Problem Solving

Introduction

Importance of Problem Solving
Approaches to problems
Basic strategy

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The Importance of Problem Solving

- Key principle of science is interplay between theories and data
  - theory pointless if it cannot be used to analyse or predict experimental results, or applied to real-world situations
- Also key skill for physics graduates
  - analytical skills and numerical problem-solving valued by employers, even when not directly physics-related
- A skill in itself
  - good knowledge of theoretical principles and definitions does not automatically translate into ability to solve problems!

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Approaches to Problems

- The "recipe" approach
  - identify specific class of problem
  - apply memorised procedure which will eventually yield solution
  - Advantages:
    - quick
    - does not require detailed understanding
  - Disadvantages
    - won't work on unfamiliar problems (even if quite closely related to a known type)
    - if applied mechanically, easy to make mistakes, and mistakes difficult to spot

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Approaches to Problems

- The "hunt the equation" approach
  - identify variables used in problem
  - find, from memorised stock, equation(s) involving the same variables, and attempt to apply these to problem
  - Advantages:
    - none that I can think of!
  - Disadvantages
    - no guarantee that this equation is the right equation
    - even if it is the right equation, no guarantee it's being applied in the right way

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Approaches to Problems

- Approach from physical principles
  - work out clear mental model of what is happening in the problem, including any necessary assumptions
  - based on this model, identify relevant physical principles
  - formulate in terms of equations, and solve
  - Advantages:
    - applicable to any problem
    - results in better understanding of situation
    - transferable skill (same principles can be applied in other fields)
  - Disadvantages
    - probably slower than the recipe approach if the problem conforms to standard type
    - not intuitive for most people: needs practice
    - does require a grasp of physical principles!

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Basic Strategy

- Model
  - Understand the problem
  - Identify the physical principles involved
- Formulate
  - Develop a mathematical description of the model (equation or set of equations)
- Solve
  - Manipulate your equations to obtain the desired quantity
- Check
  - Always check that your result makes sense
    - Dimensions correct?
    - Does sensible things in extreme/special cases?
    - Right order of magnitude, if numerical?
  - Key to correcting mistakes is to know you've made one!

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Tools

- Diagrams!
  - Always help – you do not have to be Rembrandt
  - Label everything you can identify, both knowns and unknowns
- Symbols!
  - Avoid working with numerical values: always use symbols
    - allows you to check dimensions and special cases
    - minimum number of numerical calculations → minimum number of opportunities for calculator typos
- Basic set of physical laws and principles
  - Try to keep this as small as possible – the fewer you need to remember, the more chance of getting them right

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Example 1

- A mass of 5 kg and a mass of 3 kg are connected by a light inextensible string of length 4 m. The string passes over a pulley of negligible mass. The two masses are held stationary level with each other, such that the string is just taut. The two masses are then simultaneously released. Find the acceleration of each mass immediately after release (take g = 9.8 m s^{-2}).
  - Key words:
    - light → string is effectively massless
    - inextensible → string will not stretch
    - of negligible mass → we don't need to consider the motion of the pulley

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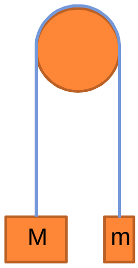
Example 1: Model

- Diagram
  - two masses, string, pulley
  - call masses M and m (use symbols)
- What will happen when masses released?
  - larger mass M will fall downwards, lighter mass m will be pulled upwards
- Why will this happen?
  - because the two masses are connected by a string
    - so tension in string must be important
- What physical principles apply?
  - Newton's second law, F = ma, and gravity, F = mg
    - String does not stretch so accelerations must be equal in magnitude (opposite in direction)



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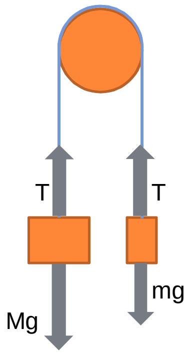
Example 1: Formulate and Solve

- Add forces to diagram
  - tensions equal in magnitude because string is massless
- Now apply Newton's laws
  - Ma = Mg – T (M goes downward)
  - ma = T – mg (m goes upward)
  - remember a is the same for both, because string does not stretch
- To solve for a, add the equations
  - (M + m)a = (M – m)g (T cancels)
- Now put in numbers
  - a = 2g/8 = g/4 = 2.7 m s^{-2}



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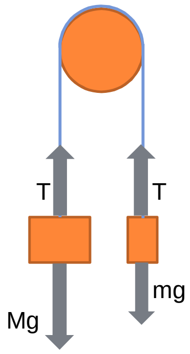
EXAMPLE 1: CHECK

