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THE SIZE AND MASS OF GALAXIES, AND THE MASS OF THE UNIVERSE

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ABSTRACT

Currently available observations strongly indicate that the mass of spiral galaxies increases almost linearly with radius to nearly 1 Mpc. This means that the total mass per giant spiral is of the order of $10^{12}\ M_\odot$, and that the ratio of this mass to the photographic light within the Holberg radius, f, is $\sim 200\ (M/L)_\odot$. Using this value of f and the luminosity function of surveyed galaxies, we determine a local mean cosmological mass density $\approx 2 \times 10^{-30}$ g cm^{-3} corresponding to $\Omega \equiv \rho/\rho_{\rm crit} \approx 0.2$. The uncertainty in this result is not less than a factor of 3.

Subject headings: cosmology — galactic structure

I. THE ARGUMENT

There are reasons, increasing in number and quality, to believe that the masses of ordinary galaxies may have been underestimated by a factor of 10 or more. Since the mean density of the Universe is computed by multiplying the observed number density of galaxies by the typical mass per galaxy, the mean mass density of the Universe would have been underestimated by the same factor. Finally, the current estimate (Shapiro 1971) for the ratio of gravitational energy to kinetic energy in the Universe is about $\Omega = 0.01$. If we increase the estimated mass of each galaxy by a factor well in excess of 10, we increase this ratio by the same amount and conclude that observations may be consistent with a Universe which is "just closed" ($\Omega = 1$)—a conclusion believed strongly by some (cf. Wheeler 1973) for essentially nonexperimental reasons.

There are two very important facts about astronomical measurements of mass that must be kept in mind. First, almost all *measurements* of mass are based on the requirement that gravitational forces of attraction balance inertial forces in a system of mutually interacting gravitating masses which is not steadily expanding or contracting: masses found by observed light multiplied by *assumed* mass-to-light ratios are not measured. Second, no gravitational information can reach an observer from matter which is distributed homogeneously on spherical shells external to the observer. Thus, mass measurements using two gravitationally interacting objects (pairs of galaxies, or one gas cloud in orbit about a galaxy) provide no knowledge about spherically distributed mass, if any, surrounding the two objects. This result, although arising from a quirk of symmetry and the inverse square law, remains approximately valid for moderately large departures from spherical symmetry. Thus, the rotation curve interior to the Sun's orbit gives little information about the disk mass exterior to the Sun ($r > 10$ kpc) and almost no information about the exterior halo mass. Similarly the studies by Page (1961) of galaxies' masses in binary pairs contain little information concerning distributed mass with a scale greater than the typically $\sim$30 kpc separation.

This ignorance would give us little cause for concern if we had reason to believe from measurements with $r < 20$ kpc that the mass $M(r)$ had "converged" to some limiting value. However, this is not the case; available evidence, summarized below, indicates that the measured masses of galaxies diverge with increasing distance even though the luminosities of the flattened components of spirals and S0 galaxies do appear to be convergent. The estimated rate of divergence of M with r varies between weak (logarithmic) to strong (linear), depending on the method of measurement. The best evidence suggests that within local giant spiral galaxies $M(r) \propto r$ for 20 kpc $\lesssim r \lesssim 500$ kpc.

The implied density distribution thus appears to be similar to that in the outer parts of isothermal gas spheres. If the observed galaxies are in fact embedded in enormous isothermal spheres comprised of optically faint mass points, the latter may be the result of the collapse phase (cf. Eggen, Lynden-Bell, and Sandage 1963) and subsequent violent relaxation (Lynden-Bell 1967) within an original assemblage of stars, globular clusters, etc., that separated from other parts of the Universe at the period of galaxy formation and now exists as a halo with a very high mass-to-light ratio surrounding the more visible, largely second-generation part of the galaxy.

