A NEW MODEL FOR THE EXPANDING UNIVERSE

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Summary

By introducing continuous creation of matter into the field equations of general relativity a stationary universe showing expansion properties is obtained without recourse to a cosmical constant.

1. *Introduction.*—Creation of matter was mentioned about twenty years ago by Jeans (1) who remarked:

"The type of conjecture which presents itself, somewhat insistently, is that the centres of the nebulae (galaxies) are of the nature of singular points, at which matter is poured into our universe from some other and entirely extraneous spatial dimension, so that, to a denizen of our universe, they appear as points at which matter is being continually created". Subsequent astrophysical developments have, however, shown little support for this particular form of creation.

More recently Dirac (2) has pointed out that continuous creation of matter can be related to the wider questions of cosmology. The following work is concerned with this aspect of the matter and arose from a discussion with Mr T. Gold who remarked that through continuous creation of matter it might be possible to obtain an expanding universe in which the proper density of matter remained constant. This possibility seemed attractive, especially when taken in conjunction with aesthetic objections to the creation of the universe in the remote past. For it is against the spirit of scientific enquiry to regard observable effects as arising from "causes unknown to science", and this in principle is what creation-in-the-past implies.

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2. *Newtonian Universes.*—We begin by mentioning the difficulties occurring in current theories of the expanding universe. A comprehensive review of cosmology, based on Einstein's general theory of relativity, has been given by Robertson (3). Milne and McCrea (4) have obtained the remarkable result that Newtonian analogues exist for all the more important models considered by Robertson. Although later we shall go over to the formalism of the relativity theory, it is convenient in these preliminary remarks to use the Newtonian equivalent models.

The work of Milne and McCrea starts from the cosmological principle applied in the narrow sense. According to the narrow cosmological principle, the distribution of material and momentum relative to an observer attached to a particular particle is identical with the distribution relative to an observer attached to any other particle, provided the comparison *refers to the same value of the time.* The latter proviso weakens the equivalence of observers. When the cosmological principle is used in its wide sense this proviso is removed and equivalence would have to be applied even if the two observers carried out their measurements at different times. It is important to notice that the cosmological principle ignores proper motions arising from local condensations of matter. That is, we refer to an approximation in which the matter within the extragalactic nebulae is regarded as being smeared into a continuous background. This approximation will be used throughout the present paper, except in the general discussion in Section 5.

Take an observer attached to a given particle and let $\boldsymbol{r}$ be a position vector measured by this observer. The relative motion of a particle situated at $\boldsymbol{r}$ at time t will be denoted by $\boldsymbol{v}(\boldsymbol{r}, t)$. Then it is easy to show (4) that the narrow cosmological principle requires the material density ρ to be a function of t only, and that $\boldsymbol{v}(\boldsymbol{r}, t)$ must be of the form $F(t)\boldsymbol{r}$, where $F(t)$ also depends only on the time. Defining $R(t)$ by

$$R(t) = \exp\left\{\int_{t_0}^{t} F(t)\,dt\right\}, \tag{1}$$

where t_0 is arbitrary, we obtain

$$\boldsymbol{v}(\boldsymbol{r}, t) = R'\boldsymbol{r}/R, \tag{2}$$

where $R' = dR/dt$. In the strictly Newtonian problem a particle at $\boldsymbol{r}$ experiences an acceleration $\boldsymbol{G}$, relative to our observer, given by

$$\boldsymbol{G} = -4\pi\gamma\rho\boldsymbol{r}/3, \tag{3}$$

where γ is the Newtonian constant of gravitation. The equations of motion are

$$\partial\rho/\partial t + \operatorname{div}(\rho\boldsymbol{v}) = 0, \tag{4}$$

$$\partial\boldsymbol{v}/\partial t + \tfrac{1}{2}\operatorname{grad}\boldsymbol{v}^2 = \boldsymbol{G}. \tag{5}$$

In deriving (5) from the standard Eulerian equation we neglect the hydrostatic pressure, which is small in the cosmological problem, and we take account of the fact that curl $\boldsymbol{v} = 0$.

