources are inside our galaxy.

The most important parameter of our model is the ratio N_g/N_e of galactic to extragalactic sources in a given flux range. If the local irregularity in N_g is to be kept within reasonable bounds this ratio cannot differ greatly from unity. For if the ratio were very small the local irregularity in N_g would not show up so clearly in the log N–log S relation. On the other hand if the ratio were very large we could not account for the optical identifications, which imply that for $S > 36$ flux units nearly all the sources are *extragalactic* (**12**).

* $\rho(P_0)$ is related to Ryle and Clarke's (**3**) ρ_0 by the relation $\rho(P_0) = \sqrt{2\pi}\,\rho_0$.

To determine N_g/N_e more precisely we proceed as follows. At 36 flux units N_e is reduced by the red shift effect in the steady state model to three quarters of its "Euclidean" value* (assuming a spectral index of 0·7 and that P_0 is 10^{25} w $(c/s)^{-1}$ ster^{-1} (Section 2.2)). Now the observed value of N at 36 flux units is 4 ± 1†. If we take the Euclidean value of N_e to be 4, its red-shifted value is 3, and N_g (as depleted by the local irregularity) lies between 0 and 2, which is consistent with the optical identifications. We now calculate the Euclidean value of N_e at other flux levels using (1), and then correct it for the red shift effect. To determine N_g we assume that at $S = 2$ flux units, the lower limit of the Scott–Ryle relation, we are looking beyond the local irregularity. We also assume that the total number of missing sources is small compared to N_g at this flux level. The computed N_g will then refer to the galactic population as a whole, rather than to the local irregularity. Now at $S = 2$ flux units the Euclidean value of N_e is 302 and the red-shifted value of N_e is 145. Since the observed N is 583 ± 40, N_g is 438 ± 40. It follows that, to within the statistical errors, the ratio of the uncorrected N_g to the uncorrected N_e lies between 1·3 and 1·6. For the sake of simplicity we shall tentatively adopt a ratio of 1·5.

We can now compute the uncorrected N_g at all flux levels using (1), and then from the observed N deduce the number of missing sources at all flux levels. The results of this computation are shown in Table I. We see that the number of missing sources increases from 5 at $S = 36$ to 21 at $S = 11$, after which it remains constant (to within the statistical errors). Thus $S \sim 11$ probably corresponds to the edge of the local irregularity, at which 21 sources are missing. This result is consistent with our assumptions that the total number of missing sources is small compared to N_g at 2 flux units, and that the edge of the irregularity occurs at a flux level greater than 2 flux units. Moreover our results imply that at $S > 20$ flux units more than half the sources should be extragalactic, which is consistent with the optical identifications in this flux range (**12**).

An interesting feature of the results in Table I is that the ratio of the actual numbers of galactic to extragalactic sources increases steadily as the flux density decreases. Hence if any particular intrinsic property of a source is more common amongst extragalatic sources than amongst galactic ones, sources with this property will become relatively less numerous as the flux density decreases. The discovery of such a trend might make it possible to construct a log N–log S relation involving a higher proportion of extragalactic sources than the present one. Such a relation would show more clearly the red shift effect.

2.2. *The physical properties of the extragalactic sources.*—From our computation of the uncorrected N_e we have

$$\frac{\sqrt{2\pi}}{3}\rho(P_0)\,P_0^{3/2} = 2{\cdot}3 \times 10^{13},$$

or

$$\rho(P_0)\,P_0^{3/2} = 2{\cdot}8 \times 10^{13}.$$

Following Ryle (**2**) and Ryle and Clarke (**3**) we can set limits on $\rho(P_0)$ and P_0 by requiring that the integrated emission of the extragalactic sources be not greater than the estimated upper limit of the extragalactic sky brightness. If the sources

* I am grateful to Mr R. W. Clarke for making available to me his calculations of red shift effects in the steady state model.

† I am grateful to Mr P. F. Scott for making available to me the details of his number counts.

have a spectral index of 0·7 (**16**), this upper limit is about 40°K (**18**) at 178 Mc/s (the frequency of the Scott–Ryle survey). In the steady state model this leads to the condition

$$\rho(P_0)\,P_0 \leqslant 60.$$

Hence

$$P_0 \geqslant 2{\cdot}2 \times 10^{23}\,\mathrm{w(c/s)^{-1}\,ster^{-1}}.$$

The actual value of P_0 has been the subject of much discussion. Hanbury Brown (**6, 13**) has suggested a value somewhat less than 10^{25} w(c/s)$^{-1}$ ster^{-1}, whereas Ryle (**14**) has proposed a value of about 4×10^{25} w(c/s)$^{-1}$ ster^{-1}. Our results do not depend critically on the choice of P_0, and we shall adopt a compromise value of 10^{25} w(c/s)$^{-1}$ ster^{-1}. This value was used in Section 2.1 to determine the steady state red shift correction. The corresponding value of $\rho(P_0)$ is 9×10^{-25} per cubic pc, so that the mean spacing between sources of the mean luminosity P_0 is 100 Mpc.

