Problem Solving Examples 2: Dimensional Analysis and Scaling

Note: this is an exercise in *dimensional analysis and scaling*. The marks are for the correct application of this technique. You will *not* get marks for solving the problems using some method other than dimensional analysis, because doing so does *not* demonstrate that you have mastered this technique.

1. An electric circuit containing an inductance L and a capacitance C will oscillate at a certain angular frequency ω. Find the dependence of ω on L and C.
   [Hint: inductance is measured in henries, H, and capacitance in farads, F. Check the toolkit sheet for quantities with related units.]

2. The drag force on a spherical body moving through a viscous fluid depends on the radius r of the body, its velocity v, and the viscosity of the fluid η. If viscosity is measured in Pa·s, determine the functional dependence of the force on r, v and η.

   Suppose a spherical raindrop of radius R has terminal velocity V. What is the terminal velocity of a raindrop of radius $2R$?

3. It is possible to calculate the mass M of a star using the period P and orbital radius R of an orbiting planet. Determine the dependence of M on P, R and G, and hence find the period, in years, of the planet Neptune, which orbits the Sun at 30 times the distance of the Earth.

4. The total power radiated by an oscillating electric dipole is a function of the frequency of oscillation ω, the dipole moment p, the speed of light c and the permittivity of free space ε_0. Show that the radiated power is proportional to the fourth power of ω, and find the dependence on the other three quantities.

   [Note that an electric dipole is two equal and opposite charges, $\pm q$, separated by a distance x. The dipole moment is the vector $\mathbf{p} = q\mathbf{x}$, the direction of $\mathbf{p}$ being defined as towards the positive charge.]

5. A pygmy shrew has a mass of about 3.5 g and has to eat its own body weight of food daily to maintain its body temperature. What fraction of its body weight (6000 kg) must an elephant eat?

   [Hint: in (nearly) the words of the classic joke, "consider a spherical shrew".]

6. A mass m is suspended from a spring with spring constant k. The mass is pulled downward from its equilibrium position by an amount x and then released. Use dimensional analysis to show that the period P of the resulting oscillations does not depend on x. Why does this argument not hold for the simple pendulum, whose period fails to depend on amplitude only if the amplitude is small?

7. The natural oscillation frequency of a plasma depends on the charge of the particles, their mass, their number density and the permittivity of free space ε_0. Show that the plasma frequency is proportional to the charge of the particles and find the dependence on the other three quantities.

8. In astrophysics, the Hertzsprung-Russell diagram is a plot of log L against log T, where L is the total power output of the star (its **luminosity**) and T is its surface temperature (calculated assuming that it radiates like a blackbody). It is found that most stars lie on the main sequence, which is a line given approximately by $\log L = 6.4 \log T + \text{constant}$.

   Estimate the radius of a main-sequence star with twenty times the Sun's luminosity, if the Sun's radius is 7×10^8 m.

9. We do not, as yet, have a working quantum theory of gravity. Calculations involving gravity are therefore only reliable at distance scales where quantum effects are not important.

   The key fundamental constant for quantum physics is Planck's constant h, or more precisely the **reduced Planck constant** $\hbar = h/2\pi$. The fundamental constants in General Relativity are the gravitational constant G and the speed of light c. At what length scale would you expect quantum effects to become important in gravitational calculations?

10. The effective range of the weak nuclear force is about 10^{-18} m. Estimate the mass of the particle exchanged in the interaction.

    Compare your result with the measured masses of the $W^\pm$ and Z bosons (80 and 91 GeV/c^2 respectively), noting that 1 GeV/c^2 = 1.8×10^{-27} kg.

    [Hints: since $E = mc^2$ is important in particle physics, you should assume that the answer might depend on c. For most applications in quantum mechanics, the reduced Planck constant $\hbar$ is more appropriate than h.]