It can be verified that

$$\rho R^3 = B, \tag{6}$$

$$\boldsymbol{v}^2 = (8\pi\gamma\rho/3 - k/R^2)\boldsymbol{r}^2, \tag{7}$$

where B, k are constants of integration, give the general solution of (4), (5). When $k=0$ the material possesses the minimum kinetic energy necessary for unlimited expansion. Unlimited expansion also occurs in the case $k<0$, since the material then has excess kinetic energy. But when $k>0$ expansion is of limited duration. These cases are conveniently described as parabolic, hyperbolic and elliptic. As a description of the expanding universe, the elliptic case is evidently inferior to the parabolic and hyperbolic cases. Moreover, the parabolic case is superior to the hyperbolic case, *since localized condensations, necessary for the formation of extragalactic nebulae, can readily occur in a parabolic but not in a hyperbolic universe.* For in the parabolic case condensations will form even if the density only exceeds the mean value by a small margin, whereas an appreciable increase of density would be required in the hyperbolic case. Accordingly it is sufficient to confine the discussion of (7) to the case $k=0$. We then have the Newtonian analogue, derived by Milne (4), of the Einstein-de Sitter relativistic model in flat expanding space (5).

With the simplification $k=0$, (7) may be integrated by using (2) and (6). This gives, for the solution representing outward motion,

$$\boldsymbol{v} = 2\boldsymbol{r}/3t, \tag{8}$$

$$\rho = 1/6\pi\gamma t^2, \tag{9}$$

the zero of time being arranged so that $R=0$ at $t=0$. Now the velocity-distance relation, found by Hubble and Humason from the observation of extragalactic nebulae, can be written as

$$\boldsymbol{v} \doteqdot (5/3) \cdot 10^{-17} \boldsymbol{r} \tag{10}$$

when time is measured in seconds. We see that for (8) to agree with (10), t must be about 4×10^{16} sec., which is about $1 \cdot 3 \times 10^9$ years. The corresponding value of ρ is about 5×10^{-28} g. per cm.^{3}.

It is important to notice that, in common with all current cosmological models, no attempt is made to show how the required distribution of material and momentum comes to be set up. The implication of (8), (9) is that if these equations are satisfied for one value of t, then they will be satisfied at all subsequent times. Thus, according to these equations, ρ, $\boldsymbol{v}^2$ were infinite at $t=0$; that is, about $1 \cdot 3 \times 10^9$ years ago. Although effects not considered in our equations would intervene to prevent strict divergence, the conclusion seems inescapable that near $t=0$ the density would be very large compared with the present value. Quite apart from its unsatisfactory nature, this conclusion is in discordance with astrophysical data which strongly suggest that physical conditions have not changed appreciably over a period of about 5×10^9 years. In this connection it may be noted that geophysical studies give about 2×10^9 years for the age of the Earth.

These difficulties suggest that alternative possibilities, based on the introduction of a cosmical constant into (3), be considered. When (3) is altered to

$$\boldsymbol{G} = \tfrac{1}{3}(-4\pi\gamma\rho + \lambda c^2)\boldsymbol{r}, \tag{3'}$$

where λ is a constant, the solution of (4), (5) is given by (2), (6) and by

$$\boldsymbol{v}^2 = (8\pi\gamma\rho/3 - k/R^2 + \lambda c^2/3)\boldsymbol{r}^2. \tag{7'}$$

This solution admits of two cases that satisfy the wide cosmological principle. First, we have the analogue of Einstein's static universe when

$$4\pi\gamma\rho = \lambda c^2 = k/R^2. \tag{11}$$

These conditions give $\boldsymbol{v}=0$, $\partial\boldsymbol{v}/\partial t=0$, $\boldsymbol{G}=0$, and R is independent of time. Since $\boldsymbol{G}=0$, localized condensations may occur in this model. Second, the analogue of the de Sitter empty universe is obtained by putting $k=0$, $\rho=0$. Then physically observable quantities, such as $\boldsymbol{v}$, are independent of time, but R, which is not directly observable, varies like $\exp\{(\lambda/3)^{\frac{1}{2}} ct\}$. Unfortunately neither of these cases satisfies observational requirements; the Einstein model has no expansion and the de Sitter model contains no material.

The Friedmann non-stationary models can be obtained by taking other values of λ, k. A specially interesting case, considered by Lemaître and Eddington, arises from a perturbed Einstein static universe. For, as Eddington (**6**) has pointed out, the Einstein model is unstable against a small change of R. A decrease in R leads to contraction, whereas a slight increase of R leads to unlimited expansion. In the Lemaître-Eddington model the latter case is assumed. The rate of expansion is at first very slow, but as the $8\pi\gamma\rho/3$, k/R^2 terms in (7′) become small compared with the $\lambda c^2/3$ term, the Lemaître-Eddington model tends asymptotically to the de Sitter empty universe.