Regardless of the validity of the speculative scenario outlined in the last paragraph, the dynamically observed masses of the best studied galaxies together with their luminosity can be used to determine a mass-to-light ratio. We shall show that, for giant spirals (M/L)

$\simeq 200\ t_{10}^{-1}(M_\odot/L_\odot)$, where M is the mass within ~0.5 Mpc, L is the easily measured light within ~0.02 Mpc of the center, and t_{10} is the age of the Universe in units of 10^{10} years. This value for M/L, combined with the observed galaxy luminosity function within several Mpc, allows us to estimate the local mass density at $2 \times 10^{-30}\ h^1 t_{10}^{-1}$ g cm^{-3} in spiral galaxies giving $\Omega \equiv (\rho/\rho_{\rm crit}) \approx 0.2 \times 10^{0\pm0.5}$ [for $h \equiv$ Hubble constant/(100 km s^{-1} Mpc^{-1}) = 0.5]. A similar conclusion, based on similar arguments, has recently been reached by Einasto, Kaasik, and Saar (1974).

II. THE MASS AND MASS-TO-LIGHT RATIO OF SPIRAL GALAXIES

Since we are explicitly noting that the mass determined for a galaxy will depend on the position of the probe measuring the mass, we will need to consider a variety of methods to obtain the function $M(r)$. The following discussion, based largely on the best observed local giant spiral galaxies, is summarized in table 1 and figure 1.

For 10 kpc $< r <$ 30 kpc, the rotational velocity of neutral hydrogen provides the best measure (optical measurements pertain primarily to gas with $r <$ 10 kpc). In a recent study of three local giant spirals (M31, M81, M101) Roberts and Rots (1973), whose results provide entries 2, 5, and 6 of table 1, pointed out that in the outer parts "the three galaxian rotation curves decline slowly, if at all" indicating that $M \propto r$. A similar result was found by Rogstad and Shostak (1972) for local Sc galaxies.

Double galaxies, whose masses may be estimated with the virial theorem, provide another estimate of mass at $r \approx$ 30 kpc. The standard study by Page (1961) gives values (entry 3) which are surprisingly an order of magnitude smaller than the rotational mass estimates at the same radius. Another sample of apparent pairs in the Virgo cluster studied by van den Bergh (1960), however, gives considerably larger estimates for M. We have (conservatively) removed three pairs from the latter study, which, from their anomalously large values of $(\Delta v)^2$, are likely to be "optical" pairs rather than bound systems, and have recalculated the mean mass (entry 4). Recently Jones (1973) has also examined the binary galaxy problem, focusing particularly on wide separations; from his figure 1 we have selected the nine pairs having separation closest to 100 kpc and calculated the mass in the conventional manner (entry 8). We plot in figure 1 the points for $M(r)$ from binaries assuming $h = 1$; the heads of the three arrows indicate the positions of the corresponding points if $h = 0.5$. It is clear that the discrepancy between Page's binaries and the rotationally derived masses is not due to an uncertainty in h; it may arise from the particular selection rules used to determine the sample of pairs examined.[1]

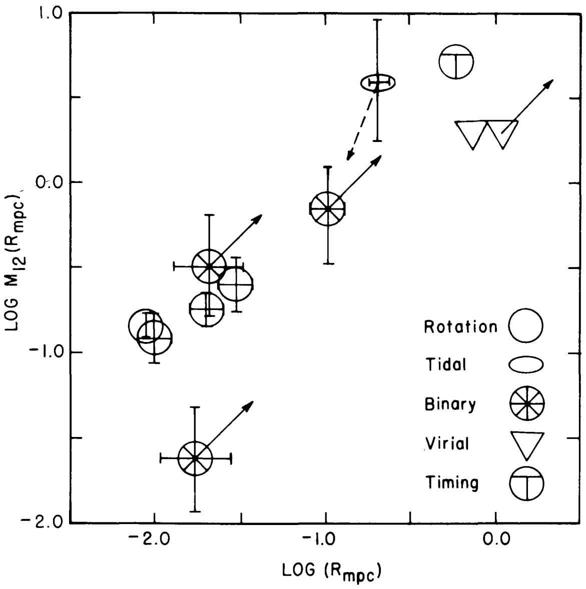

FIG. 1.—Mass (unit $10^{12}\ M_\odot$) of local giant spiral galaxies within a distance (R/1 Mpc) of their centers, as determined by various methods.