- Final equation was (M + m)a = (M – m)g
- Are dimensions right?
  - ☑ yes, both sides have dimensions (mass)×(length)/(time)^2
- Does equation behave sensibly at special values?
  - ☑ If m = 0, mass M should be falling freely, so we should get a = g
  - ☑ If M = 0, mass m should be falling freely downward, not up as we assumed, so we should get a = –g
  - ☑ If M = m, masses balance, so they should just hang there: a = 0
  - ☑ If m > M, expect m to go down and M up, so a < 0
- Is the numerical answer sensible?
  - ☑ expect presence of m to reduce rate at which M falls, so should get a < g for 0 < m < M



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EXAMPLE 2

- A mass of 5 kg and a mass of 3 kg are connected by a light inextensible string of length 4 m. The string passes over a pulley of negligible mass and size, 2.5 m above ground level. The 3-kg mass is held stationary level with the pulley, and then released. Find the velocities of the two masses immediately after the string becomes taut (take g = 9.8 m s^{-2}).

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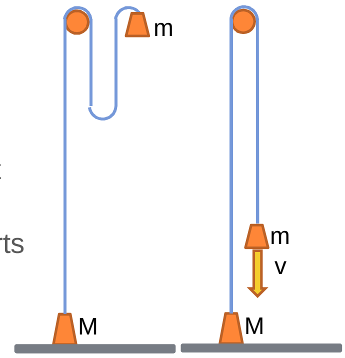
EXAMPLE 2 ANSWER



- Model
  - initial state as diagram
  - release m, m falls until string taut (second diagram)
  - when string becomes taut, it exerts force on mass m
    - m's velocity is reduced by Δv
    - change in momentum mΔv
    - impulse mΔv transferred to string
  - as string is massless and inextensible this impulse must be transmitted to mass M
    - mass M gains momentum mΔv = MV (V upward speed of M)
    - as string does not stretch V = v – Δv (both masses must have same speed)

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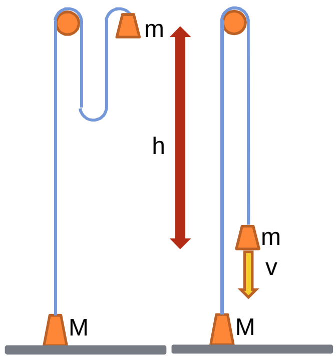
EXAMPLE 2 ANSWER



- Formulate
  - mass m falls through height h
    - therefore loses PE mgh
    - therefore gains KE $\frac{1}{2}mv^2 = mgh$
    - so $v = \sqrt{(2gh)}$
  - after string becomes taut
    - momentum of M = MV
    - impulse exerted by m = m(v – V)
    - these must be equal, so mv = (M +m)V, V = mv/(m+M)
- Solve
  - numbers:
    - h = 4 – 2.5 = 1.5 m, so $v = \sqrt{(2\times9.81\times1.5)} = 5.42$ m s^{-1}
    - m = 3 kg, M = 5 kg, so V = (3×5.42)/(3+5) = 2.0 m s^{-1}

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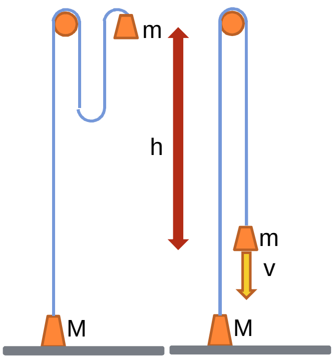
Example 2 Answer



- Check
  - dimensions
    - v = √(2gh)
      - ☑ $[L]\,[T]^{-1} = [L]^{1/2}[T]^{-1}[L]^{1/2}$
    - V = mv/(m+M)
      - ☑ $[L]\,[T]^{-1} = [M]\,[L]\,[T]^{-1}\,[M]^{-1}$
  - special values
    - no slack in string, mass m does not fall when released, M does not move
      - ☑ h = 0 gives v = V = 0
    - mass m → 0, just a loose end of string hanging over pulley, M does not move
      - ☑ m = 0 gives V = 0
    - mass M → 0, no braking effect, m falls freely (trailing string)
      - ☑ M = 0 gives V = v

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Summary

- Two valid approaches to problem solving:
  - "plug in numbers" if problem of standard type
    - unfortunately of limited usefulness in real world
  - construct mental **model** of system, identify relevant physical principles, **formulate** these as equations, **solve** the equations, and **check** solution
    - this one <u>does</u> work in real world (hence this course!)
- Useful tools:
  - diagrams
  - dimensional analysis (next lecture)
  - special values (next lecture)

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