Objections to the Lemaître-Eddington universe are subtler than the difficulties occurring in other models. The force $\boldsymbol{G}$ is repulsive except during the pre-expansion stage. Thus, localized condensations can only be formed during this stage (**7, 8**). This conclusion is in contradiction with astrophysical data which indicate that, quite apart from their general expansion, the extragalactic nebulae are in a vigorous state of dynamical evolution. Furthermore, there are aesthetic objections. For the Lemaître-Eddington universe depends on the introduction of a cosmical constant which, as Einstein has remarked, is an unsatisfactory device. Moreover, the wide cosmological principle is not satisfied.

In this connection it may be noted that Lemaître now prefers a model with point-source creation. The constants λ, k and the initial velocity of expansion are adjusted so that there is a stage in the expansion approximating to the Einstein static universe. Later stages in the expansion are then the same as in the Lemaître-Eddington model.

The aim of the present paper is to overcome the difficulties outlined above. Using continuous creation of matter, we shall attempt to obtain, within the framework of the general theory of relativity, but without introducing a cosmical constant, a universe satisfying the wide cosmological principle that shows the required expansion properties and in which localized condensations are continually being formed.

3. *The Mathematical Formalism.*—Following a procedure similar to that based on Weyl's postulate (Robertson **3**), we first obtain a simple form for the Riemannian quadratic metric

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu.$$

Take the pencil of geodesics passing through a definite point O in the space-time continuum. Three independent coordinates are required to specify a particular geodesic belonging to this pencil. The first restriction placed on the metric is that there shall be a unique geodesic (remembering that we are considering the smoothed out problem) joining O to a general point P of space-time. Then P can be described by four coordinates in the following way. The three coordinates defining the unique geodesic give the "space" coordinates x_1, x_2, x_3 of P, while the absolute length (interval) measured along this geodesic from a fixed reference point to P gives the product ct, where c is a constant and t is the "time" coordinate. Next we use the geometrical property that the reference points on the various geodesics can be chosen so that the time sections, defined by equations of the form $t = \text{constant}$, are orthogonal to the geodesics themselves. Accordingly, the metric can be written as

$$ds^2 = c^2\, dt^2 + g_{ij}(t, x_1, x_2, x_3)\, dx_i dx_j; \qquad i, j = 1, 2, 3. \tag{12}$$

The discussion of the previous paragraph is of a geometrical character. Connection with the physical world is introduced by the requirement that the narrow cosmological principle shall be satisfied in every time section. This condition can be converted into geometrical terms by using results due to Lie, Killing, and Fubini (see Robertson **3**), which show that (12) can then be reduced to the more specialized form

$$ds^2 = c^2\, dt^2 - R^2(t) h_{ij}(x_1, x_2, x_3)\, dx_i dx_j; \qquad i, j = 1, 2, 3, \tag{13}$$

where the subspace

$$du^2 = h_{ij}\, dx_i dx_j \tag{14}$$

has constant Riemannian curvature. We now introduce the final simplification of taking this curvature to be zero. Then it can be shown (**3**) that the coordinates describing the geodesics through O can be chosen to give

$$ds^2 = c^2\,dt^2 - R^2(t)(dx_1^2 + dx_2^2 + dx_3^2). \tag{15}$$

In this connection it may be noted that if a stationary universe showing expansion is to be obtained, then formal similarities with the de Sitter empty model must be expected. In Section 2 it was seen that $k=0$ in the Newtonian form of this model. Since the condition $k=0$ is the Newtonian equivalent of zero Riemannian curvature, the reason underlying the final reduction of (13) will be readily understood.

For convenience in later work we note that the only non-vanishing Christoffel symbols are

$$\{ij, 0\} = RR'\delta_{ij}/c^2, \qquad \{i0, j\} = R'\delta_{ij}/R, \qquad i, j = 1, 2, 3, \tag{16}$$

where δ_{ij} is the Kronecker symbol. The only non-vanishing components of the Ricci tensor $G_{\mu\nu}$ are

$$\begin{aligned} G_{ij} &= -(RR'' + 2R'^2)\delta_{ij}/c^2, \qquad i, j = 1, 2, 3, \\ G_{00} &= 3R''/R, \end{aligned} \tag{17}$$