Dwarf spheroidal companions of our Galaxy are tidally limited, as noted by Hodge (1966). From the

TABLE 1

$M(r)$ FOR SPIRAL GALAXIES

Object	#	$\log_{10}(R_{\rm Mpc})$	$\log_{10}(M_{12})$	Method
1. Galaxy..................	1	−2.05±?	−0.84±0.07	Rotation
2. Local giants..............	3	−2.00±0.10	−0.92±0.14	Rotation
3. Field doubles.............	20	−1.77±0.20	−1.62±0.30	Binary
4. Virgo doubles.............	23	−1.68±0.20	−0.5 ±0.3	Binary
5. Local giants..............	3	−1.69±0.10	−0.74±0.10	Rotation
6. Local giants..............	3	−1.52±0.10	−0.60±0.16	Rotation
7. Dwarf spheroidals.........	3	−1.14±0.04	−0.60±0.55	Tidal
8. Field doubles.............	9	−0.98±0.11	−0.15±0.32	Binary
9. Dwarf spheroidals.........	3	−0.69±0.03	+0.59±0.35	Tidal
10. M81 group...............	7	−0.15±?	+0.30±?	Virial
11. M31 group...............	6	−0.22±?	+0.72±?	Timing
12. Local groups............	200	+0.04±?	+0.30±?	Virial

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[1] In a subsequent assay at the problem, Page (1962) revised the masses of spirals *downward* slightly thus increasing the discrepancy; we did not use this later work because galaxy separations were not quoted.

straightforward tidal theory one can derive the mass of our Galaxy interior to the perigalacticon of the tidally truncated companion galaxy. The six galaxies, considered in two groups, thus measure the mass of our galaxy at (roughly) two radii (see entries 7 and 9). Only the distant group is plotted in figure 1 due to the large error in the estimate (determined from quoted errors in Hodge 1966) from the other group. The ellipse at the tail of the arrow is for a circular orbit; the head points to the determination if, in fact, the orbits are highly eccentric with perigalacticon at one-half the present radius for each companion galaxy.

At large separation three further values of $M(r)$ are given. The mass for the M81 group (entry 10) is based primarily on the two outlying members. If these two are omitted, the radius and mass decrease by a factor 10. The Local (M31) group mass determination (entry 11) is based on the requirement that an initially separating pair of galaxies reverse and, after 10^{10} years, achieve the relative velocity and position reached by our Galaxy and Andromeda; for the method see Peebles (1971) and references therein. The numerical result given in table 1 was based on a distance to Andromeda of 700 kpc, an approach velocity of 300 km s^{-1} (heliocentric), the standard solar motion, and an assumed local galactic rotation rate of 230 km s^{-1}, the quoted mass scales roughly as t_{10}^{-1}, due to dependence on the allowed orbital time. Gunn (1973), using the identical method but somewhat different kinematical parameters, derived a smaller mass ($\log M_{12} = 0.46$) for the Local Group. Kahn and Woltjer (1959) and Oort (1970) with similar methods have derived masses in the range (1–10) $\times 10^{12}$ $M_\odot$. Finally, Geller and Peebles's (1973) study of wide groups (entry 12) was designed to statistically compensate for projection effects.

Although the values of $M(r)$ given in table 1 are among the best that could be obtained from the current literature, the uncertainties, which have been roughly indicated by error bars, are obvious. However, the general trend seen in figure 1, of significantly increasing mass with increasing radius, is almost certainly real. It has been found in other studies of galaxy masses (cf. Rood, Rothman, and Turnrose 1970; Field and Saslaw 1971; Rood 1974), where it was sometimes expressed as an increase in the "virial discrepancy" with increasing size of group.

Let us designate by M/L the ratio of the total galactic mass to the photographic light within the easily measured radius of $\sim$20 kpc (essentially the Holmberg radius) in solar units. Then, for the Local Group and the M81 group we have $f_{\rm sp} \equiv (M/L)_{\rm spiral} \approx (5 \times 10^{12})/(2.5 \times 10^{10}) = (2 \times 10^{12})/(1 \times 10^{10}) = 200$. Since these mass determinations do not depend significantly on the Hubble constant but the result for the Local Group does depend on t_{10}, we can write this

$$f_{\rm sp} \simeq 200 h^0 t_{10}^{-1} \qquad (1)$$

compared with the mass-to-light ratio of elliptical galaxies

$$f_{\rm el} \simeq 300 h^1$$

derived by Rood *et al.* (1972) from their study of the Coma cluster.

