PROBLEM SOLVING EXAMPLES 1

All of these are old exam questions. They're expected to take about 15 minutes each. Note that this is a course on *problem solving*, so marks are given for *correct technique*, not simply for correct answers!

1. A rollercoaster slides down a frictionless track and then enters a vertical loop of radius r. Show that it must be released from a minimum height of $5r/2$ above the bottom of the loop if it to remain in contact with the loop at all points.

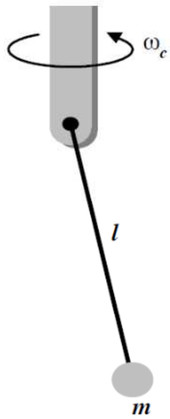
2. What is the effective spring constant of a system of two springs, each of spring constant k, connected (i) in series (one spring connected to the end of the other), (ii) in parallel (side by side)?

3. Two small balls of mass m_1 and m_2 are released simultaneously above a horizontal surface with a small vertical separation between them. What ratio of the masses must be chosen to ensure that, following their collision, all the kinetic energy is transferred to the upper ball? With this ratio of masses, find the height to which the upper ball bounces compared to its initial release height.

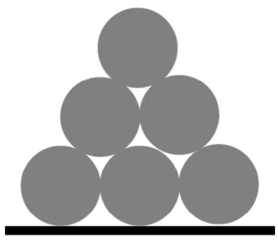
   [Air resistance can be neglected and all collisions assumed to be elastic. Assume both balls fall through the same distance.]

4. The pivot of a simple rigid pendulum of length l and mass m is made to rotate about a vertical axis at angular velocity ω_c, as shown in the figure. By considering the forces acting when there is a small displacement from the vertical, calculate the maximum value of ω_c for which the pendulum can hang vertically below the pivot.

   [Note: if you are unfamiliar with the term, angular velocity about an axis is simply rate of change of angle, $d\theta/dt$. For an object moving at speed v in a circle of radius r, angular velocity $\omega = v/r$.]



5. Identical cylinders are stacked to form a pyramidal structure. For the structure shown in the diagram, show that the coefficient of friction between the cylinders and the horizontal surface must have a minimum value of ½ tan 30° if the structure is to be stable. Does this minimum for the coefficient of friction increase or decrease as the number of rows in the structure increases?



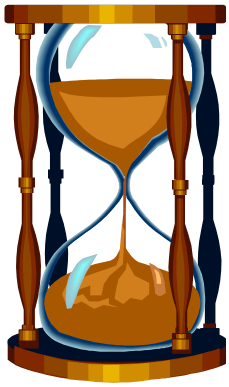
6. A trolley has length L and rests on frictionless wheels. At one end of the trolley is a gun which fires a bullet that subsequently travels the length of the trolley before being stopped in a target at the far end of the trolley. If the mass of the trolley, gun and target is M, and the mass of the bullet is m, show that the distance travelled by the trolley during the recoil event is

$$d = \frac{L}{\frac{M}{m} + 1}.$$

7. A rubber ball is launched vertically upwards from ground level with initial velocity v_0. When the ball falls back to the ground it bounces, retaining a fraction f of the kinetic energy that it had just before it hit the ground. The ball continues to bounce vertically, retaining the same fraction f of its kinetic energy at each bounce (i.e., each time, KE after bounce = f × KE before bounce). Show that the total distance s travelled by the ball summed over all its bounces is

$$s = \frac{v_0^2}{g(1-f)},$$

where g is the acceleration due to gravity.



8. Fluid in an hour-glass leaks through a small neck of area a. By equating the loss of potential energy of the fluid to the gain in kinetic energy, or otherwise, find how the radius r of the glass vessel must depend on the height h above the neck if the level of the fluid is to fall at a constant rate. (You may assume $a \ll \pi r^2$.)

9. A room has two windows, each 1.2 m wide by 0.8 m high. The interior temperature is 22°C, while outside it is 4°C. One window is single glazed with a sheet of glass 10 mm thick, while the other is double glazed, the two layers of glass each being 5 mm thick and separated by 3 mm of air. The coefficient of thermal conductivity of glass is 1.1 W m^{-1} K^{-1}, while that of air is 0.025 W m^{-1} K^{-1}. Compare the rate of heat loss through the two windows. (Assume that all heat is lost by conduction, and that there is no convection within the air gap.)

   [Note that the thermal conductivity is the rate of heat loss through unit area and unit thickness of material when the two sides have a temperature difference of 1 K. It is exactly analogous to electrical conductivity.]

10. A spaceship sits on a planet where the ambient temperature is 100 K and the atmospheric pressure is very low. Inside the spaceship the temperature is 293 K and the outer face of a glass window is at 270 K. How thick is the window? (The thermal conductivity of glass is 1.1 W m^{-1} K^{-1}.)

11. Two small metal spheres are each suspended by a light thread of length L from a common point. The spheres have mass m and each is given a charge $+Q$. If the angle made by each thread to the vertical is θ, show that when θ is small

$$\theta^3 \cong \frac{Q^2}{16\pi\varepsilon_0 L^2 mg},$$

where g is the acceleration due to gravity.

12. According to contemporary records, the Montgolfier hot air balloon – the first man-made flying machine – had a volume of about 2000 m^3 and weighed 780 kg. On its maiden voyage, it carried two people. Estimate the temperature to which the air inside the (rigid) balloon had to be heated in order to achieve lift-off.