and the spur of this tensor is given by

$$G = 6(RR'' + R'^2)/R^2c^2. \tag{18}$$

We now diverge from the usual procedure by introducing at each point P of space-time a vector C_μ of fixed length directed along the geodesic from O to P. The sense of this vector is always taken as being away from O. Thus, in terms of our coordinate system we have, at each point, a vector with components proportional to (1, 0, 0, 0). The constant of proportionality which will be written as $3c/a$, determines the length of the vector. Accordingly, the vector field is given by

$$C_\mu = 3c/a(1, 0, 0, 0). \tag{19}$$

By differentiation a symmetrical tensor field $C_{\mu\nu}$ is obtained. That is

$$C_{\mu\nu} = \partial C_\mu/\partial x^\nu - \{\mu\nu, \alpha\}C_\alpha. \tag{20}$$

Since $\partial C_\mu/\partial x^\nu$ is everywhere zero, we obtain, by using (16), the following non-vanishing components of $C_{\mu\nu}$,

$$C_{ij} = -3RR'\delta_{ij}/ac; \qquad i, j = 1, 2, 3. \tag{21}$$

The essential step in the present work is the introduction of the tensor $C_{\mu\nu}$ into the Einstein field equations. Thus we write

$$G_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}G + C_{\mu\nu} = -\kappa T_{\mu\nu}, \tag{22}$$

where $\kappa = 8\pi\gamma/c^4$ and $T_{\mu\nu}$ is the material energy tensor. Neglecting both kinetic energy terms and the hydrostatic pressure, the only non-vanishing component of $T_{\mu\nu}$ is

$$T_{00} = \rho c^4, \tag{23}$$

where ρ is the proper density of matter. The $C_{\mu\nu}$ term in (22) plays a rôle similar to that of the cosmical constant in the de Sitter model, with the important difference, however, that there is no contribution from the C_{00} component. As we shall see, this difference enables a universe, formally similar to the de Sitter model, to be obtained, but in which ρ is non-zero.

The field equations (22) now give

$$2RR''+R'^2-3cRR'/a=0, \tag{24}$$

$$3R'^2=\kappa\rho c^4R^2. \tag{25}$$

Choosing the zero of time so that $R=1$ at $t=0$ it is easy to see that

$$R=e^{ct/a}, \qquad \kappa\rho c^2=3/a^2 \tag{26}$$

is a solution of (24), (25). Thus we obtain a metric identical in form with the de Sitter model, but in the present universe the proper density of matter is a constant non-zero quantity.

The solution (26) is stable against fluctuations in the value of R'/R. Thus, suppose $R'/R=\alpha c/a$ at $t=0$, where $\alpha\neq 1$. Then (24) can be integrated to give

$$\left.\begin{aligned} 3ct/4a&=\tanh^{-1}(2aR'/cR-1)-\tanh^{-1}(2\alpha-1) \text{ if } \alpha<1,\\ 3ct/4a&=\coth^{-1}(2aR'/cR-1)-\coth^{-1}(2\alpha-1) \text{ if } \alpha>1. \end{aligned}\right\} \tag{27}$$

It can be seen from (27) that R'/R approaches c/a in a time of order a/c even if α is appreciably different from unity. Thus we see that a/c is a natural unit of time.

Taking the divergence of (22) we obtain

$$(C^{\mu\nu})_\nu=-\kappa(T^{\mu\nu})_\nu, \tag{28}$$

since the divergence of $G_{\mu\nu}-\frac{1}{2}g_{\mu\nu}G$ vanishes on account of the four identities. Using (16), together with the relation

$$(C^{\mu\nu})_\nu=\partial C^{\mu\nu}/\partial x^\nu+\{\alpha\nu,\,\mu\}C^{\alpha\nu}+\{\alpha\nu,\,\nu\}C^{\mu\alpha}, \tag{29}$$

it can be shown that

$$\left.\begin{aligned} (C^{i\nu})_\nu&=0, \qquad i=1,\,2,\,3,\\ (C^{0\nu})_\nu&=-9R'^2/ac^3R^2. \end{aligned}\right\} \tag{30}$$

Thus

$$\left.\begin{aligned} (T^{i\nu})_\nu&=0, \qquad i=1,\,2,\,3,\\ \kappa(T^{0\nu})_\nu&=9R'^2/ac^3R^2. \end{aligned}\right\} \tag{31}$$

By working out $(T^{0\nu})_\nu$ the second of equations (31) gives

$$\kappa\left\{\frac{\partial\rho}{\partial t}+3\rho\frac{R'}{R}\right\}=\frac{9R'^2}{ac^3R^2}, \tag{32}$$

which reduces to $\partial\rho/\partial t=0$ when (26) is satisfied. Equation (32) could have been obtained directly from (24) ,(25).