The very large mass-to-light ratio and the very great extent of spiral galaxies can perhaps most plausibly be understood as due to a giant halo of faint stars. Such a structure, which appears superficially so improbable, has been proposed recently (Ostriker and Peebles 1973) for quite different reasons concerning dynamical stability. The least troublesome way of ensuring the stability of a cold disk of stars against nonaxisymmetric disturbances is to suppose that the spherically distributed (halo) mass *interior* to $r \sim 10$ kpc is substantial. This requires the interior halo to have a large mass-to-light ratio ($f \sim$ 50–100). If, further, the spherical halo *exterior* to 10 kpc is as extensive as the halo of an elliptical (cf. here Kormendy and Bahcall 1974 with Arp and Bertola 1971) and the mass-to-light ratio stays large, then the mass associated with the outer parts of spiral galaxies must be large on these grounds.

III. COSMOLOGICAL IMPLICATIONS

The total space density associated with galaxies is

$$\begin{aligned}\rho_{\rm gal} &= f_{\rm sp} j_{\rm sp} + f_{\rm el} j_{\rm el} \\ &= 4.0 \times 10^{-30} h^1 t_{10}^{-1} + 0.4 \times 10^{-30} h^2 \text{ g cm}^{-3}, \qquad (2)\end{aligned}$$

where for $j_{\rm sp}$, $j_{\rm el}$, the mean volume emissivities of the light from spiral and elliptical galaxies, we have taken 3×10^8 $hL_\odot$ Mpc^{-3} and 2×10^7 $hL_\odot$ Mpc^{-3}, respectively, from Shapiro's (1971) analysis of the de Vaucouleurs catalog. Since the critical density is 1.9×10^{-29} h^2 g cm^{-3}, we have

$$\Omega_{\rm gal} = (\rho_{\rm gal}/\rho_{\rm crit}) = \frac{0.21}{h t_{10}} + 0.02 \geq 0.2 \qquad (3)$$

since $h t_{10} < 1$.

The uncertainty of this result, due to inaccuracy in f_s alone, is clear from the preceding discussion. The total uncertainty, which is not less than a factor of 3, contains an important contribution due to our poor knowledge of j_s as well. Gunn (1973) has pointed out that, since the de Vaucouleurs catalog is based on the neighborhood of our Galaxy, it is influenced by the local supercluster and may overestimate j_s by as much as $10^{0.5}$. However, estimates of $(j_{\rm sp} + j_{\rm el})$, based on counts of galaxies in deeper surveys, are consistent within a factor $\sim$1.5 of the values adopted here. In particular, Oort's (1958) analysis of the mean emissivity within a sphere of radius $\sim$300 h^{-1} Mpc also gives $j \approx 3 \times 10^8$ h, the value we have adopted from the survey of our local region. Furthermore, the brightness of the night sky (see references in Peebles 1971 and Mattila 1973), which integrates over all galaxies including those of low surface brightness or small angular size that might be missed in the standard survey, would permit $(j_{\rm sp} + j_{\rm el})$ larger than the adopted value by a factor 3–10; however, if the uncatalogued galaxies have the same spatial distribution as the counted ones, then Shectman's (1973) analysis of the fluctuations in the background indicates an emissivity of $j \lesssim 5 \times 10^8$ h.

There are various other cosmological arguments which can be given for either a low- or high-density Universe. Most are well known and will not be repeated here. Two new points seem especially significant. The *Copernicus* determination of interstellar deuterium (Rogerson and York 1973) coupled with conventional (or unconventional) big-bang nuclear calculations (cf. Wagoner 1973) strongly indicates an empty ($\Omega = 0.01$) Universe *unless* subsequent production of deuterium has been significant (Colgate 1973; Hoyle and Fowler 1973). Finally, the great extent of rich clusters of galaxies (Yahil 1974; Rood *et al.* 1972) and the correlations observed among galaxy positions, when interpreted from the viewpoint of gravitational instability (Peebles 1974), appear to indicate that $\Omega \approx 1$.

The arguments presented above indicate that *within the current observational uncertainties* the masses associated with ordinary spiral galaxies may make a cosmologically interesting contribution. Further observational work on (*a*) the nightsky background light due to unresolved galaxies, (*b*) optical or radio searches for giant halos surrounding ordinary galaxies, and (*c*) the puzzling problem of binary mass determinations would be especially rewarding at this time.

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