Table I

S	N	Statistical Error	Steady state red shift factor	N_e	Actual N_e	N_g (1·5 N_e)	Actual N_g	Deficit in N_g
2	583	40	0·48	302	145	453	438	15
2·2	530	31	0·49	265	130	397	400	−3
2·5	418	26	0·50	216	108	324	310	14
3	297	22	0·51	166	85	249	212	37
3·5	228	20	0·53	129	68	193	160	33
4	185	17	0·55	108	59	162	126	36
4·5	157	16	0·55	90	49	135	108	27
5	132	16	0·56	75	42	112	90	22
6	95	6	0·58	59	34	88	61	27
7	78	5	0·59	46	27	69	51	18
8	59	5	0·61	38	23	57	36	21
9	46	4	0·62	32	20	48	26	22
10	36	4	0·64	27	17	40	19	21
10·5	31	3	0·64	25	16	37	15	22
11	30	3	0·64	24	15	36	15	21
11·5	27	3	0·64	22	14	33	13	20
12	26	3	0·65	21	14	31	12	19
12·5	24·4	3	0·65	19·6	12·7	29·4	11·7	17·7
13	23·3	3	0·66	18·4	12·1	27·6	11·2	16·4
14	20·2	2	0·66	16·5	10·9	24·7	9·3	15·4
15	18·2	2	0·67	14·9	10·0	22·3	8·2	14·1
15·5	16·6	2	0·67	14·2	9·5	21·3	7·1	14·2
16	15·1	2	0·68	13·5	9·2	20·2	5·4	14·3
17	12·8	2	0·69	12·3	8·5	18·4	4·3	14·1
17·5	11·5	2	0·69	11·8	8·1	17·7	3·4	14·3
18	11	2	0·70	11·3	7·9	16·9	3·1	13·8
18·5	10·3	2	0·70	10·9	7·6	16·3	2·7	13·6
20	9·5	1·5	0·71	9·4	6·7	14·1	2·8	11·3
21	9·0	1·5	0·71	8·8	6·2	13·2	2·8	10·4
24	7·7	1·5	0·72	7·4	5·3	11·1	2·4	8·7
25	6·4	1·2	0·73	6·9	5·0	10·3	1·4	8·9
29	5·1	1·1	0·74	5·5	4·1	8·2	1·0	7·2
32	4·6	1·1	0·74	4·7	3·5	7·0	1·1	5·9
36	4	1	0·75	4	3	6	1	5

2.3. *The physical properties of the galactic sources.*—For the galactic sources we have

$$\rho(P_0)\,P_0^{3/2} = 4\cdot2 \times 10^{13},$$

if their luminosity function is assumed to be of the form (2). Now the minimum sky brightness temperature at 178 Mc/s is 80°K (**15**). In estimating from this an upper limit on the integrated emissivity of the galactic sources, we must allow for the difference in spectrum between these sources and the galactic radiation. Most of the sources have a constant spectral index of about 0·7 (**16**), while the spectral index of the Galaxy varies between about 0·0 at 38 Mc/s and 0·9 at 178 Mc/s (**18**). If we assume that at 38 Mc/s the sources contribute one-half of the total sky brightness*, we have the condition

$$\rho(P_0)\,P_0 r = 5 \times 10^{10},$$

where r is the thickness of the distribution of galactic sources.

Hence

$$\frac{P_0}{r^2} = 0\cdot08 \text{ flux units,} \tag{3}$$

and

$$\rho(P_0) r^3 = 7 \times 10^4. \tag{4}$$

As pointed out by Ryle (**2**) and Ryle and Clarke (**3**), any galactic model of radio sources must be consistent with their observed isotropy. If we assume that most of the sources are in the disk of the Galaxy, isotropy will be observed until we reach flux levels at which we can see beyond the edge of the disk. With our assumed luminosity function, there would be a 16 per cent drop in the number of galactic sources (at the galactic pole) at the flux level for which a source of luminosity 10 P_0 would just be visible at a distance r. This flux level is, from (3), 0·8 flux units. At lower galactic latitudes the number counts would be maintained down to a lower flux level. Now the Scott–Ryle survey extends down to 2 flux units, where the drop in N_g at the galactic pole is only 8 per cent. Accordingly the expected degree of anisotropy would be small. Ryle and Clarke actually used Hewish's (**19**) statistical calculations to show that two particular regions of the sky yield isotropy down to a flux level of 0·15 flux units. This result is also consistent with our model since the two regions concerned were situated at galactic latitudes 45° and −55°. They are thus placed nearly symmetrically with respect to the galactic disk, and the expected anisotropy would be small. On the other hand, a significant anisotropy at 0·15 flux units would be expected for regions at substantially different galactic latitudes. This property of our model still remains to be tested.