4. *The Physical Interpretation of the Formalism.*—We now consider the relation of physical measurement to the coordinate system used above. Since $(C^{0\nu})_\nu\neq 0$, matter is being created. But because $(C^{i\nu})_\nu=0$ for $i=1,\,2,\,3$, the matter possesses zero momentum in our system of coordinates. Thus a particle created at a point P moves away from O along a geodesic joining O and P. In principle, changes in the coordinate t are given by the proper time measured by an observer attached to such a particle. In practice, however, an observer on the Earth has not exactly a geodesic through O as world-line. Deviations arise when local condensations are formed, since electromagnetic forces are introduced in the condensation process (through atomic collisions). Remembering, however, that local condensations only have a small effect on the large-scale problem, the proper time measured by an observer on the Earth gives a good approximation for changes in the coordinate t.

Similarly we neglect the effect of local condensation on the measurement of the space coordinates. The coordinates x_1, x_2, x_3 can be chosen so that

$$x_1 = r \sin\theta \cos\phi, \qquad x_2 = r \sin\theta \sin\phi, \qquad x_3 = r \cos\theta, \tag{33}$$

where r, θ, ϕ are spherical polar coordinates measured relative to a terrestrial observer. The angular coordinates θ, ϕ are easily determined by the usual sextant type of observation, but the determination of r raises an important question. At a particular time t the absolute distance (interval) from the Earth of an event occurring at a point with space coordinates r, θ, ϕ is $R(t)r$ and not the "parametric" distance r. Now, according to the postulates of the relativity theory, a measuring rod has the same absolute length for all values of t. It follows therefore that distances obtained by surveying methods give $R(t)r$ and not r.

Large distances are in practice obtained from the apparent intensity of a standard light source. In the Galaxy and the nearer extragalactic nebulae Cepheid variables are used as the standard, while at the greatest distances the whole emission from a galaxy is employed. This method has been discussed by Tolman (**9**) and by Whittaker (**10**), who find that, when the time sections have zero Riemannian curvature, such measurements also give $R(t)r$.

It is seen therefore that, neglecting the effect of local condensations, normal methods of measurement determine proper volumes and not "coordinate" volumes. Thus on the basis of (26) we expect that measurements of the mean intergalactic density would give a value independent of t.

It will be noticed that practical measurement of r, θ, ϕ is entirely based on light tracks. In particular, it is a necessary consequence of the sextant method of determining θ, ϕ that these coordinates are constant along a light track. Thus the path of a light pulse emitted by the observer satisfies the equation

$$\frac{dr}{dt} = \frac{c}{R(t)}. \tag{34}$$

Accordingly, a light pulse emitted by the observer at time t_1 reaches at time $t(>t_1)$ a point whose r coordinate is given by

$$r = c\int_{t_1}^{t} \frac{dt'}{R(t')}. \tag{35}$$

Using (26) this becomes

$$r = a(e^{-ct_1/a} - e^{-ct/a}), \tag{36}$$

which tends to $ae^{-ct_1/a}$ as $t \to \infty$. This means that a light signal emitted by the observer at time t_1 can never reach points with r coordinates greater than $ae^{-ct_1/a}$. Similarly a light signal emitted at time t_1 from a point with r coordinate greater than $ae^{-ct_1/a}$ cannot reach the observer. This result may be stated in a more significant form in terms of absolute distances. In any time section a light signal emitted at an absolute distance greater than a from a point on a geodesic through O can never reach an observer with this geodesic as world-line. Thus a may be described as the radius of the "observable" universe.

According to (26) the total mass of material within the observable universe is independent of t and is given by

$$4\pi a/\kappa c^2. \tag{37}$$

This result deserves comment. Ignoring local condensations, each particle has a geodesic through O as world-line. Now the r coordinate is constant along such a geodesic. Thus a particle with space coordinates r, θ, ϕ passes out of the observable universe at a time t given by

$$r = ae^{-ct/a}. \tag{38}$$

Particles passing out of the observable universe are compensated by the creation of new particles in accordance with (31). It is only through the creation of matter that an expanding universe can be consistent with conservation of mass within the observable universe.

Next we turn to a comparison of (26) with the observed velocity-distance law. Consider light emitted from an oscillatory source with a geodesic through O as world-line, and let this geodesic be characterized by the space coordinates r, θ, ϕ. Write t_1, $t_1 + dt_1$ for the times of two successive maxima of the source, where $t_1 + dt_1$ is taken such that

$$r < ae^{-c(t_1 + dt_1)/a}. \tag{39}$$

Then light rays emitted at times t_1, $t_1 + dt_1$ reach the observer at times t, $t + dt$ given by

$$r = c\int_{t_1}^{t} \frac{dt'}{R(t')} = c\int_{t_1+dt_1}^{t+dt} \frac{dt'}{R(t')}. \tag{40}$$

For small dt_1 (26) gives

$$dt/dt_1 = e^{c(t-t_1)/a}. \tag{41}$$

But dt/dt_1 is the ratio of the frequency, ν say, of the source itself to the apparent frequency, $\nu + d\nu$ say, of the light reaching the observer. Thus we have

$$-\frac{d\nu}{\nu} = \frac{r}{a} e^{ct/a}. \tag{42}$$

This apparent shift of frequency is towards the red, and corresponds to an apparent Doppler velocity

$$(cr/a)e^{ct/a}. \tag{43}$$

Accordingly a light source moving along a geodesic through O has an apparent recessional velocity at time t given by multiplying the absolute distance, at time t, between the source and the observer by the constant c/a. Remembering that physical measurements determine absolute distance, it follows by comparison with (10) that, when absolute distance is measured in centimetres and t is measured in seconds, the constant c/a is given by

$$c/a \doteqdot (5/3) \times 10^{-17} \text{ sec.}^{-1}. \tag{44}$$

This equation, together with $c = 3 \times 10^{10}$ cm. per sec., leads to the following results:—

(i) the radius a of the observable universe is about $1 \cdot 8 \times 10^{27}$ cm.,
(ii) the unit of time a/c is about 6×10^{16} sec., or about 2×10^9 years,
(iii) the proper density ρ is about 5×10^{-28} g. per cm.^{3},
(iv) the mass within the observable universe is about $1 \cdot 2 \times 10^{55}$ g.

At a given time the present model is analogous to a Newtonian universe in which the instantaneous values of $\boldsymbol{v}^2$, ρ are given by

$$\boldsymbol{v}^2 = c^2r^2/a^2, \qquad \rho = 3c^2/8\pi\gamma a^2. \tag{45}$$

Eliminating a^2 we can write

$$\boldsymbol{v}^2 = 8\pi\gamma\rho r^2/3, \tag{46}$$

which is similar to the equation for $\boldsymbol{v}^2$ occurring in the case discussed in Section 2, in which $k=0$, $\lambda=0$. The important feature of this case was that condensations were formed wherever the local density increased slightly above the mean value. It is therefore to be expected that localized condensations will continually arise in the universe discussed above.

The following general picture of the condensation process suggests itself. Extragalactic nebulae are continually passing out of the observable universe, but the total number of nebulae within the observable universe remains approximately constant on account of the formation of new condensations. Thus a nebula, or a cluster of nebulae, condensing at an absolute distance d from the observer passes out of the observable universe after a time of about

$$2 \times 10^9 \log_e (a/d) \text{ years.} \tag{47}$$

The smallest value that can be taken for d corresponds approximately to the observed mean intergalactic distance of about $1\cdot 5 \times 10^{24}$ cm. Accordingly, the oldest condensations within the observable universe (other than the condensation in which the observer happens to be situated, which may be of any age) has an age of about $1\cdot 5 \times 10^{10}$ years. It is attractive to associate these condensations with the great nebular clusters. The youngest condensations, which have ages of about 2×10^9 years, are conveniently associated with the single field nebulae. Astrophysical evidence indicates that the "local group" of nebulae containing our Galaxy has an age of about 5×10^9 years. It is therefore satisfactory that this group shows a moderate degree of aggregation.

Hubble's estimation of about 5×10^{-31} g. per cm.^{3} for the mean density of luminous material, taken together with our estimate of 5×10^{-28} g. per cm.^{3} for the average density of *all* material, suggests that only about one part in a thousand of the intergalactic medium is at present in a condensed state.