In order to derive the main parameters of the local irregularity we note that the galactic population is first observed at a flux level somewhat greater than 36 flux units, say at $S \sim 50$ flux units, and that the number of missing sources is probably constant beyond $S \sim 11$ flux units. If r_0 is the average spacing of sources of luminosity P_0 throughout the galaxy, we have from (4) that

$$r/r_0 \sim 40.$$

* We need not make a substantial allowance for the halo contribution since Baldwin (**17**) has recently shown that this contribution is much smaller than was previously supposed.

The nearest such sources are at a distance d_0 given by

$$P_0/d_0^2 \sim 50 \text{ flux units.}$$

Hence from (3) we have

$$d_0/r_0 \sim 1{\cdot}6. \tag{5}$$

Also the irregularity ends at a distance D given by

$$P_0/D^2 \sim 11 \text{ flux units,}$$

so that

$$D/r_0 \sim 3{\cdot}4. \tag{6}$$

Hence the local irregularity, as characterized by (5) and (6), is of reasonable dimensions when compared to the average spacing of sources throughout the Galaxy. If we assume that the thickness r of the distribution of sources is of the same order as the thickness of the stellar disk, namely 200 pc, then we have

$$r_0 \sim 5\,\text{pc}, \quad d_0 \sim 8\,\text{pc}, \quad D \sim 17\,\text{pc}$$

so that the irregularity is about 34 pc across, which is reasonably small compared to the size of the Galaxy, or even of a spiral arm. Presumably the Sun is not at the centre of this irregularity, so that some degree of anisotropy should be present, which might be detectable above the statistical fluctuations at flux levels exceeding, say, 10 flux units. It would therefore be useful to have precise limits on the degree of isotropy observed over the whole sky.

Now that we have an estimate of the distance scale of the galactic sources we can derive their size scale from their observed angular diameters (**13**). At $S > 12$ flux units 50 per cent of all the sources have an angular diameter $\geqslant 23''$, and 90 per cent have an angular diameter $\geqslant 3''$. According to Table I about 50 per cent of the sources in this flux range are galactic, and we will tentatively assume that these have a typical size of about $5''$. If $r \sim 200$ pc the sources of luminosity P_0 are mostly at a distance of about 15 pc in this flux range, so that their actual diameters are about 3×10^{-4} pc. Moreover, with this choice of r, $P_0 \sim 3 \times 10^{10}\,\text{w(c/s)}^{-1}$ ster^{-1}, and the minimum energy in magnetic field and relativistic particles is about 10^{39} ergs for these sources (cf. **20**). Whether sources with these properties can be expected to exist in the Galaxy will be discussed elsewhere.

3. *The number-flux density relation for $S \leqslant 2$ flux units*

The recent observations of Ryle and Neville (**8**) provide an opportunity of testing our model of number counts at values of flux density between 1·2 and 0·25 flux units. The observations were carried out at a galactic latitude of about 30°, so that there is an appreciable correction at the lowest flux levels for the finite thickness of the distribution of galactic sources. The comparison between theory and observation is shown in Table II*, which includes this correction. It will be seen that the discrepancy between theory and observation is less than the statistical errors of the observations. Thus even at the flux level where the red shift effect reduces the number of extragalactic sources by about a factor four, our mixed model based on the steady state theory agrees with observation.

* I am grateful to Miss Ann C. Neville for making available to me before publication the details of her number counts.

TABLE II

S	N	Statistical error	Steady state red shift factor	N_e	Actual N_e	N_g ($1{\cdot}5N_e$)	Actual N_g	Deficit in N_g^a
0·25	10470	1320	0·27	6913	1867	10369	8917	314
0·5	4485	745	0·33	2416	805	3624	3443	−237
0·75	2475	460	0·38	1328	505	1992	1960	−10
1·2	1365	340	0·40	657	263	985	985	−117

I am grateful to Mr R. W. Clarke, Miss Ann C. Neville, and Mr P. F. Scott for giving me helpful information, and to Professors F. Hoyle and M. Ryle for valuable discussions.

Department of Applied Mathematics and Theoretical Physics,
Free School Lane,
Cambridge:
1963 *March.*

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