5. *General Remarks.*—The work of Sections 3, 4 completes the main argument concerning the smoothed-out problem. The discussion becomes much more complicated when the effects of localized condensations are taken into account. Although such a discussion is outside the scope of the present paper, it seems desirable to make brief mention of the questions that arise.

The metric can always, in the present model, be reduced to (12), but the simplification leading to (13) neglects local condensation and, therefore, cannot be used in a strict theory. A symmetrical tensor field $C_{\mu\nu}$ can still be introduced, however, and field equations of the form (22) can be constructed. This question will be considered in a further paper.

In the usual formulation of cosmology (3) Weyl's postulate plays an important rôle. Neglecting the effect of localized condensations, this postulate requires material particles to move along geodesics that form a pencil. Although the work of Section 3 is formally similar to that based on Weyl's postulate, there is a difference of principle. For the status of Weyl's postulate in its original form is not clear. It cannot represent a "Law of Nature", since particles in local condensations do not in fact conform with the postulate. In the present model, on the other hand, there is a modified form for Weyl's postulate that can be applied even when local condensations are taken into account. This modified form defines the creation properties of matter. That is, at the time of its creation a particle follows a geodesic passing through O. But there is no requirement that it shall continue to do so and in fact the particle will subsequently move along a path not passing through O if it should be perturbed by an electromagnetic field. Weyl's postulate is evidently of deeper significance when expressed in this modified form. It must be admitted, however, that the nature of the point O remains mysterious. In this connection we note that an event occurring at O cannot be observed at a point with finite t. For in order to reach O, time must be followed back to $t = -\infty$.

The present model has both an infinite future and an infinite past, and in the approximation in which the effects of local condensations are neglected, the wide cosmological principle is satisfied.

The origin of the fluctuations of density, necessary for the formation of local condensations, raises a question of interest. Such fluctuations evidently arise if condensations with proper motions are already present. But the presence of condensations is not a necessary condition, for density fluctuations will occur, even in the absence of condensations, provided matter is created in a quantum type of process. Quantum processes are not, of course, mentioned in the macroscopic treatment of Section 3. Nevertheless, quantum effects must be considered in the physical interpretation of the theory, since it is only through discontinuous processes that statistical fluctuations can be imposed on the universe in the first place. Thus, fluctuations, leading ultimately to approximate thermodynamic conditions being realized in extremely localized regions (in stars, for example), probably arise on account of the discrete particle nature of matter. These considerations are important in relation to the measurement of the coordinates. For in a universe containing no statistical fluctuations and satisfying the wide cosmological principle, there is no possibility of introducing the rod and clock equipment postulated in the relativity theory.

A further interesting feature is that the total "entropy" within the observable universe does not increase with time. Although entropy increases in a localized region, the total entropy remains approximately constant because local condensations carry entropy out of the observable universe. Thus thermodynamics has only localized application. There is no general thermodynamic degeneration of the observable universe as a whole.

It is not possible in the present state of nuclear physics to make a definite statement on the identity of the created particles. Neutron creation appears to be the most likely possibility. Subsequent disintegrations might be expected to supply the hydrogen required by astrophysics. Moreover, the electrical neutrality of the universe would then be guaranteed.

Finally, we notice that a dimensionless number is obtained by dividing the length $a/3$ appearing in (19) by the "range" k of nuclear forces. Using laboratory data for k, together with the determination given above for $a/3$ (on the basis of a comparison with the Hubble-Humason velocity-distance relation), we obtain about 4×10^{39} for this number. This value is close to the dimensionless number $2\cdot3 \times 10^{39}$ given by the ratio of the electrical to the gravitational force between a proton and an electron. Allowing for uncertainty in the determination of $a/3$, and in the interpretation of k, this coincidence suggests that the relation

$$a = 3ke^2/\gamma mM, \tag{48}$$

where m, M are the masses of the electron and proton, is of deep significance. Since the creation process is likely to be of an essentially quantum character, the relation (48) may be regarded as being entirely concerned with microscopic properties. Weyl has interpreted a relation similar to (48) as giving a connection between the radius a of the observable universe and the physical constants, and Eddington, in particular, has attached much importance to this interpretation.

But this view, which Eddington has adopted as the basis of his work on physica theory, may well prove to be a misconception, for according to the above discussio the radius of the observable universe is subsidiary to, and is determined by, th creation constant. Indeed, the possibility of interpreting (48) as an equatio of microscopic physics gives perhaps the best indication that the work of Section is not simply a formal device.

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